生态仿真优化实验室
Simulation Optimization Lab

西北农林科技大学
Northwest A&F University

Complex Forest Ecosystem:
Theory and Methodology

Dr. Tianjian Cao

Professor of Forest Management

cao@nwafu.edu.cn

www.optifor.cn

Introduction

- This course is designed for master students who are preparing to engage in an interdisciplinary study in the fields of
  - silviculture,
  - forest health,
  - forest ecology,
  - forest planning,
  - and forest ecosystem management.

Introduction

- The aim of this course is to help students acquire skills in predicting and assessing dynamic forest ecosystems in terms of

- Data

  ↓

- Models

  ↓

- DSSs

  ↓

- Applications

Purposes

- The purpose of this course is to show students how to link their knowledge in forest sciences with an interdisciplinary approach.

- Prerequisites
  - Forest Ecology
  - Forest Mensuration and Management
  - Silviculture

  - Forest Biometrics
  - Operations Research

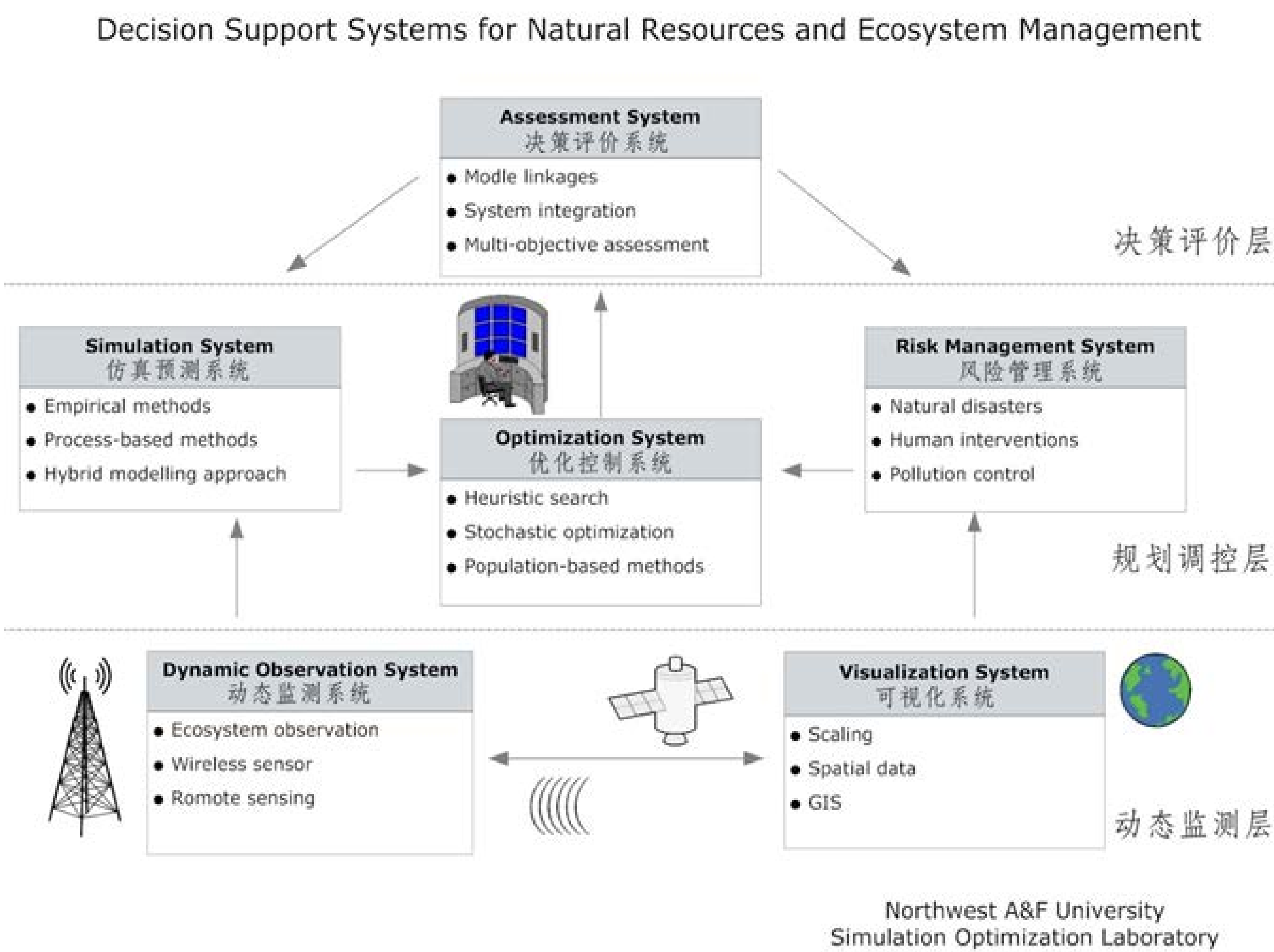
Grading

- Quiz, 30%
- Final, 70%

- Part I：Forest Inventory
- Part II：Forest Planning
- Lecture hours：32+32 hrs
- Credits：2+2 crs

- Instructor：Prof. Dr. Tianjian Cao
- E-mail: cao@nwafu.edu.cn

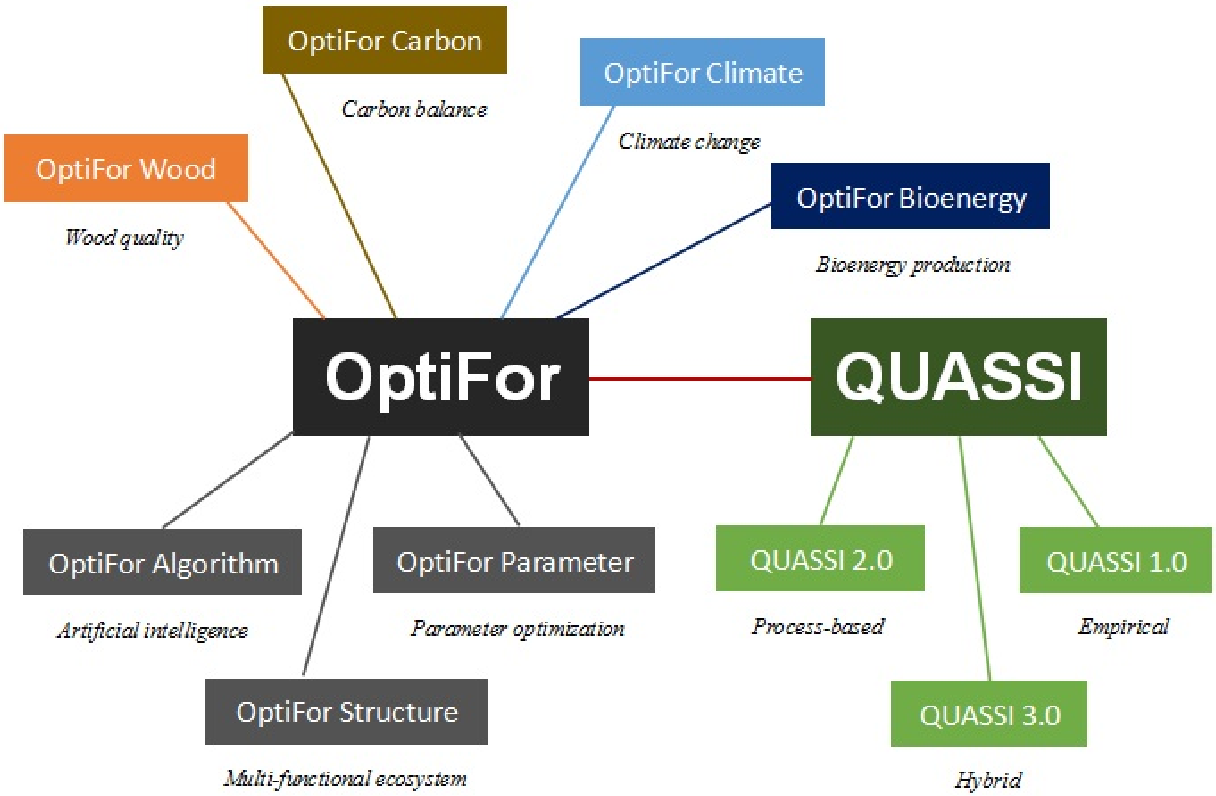
Simulation Optimization Laboratory



Simulation Optimization Laboratory

- EST. since 2008.
  - www. optifor.cn
- Interdisciplinary studies
  - Biometrics, operations research
  - Forest sciences, ecology
- Research interests
  - Uncertainty in dynamic ecosystems
  - Simulation-optimization systems

Research areas



The OptiFor Family

www.optifor.cn

Research areas, con't

- OptiFor 1.0
  - OptiFor Wood
  - OptiFor Carbon
  - OptiFor Climate
  - OptiFor Bioenergy
- OptiFor 2.0
  - OptiFor Algorithm
  - OptiFor Parameter
  - OptiFor Structure
- QUASSI 1.0
  - Empirical models
- QUASSI 2.0
  - Process-based models
- QUASSI 3.0
  - Hybrid models

Research network

Quang V. Cao
Biometrics

Shuaichao Sun
Competition

Hailian Xue
Algorithms

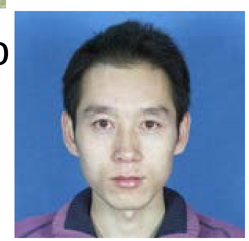
Lauri Valsta
Optimization

Tianjian Cao
Simulation-optimization

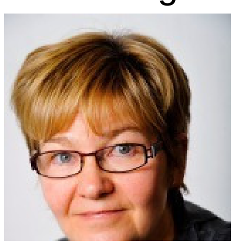
Jerry Vanclay
Forest modeling

Kari Hyytiäinen
Model linkages

Xianglin Tian
Hybrid modeling



Bin Wang
Bayesian methods



Annikki Mäkelä
Process models

Timo Pukkala
Forest planning

Mengzhen Kang
Structure-function

Shubin Si
Complex system

Aims and Objectives

- Long-term and short-term aims
  - Long-term：Uneven-aged stand growth models
  - Short-term：Methodology development

- Theory and application study objectives
  - Theory studies：Complex ecosystem theory and optimization methods
  - Application studies：Adaptive management of natural forest ecosystems

Computer programs to be used in the course

- SPSS
- SigmaPlot
- Solver
- Simile
- OptiFor
- QUASSI

Part I: Forest Inventory

Data and Models

Caveats

- *Impossible to validate any model (Popper 1963).*
- *All models are false, but some models are useful. Our job is to identify the useful ones useful. (G.E.P. Box).*
- *No model can be evaluated in the absence of some clearly stated objective (Goulding 1979).*
- *No criterion is universal, so some subjectivity must always remain.*
- *Empiricism cannot be avoided, and keeps models grounded in reality.*

Source: Monserud (2003), presentation in Symposium of Systems Analysis in Forest Resources, October 7-9, 2003, Stevenson, Washington

Contents

- Data collection
- Sampling design
- Calculating supplementary data
- Site and competition variables
- Simulation modeling techniques
- Whole-stand and individual-tree models
- Empirical and mechanistic models

Pre-exam

- Suppose you are runing a project concerning forest inventory,

- Try to plan the project, including
  - data requirements,
  - methods applied,
  - budget,
  - timetable,
  - etc..

Why forest inventory?

- Forest managers
- Forest owners
- Forest policy makers

- Investment banks
- Timber industry
- Paper making industry

- Forest ecologists
- Forest parks
- NGOs

Forest inventory systems

- National forest inventory,
- inventory for forest management planning,
- inventory for silvicultural operations

- National and local standards for forest inventory

Ecosystem observation systems

- LTER www.lternet.edu
- GTOS www.fao.org/gtos
- ECN www.ecn.ac.uk
- GEMS www.unep.org/gemswater
- CNERN www.cnern.org
- CFERN www.cfern.org

Sufficient data

- Field work can provide necessary and sufficient data efficiently.
- However, it may take several years to obtain the necessary data from permanent plots,
- and few of us can wait that long.
- “Nothing perfect except in our memories…”

Procedure of forest inventory

- Definition

    => Collection

    => Validation

    => Storage

    => Analysis

    => Synthesis

Data collection

- The life cycle of a datum spans its definition, collection, validation, storage, analysis and synthesis.
- All stages are equally important, and an efficient data management system requires a healthy balance between them.
- The first step is to define information needs and devise data collection procedures to satisfy those needs.

Differing data needs

- Stem analyses do not provide reliable growth data for many tree species, e.g., in tropical moist forests.
- Permanent plots can never be completely replaced by temporary plots.

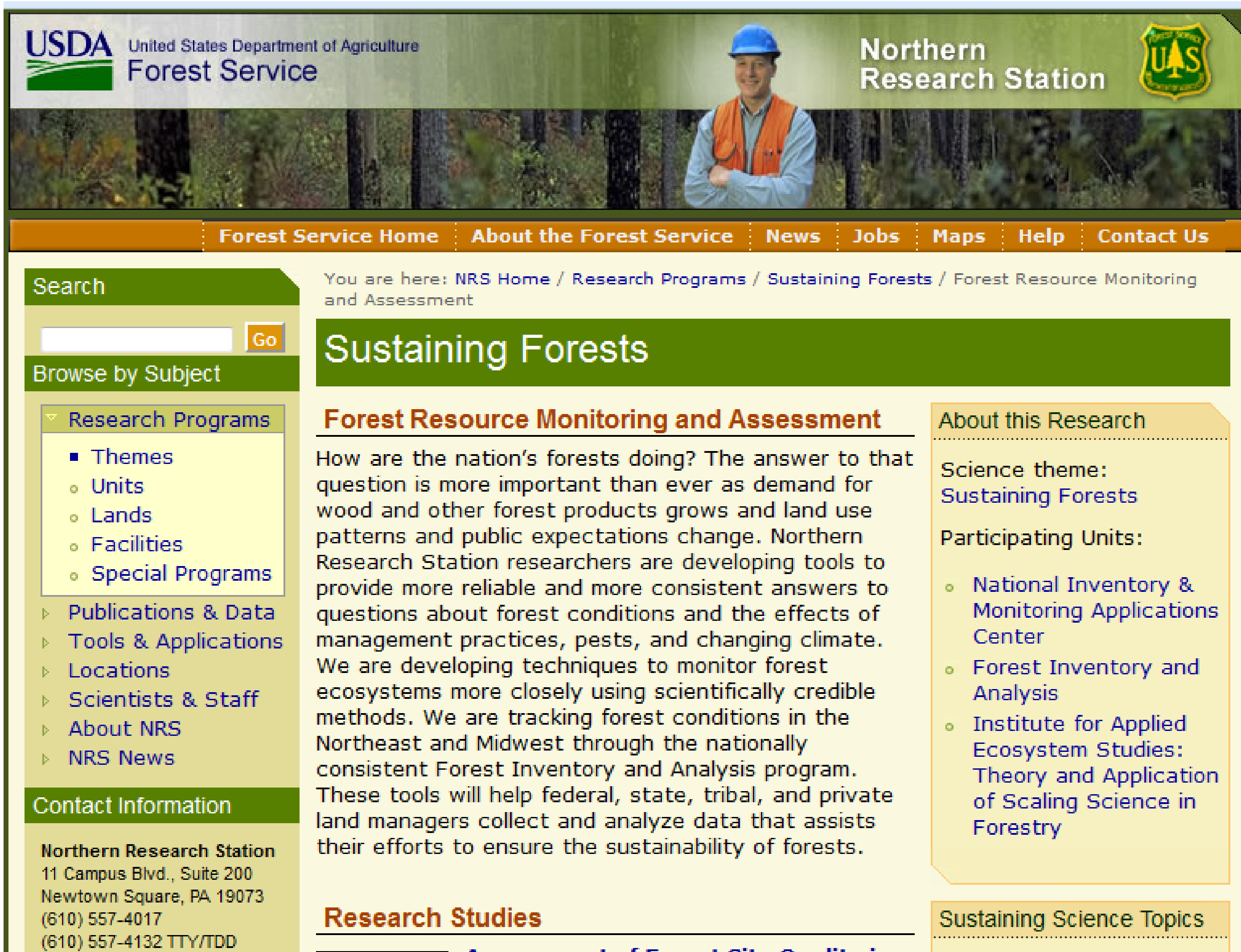
- Resource Inventory
- Continuous Forest Inventory for yield control
- Growth modelling
- Long term monitoring of environmental change

IUFRO Division 4

- http://www.iufro.org/science/divisions/division-4/

- 4.00.00 - Forest Assessment, Modelling and Management
  - 4.01.00 – Forest mensuration and modelling
  - 4.02.00 – Forest resources inventory and monitoring
  - 4.03.00 – Informatics, modelling and statistics
  - 4.04.00 – Forest management planning
  - 4.05.00 – Managerial economics and accounting

Forest resource monitoring and assessment



Mechanism

- Monitoring
  - Monitoring systems
  - Monitoring methodology and techniques
- Assessment
  - Assessment index
  - Assessment methodology
  - Single objective vs. multive objective
- Monitoring and Assessment Systems
  - UNDP: United Nations Development Programme
  - UNEP: United Nations Environment Programme
  - IPCC: Intergovernmental Panel on Climate Change
  - FAO: Food and Agriculture Organization

4.01.00 – Forest mensuration and modelling

- 4.01.01 – Design, performance and evaluation of experiments
- 4.01.02 – Growth models for tree and stand simulation
- 4.01.03 – Instruments and methods in forest mensuration
- 4.01.04 – Effects of environmental changes on forest growth
- 4.01.05 – Process-based models for predicting forest growth and timber quality
- 4.01.06 – Analysis and modelling of forest structure

Definitions

- forest resources
- forest inventory
- forest resource monitoring
- forest management planning

Definitions, con’t

- Timber value vs. non-timber value

  Timber: sawlogs, pulpwood, energy wood, ...

  Non-timber: mushrooms, hunting, biodiversity, carbon sequestration, amenities, preventing erosion...

- Quantitative method vs Qualitative method

  Quantitative: connected with the amount or number of sth rather than how good it is

  Qualitative: connected with how good sth is, rather than how much of it there is

Data requirements

- LUCC: Land Use and Cover Change
- Forest growth and yield
- Forest operations
- Forest biodiversity
- Forest carbon sequestration
- Forest health
- Forest ecosystem management
- etc.

Theory

- Sampling theory and sampling design
- Temporary vs. permanent plots
- Stem analysis
- Empirical vs. mechanistic data
- RS and GIS data
- Forest growth and yield models

Sampling theory

- Sampling plots (Kiaer 18??),
- Stochastic sampling (Bowley 1912)
- Stochastic > Typical, Neyman (1934)
- Confidence interval, Neyman (1934), Bellhouse (1988)
- Reducing inventory cost, Loetsch et al. (1973)
- Early sampling in North America, Germany, Nordic countries, visual inspection
- Since 1900 to 1920， applications of statistics

Sampling theory, con't

- Stripe sampling, 1830s
- Systematic > Stochastic, Hasel (1938), Osborne (1942)
- Stratified random sampling, Finney (1948)
- Point sampling（Bitterlich 1947）
- PPS sampling, Grosenbaugh（1952）

Sampling theory, con't

- CFI (Continues Forest Inventory) system, Scott（1947）
- Permanent + temporary plots, Bickford (1959)
- Model-based methods?（Matern 1960; Mandallaz 1991; Kangas 1993; Gregoire 1998）。
- Satellite images, (Czaplewski 1999)
- Remote sensing, LiDAR

Forest inventory in China

- NFI (National Forest Inventory)
  20m * 30m，2km * 4km
  tot. 230,000 plots

- Forest management planning
  stand level

- Silvicultural operations
  harvest scheduling

Forest Inventory in Europe

- Sweden
  - Since 1923， based on systematic sampling, including permanent plots（since 1983）and temporary plots.

- Germany
  - 44000 plots,150m * 150m，4km * 4km or 2km * 2km

Forest Inventory in North America

- Canada
  - Province government77%，central government 16%，7% private
  - Province level，10-15 years
  - Country level，based on province forest inventory
  - Industrial forests（forest management，silvicultural operations）
- The United States
  - FIA，since1930
  - P1，satellite images，forest or non-forest land
  - P2，one plot every 2439 ha （forest ecosystem）
  - P3，one plot every 39024 ha （ecological data）

Plot layout

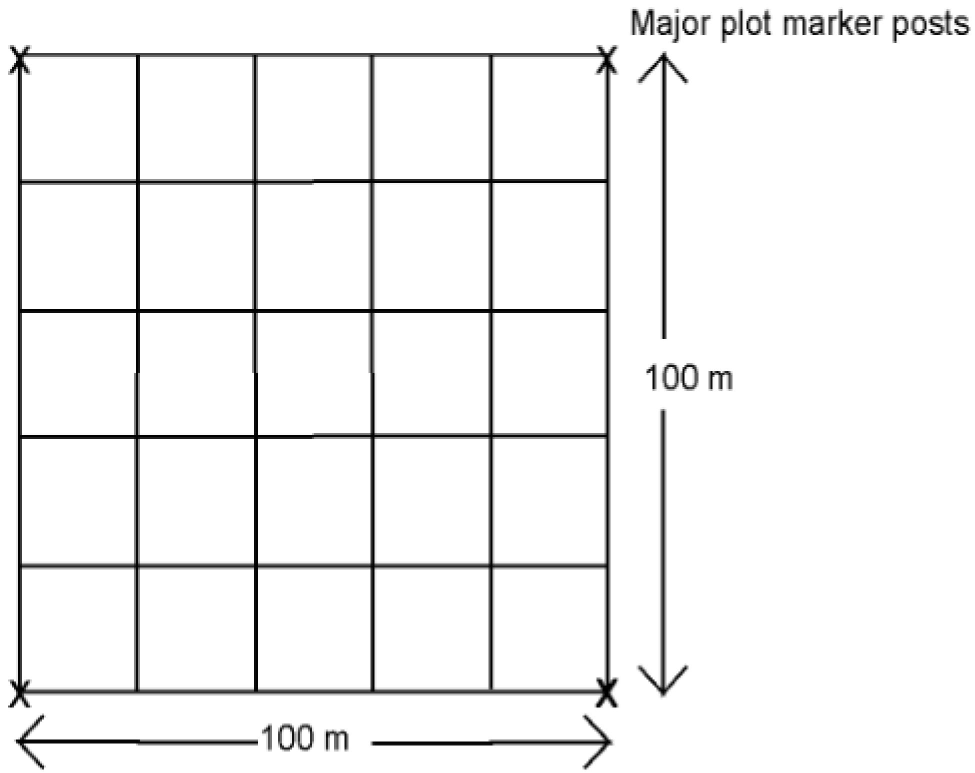


Fig. 5.4. Recommended plot layout for permanent sample plots.

Field form

Plot No ..... Subplot ..... Page ... of ...
Subplot Dimensions ...... x ...... Date ../../....
Orientation ..... Coordinates . . . .
Location . . . .
. . . .
Assessing Officer . . . .

Tree number Coordinates					
Family Genus Species Common name					
DBH Point of measure Valid/approx					
Alive/dead/cut/missing Erect/leaning/fallen Broken/injury					
Tree height Bole height Crown position Crown form Crown diameter					
Merchantable length Stem straightness Stem defects					
Notes: Flower/fruiting Pests/disease					

Example: Stratified random sampling

	A(ha)	n	V(m3/ha)	SD(m3/ha)	Vtot(m3)	SD(m3)
Bare land	18.0	18	42	6.7	756	121
Young stands	33.3	35	167	10.0	5557	344
Old stands	48.7	49	268	13.3	13033	649

- V_ave = 0.180*41.976+0.333*166.84+0.486*267.67=193m3/ha
- SD=sqr(0.180^2*45.437+0.3336^2*100.27+0.487^2*177.72)=7.4 m3/ha
- Vtot=755.6+5557.1+13033.4=19346 m3
- SD_tot=sqr(14722+111246+421372)=740 m3

Example: remote sensing as additional info.

- Suppose you have1000 ha forest， including 5 areas with NIR information， average value 0.2482， SD 0.0364. 1000 observation database

$$V_i = 322.7473 - 714.951*NIR_i + \epsilon_i$$

- epsilon_i SD 38.66m3/ha。

Example , con't

Area	Size(ha)	NIR	V(m3/ha)	SD
1	94	0.22893	155.5	43.82
2	69	0.25104	140.7	40.43
3	123	0.26008	139.2	42.34
4	537	0.28201	120.4	45.40
5	177	0.31497	92.5	44.35
mean		0.27802	122.5	47.84
sum	1000			

◦

Example, con't

Area	n	NIR	y_i	s_e	y_iSYN	y_iREG	y_iSUR	s_e
1	7	0.22893	169.2	13.79	115.6	151.4	158.5	11.387
2	5	0.25104	92.5	25.35	115.6	134.1	102.0	31.773
3	6	0.26008	120.8	20.36	115.6	127.0	123.0	20.389
4	22	0.28201	113.5	9.45	115.6	109.8	116.3	6.946
5	10	0.31497	91.4	12.21	115.6	84.0	83.1	8.346
mean		0.27802	115.6	6.91	115.6	112.9	112.9	5.216
sum	50							

- y_i area i, traditional part estimation
- y_iSYN area i, average estimation
- y_iREG area i average value*coefficient
- y_iSUR tot.

Quiz 2. SD

- average volume per ha：

  y_ave = 1/n * sigma_i=1_n(y_i)

  = 1/102 * sigma_i=1_102(y_i) = 193 m3/ha

- f = n/N

  f = sigma_i=1_n(a_i/A) = sigma_i=1_102(a_i/100) = 0.02

s_y^2 = (1/(n-1))*(sigma_i=1_n(y_i^2)-

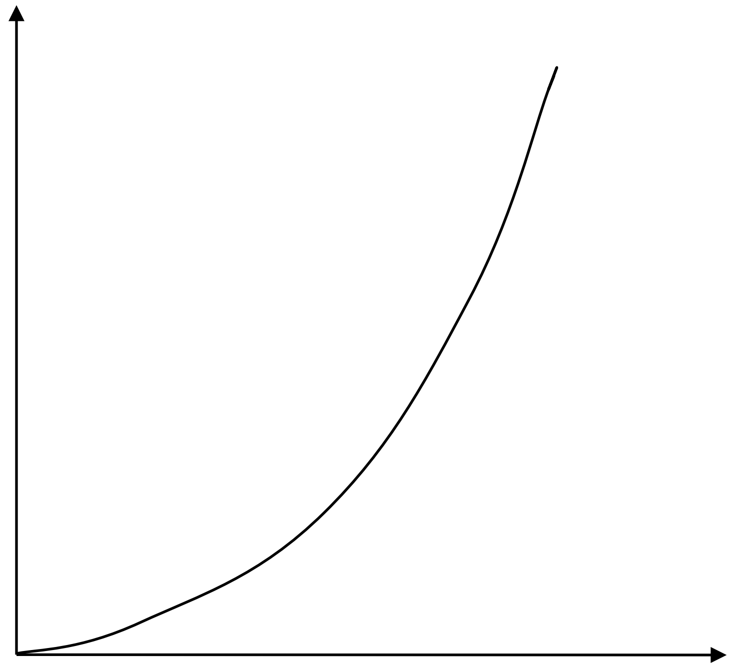
(sigma_i=1_n(y_i^2))^2/n) = 12601 (m3/ha)^2

Question: s_y_hat = sqr((1-n/N)(s_y^2/n))=?

Sigma epsilon, or Pi epsilon?

- Sampling errors
- Design errors
- Inventory errors
- Model errors
- Prediction errors
- Planning errors
- Decision errors

- Tot_e = f(e_i, t)



Sampling techniques by data requirements

Table 5.1. Different applications require different sampling techniques.

Plot characteristics	Principal objective of permanent plot system			
	Resource inventory	Continuous forest inventory	Growth modelling	Site monitoring
Permanence	Temporary	Permanent	Permanent	Permanent
Area	Variable, $\propto$ tree size	Fixed	Fixed	Fixed
Within-plot variance	Hetero-geneous	Homo-geneous	Homo-geneous	Homo-geneous
Placement	Stratified random	Systematic	Stratified random	Purposive or systematic
Sample unit	Plot	Plot	Tree	Plant parts

Sampling design

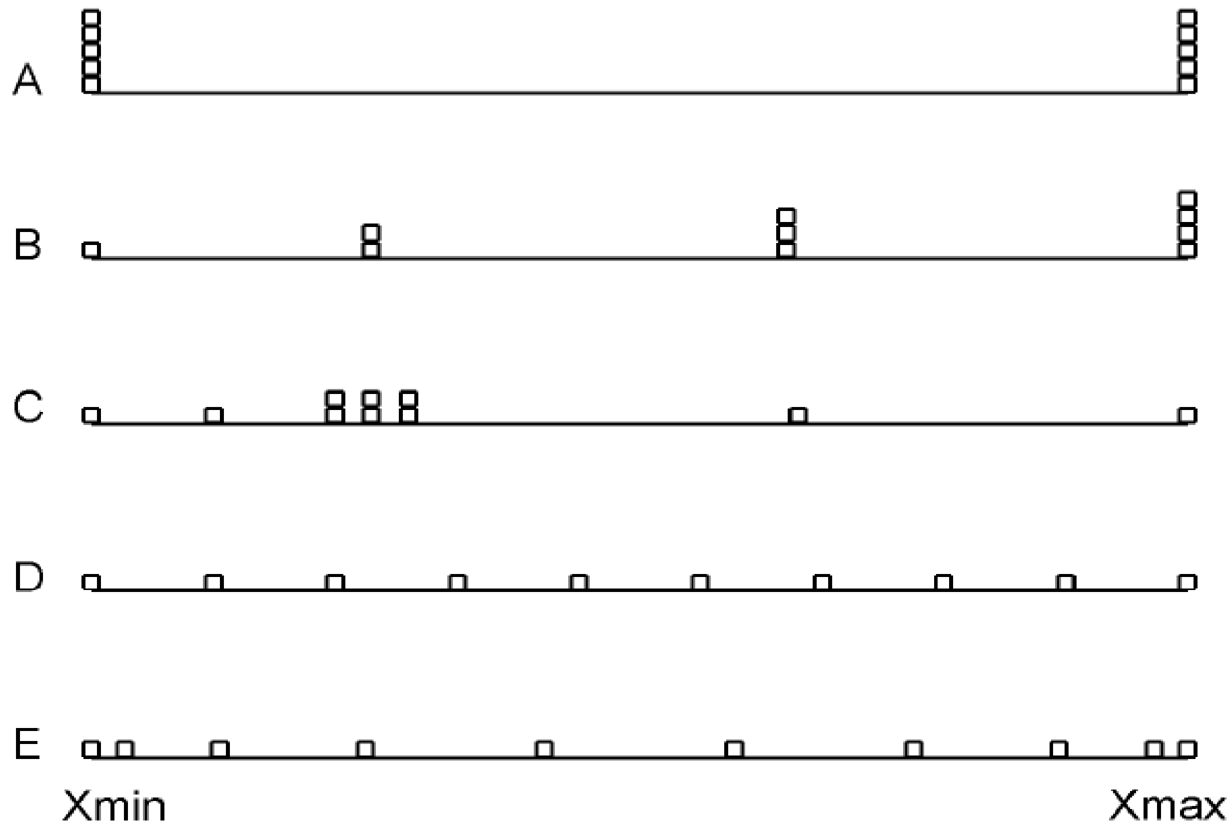


Fig. 5.2. Efficient placement of ten samples to (A) estimate slope of a straight line, (B) detect curvilinearity where variance ∝ X, (C) calibrate an optimum, (D) detect a threshold, and (E) fit a curved relationship.

Sampling principles

- Temporal distribution
  - Short time periods may give rise to biased growth estimates, and longer periods of observation offer a better basis.
- Spatial distribution
  - Permanent plots should sample an adequate geographical range, including latitude, longitude, elevation and other topographical features such as ridge and valley locations.
- Site factors
  - the sampling system should ensure that the full range of site factors (e.g. soil type and depth) is included in the permanent plot system.
- Stand conditions
  - stand structure and composition, should be manipulated experimentally to provide the best database.

Interpolation vs. Extrapolation

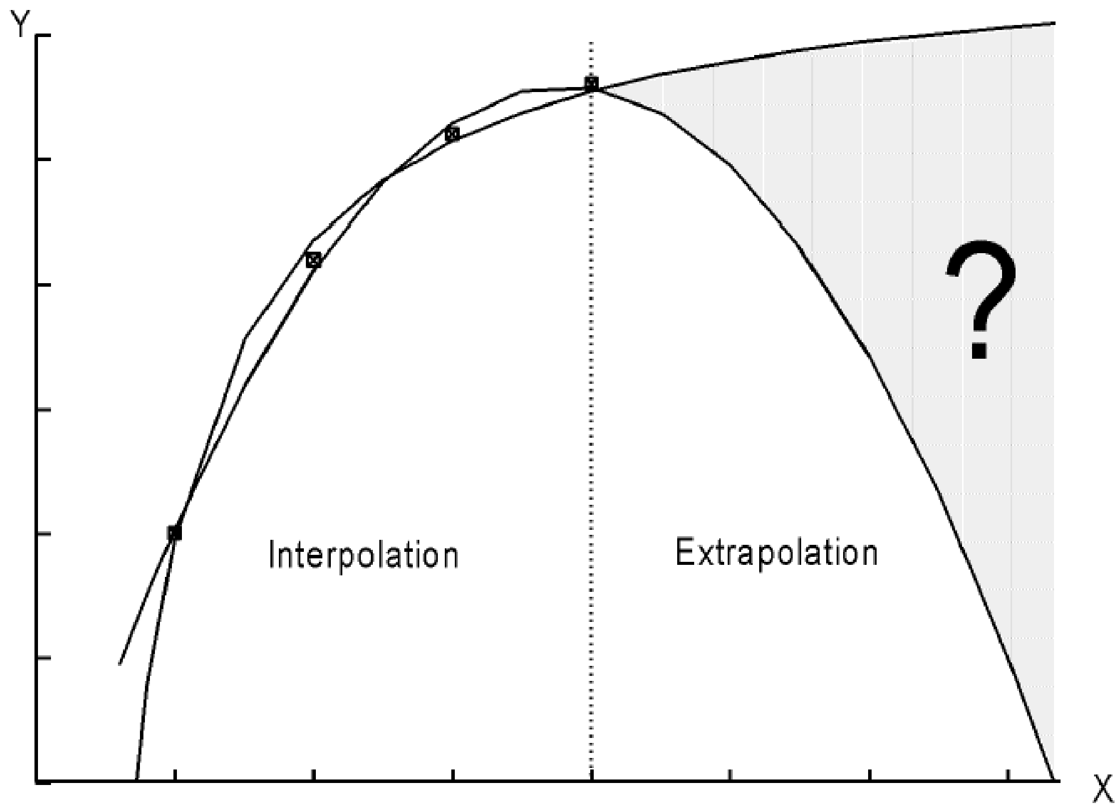


Fig. 5.1. Interpolation is safer than extrapolation. Both these lines have an R^2 better than 0.996, but provide no basis for making a prediction outside the range of the data.

Database weaknesses

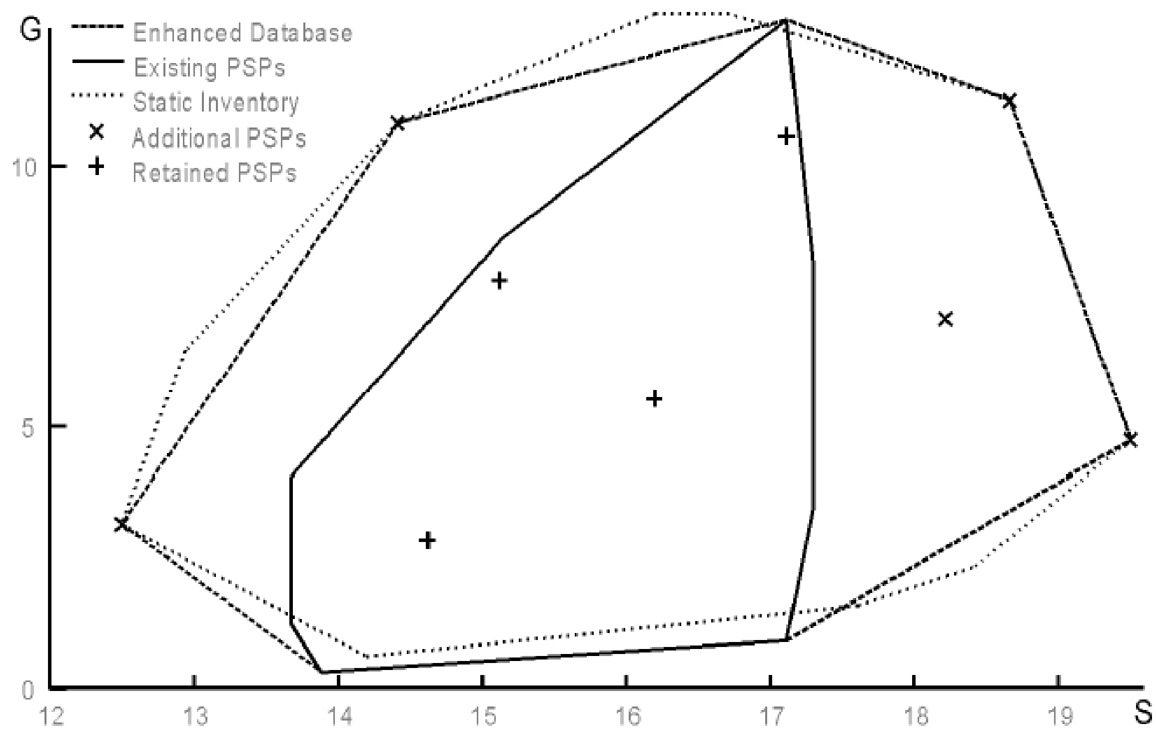


Fig. 5.3. Database weaknesses revealed by comparing dynamic and static inventory data. Five new PSPs (×) would improve the database for modelling (*Callitris* forest in Queensland, redrawn from Beetson *et al*. 1992).

Experiments design

- Passive monitoring data: survey data from forest areas under routine management.
- Treatment response data: from paired treatment and control plots (controlled experiments).

- Number of plots
- Size and shape of plots
- What to measure
- When to remeasure
- Data administration

Number of plots

$$n_0 = \left(\frac{z_{\alpha/2}}{d}\right)^2 S^2$$

$$n = \frac{n_0}{\left(1 + \frac{n_0}{N}\right)}$$

- Known:
- Forest area = 100 ha
- Inventory aim: Vtot (m3/ha)
- Confidence interval = 95%
- d = 15 m3/ha
- S = 50 m3/ha
- z_alpha/2 = 1.96

- No. of plots (20m*20m) = ?

Solutions

- $n_0 = (z_{\alpha/2} / d)^2 * S^2 = (1.96/15)^2 * 50^2$

- $N = 100/(20*20/10000) = 2500$

- $n = n_0/(1+n_0/N) = 42.68/(1+42.68/2500) = 41.96$

Variables should be measured

- At the initial enumeration
  - Plot location, dimensions, orientation and area,
  - Species and coordinates of all trees on the plot,
  - Topographic details, including altitude, aspect, slope, position on slope,
  - Forest type and floristic attributes,
  - Physical soil characteristics (depth, texture, colour, parent material), and
  - Uniformity of the site

Variables, con’t

- At the first measure, immediately after any harvest, and periodically (e.g. every second or third measure):
  - Sufficient tree heights for the determination of site productivity (or data necessary for alternative estimates of site productivity),
  - Merchantable heights and defect assessments of all stems (including non-commercial species, as utilization standards may change with time),
  - Crown characteristics (position, length, width, form, etc.);

Variables, con’t

- At every measure, assess all stems (including non-commercial; every stem from the previous measure must be reconciled) for:
  - Diameter (over bark, breast high or above buttress), height to measure point, and validity (to indicate defects at measure point and anomalous but correct increments),
  - Status (alive, dead, harvested, treated) and stance (erect, leaning, fallen, broken), and
  - Tree coordinates (recruits only);

Variables, con't

- As necessary, record the occurrence of:
  - Logging, treatment and other activities, and the prescription used,
  - Scars and other damage with may affect measurements or growth,
  - Meteorological phenomena (drought, flood, etc.),
  - Mast years (heavy seed crops),
  - Pests, diseases, fire, or any othe aspect which may affect growth.

Data problems

- The greatest problem facing many agencies is that the data necessary for growth model development are not available.

- Unreliable measurements
- Changes to procedures
- Mistaken or undetermined species identities

- No data set can be perfect, but many will be found to contain deficiencies that will frustrate future analyses.

Density experiment design

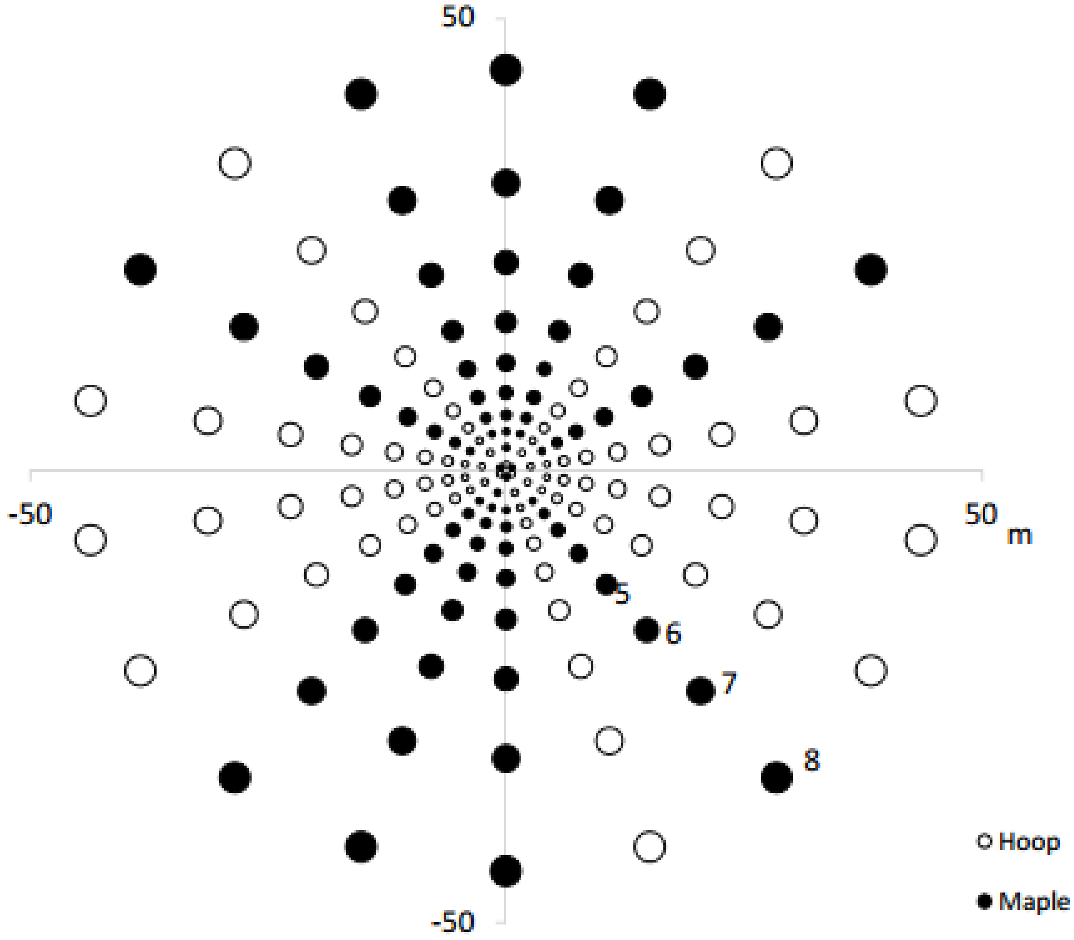


Figure 2. Google Earth image of Mt Mee Nelder trial, 20 July 2009 (© 2012 Google, © 2012 GeoEye, 27.096°S, 152.734°E), showing the two species, survival, and proximity of other plantings.

Planting experiment design

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	o	o	o	o	o	o	o	o	o	+	o	+	+	+	+	+	+	+	+	+
2	o	o	o	o	o	o	o	o	+	o	+	o	+	+	+	+	+	+	+	+
3	o	o	o	o	o	o	+	o	o	+	o	+	+	o	+	+	+	+	+	+
4	o	o	o	o	o	o	o	o	+	o	+	o	+	+	+	+	+	+	+	+
5	o	o	o	o	o	o	o	o	o	+	o	+	+	+	+	+	+	+	+	+
6	o	o	o	o	o	o	o	o	+	o	+	o	+	+	+	+	+	+	+	+
7	o	o	x	o	o	o	□	o	o	□	x	+	+	x	+	+	+	□	+	+
8	o	o	o	o	o	o	o	□	+	o	+	o	x	+	+	+	+	+	+	+
9	o	x	o	x	o	x	o	x	o	+	o	+	□	+	□	+	□	+	□	+
10	x	o	x	o	x	o	□	o	x	□	x	□	+	x	+	□	+	□	+	□
11	o	x	o	x	o	x	+	x	o	+	o	+	□	o	□	+	□	+	□	+
12	x	o	x	o	x	o	x	o	x	□	x	□	+	□	+	□	+	□	+	□
13	x	x	x	x	x	x	x	+	□	x	□	x	o	□	□	□	□	□	□	□
14	x	x	o	x	x	x	+	x	x	+	o	□	□	o	□	□	□	+	□	□
15	x	x	x	x	x	x	x	x	□	x	□	x	□	□	□	□	□	□	□	□
16	x	x	x	x	x	x	x	x	x	□	x	□	□	□	□	□	□	□	□	□
17	x	x	x	x	x	x	x	x	□	x	□	x	□	□	□	□	□	□	□	□
18	x	x	x	x	x	x	□	x	x	□	x	□	□	x	□	□	□	□	□	□
19	x	x	x	x	x	x	x	x	□	x	□	x	□	□	□	□	□	□	□	□
20	x	x	x	x	x	x	x	x	x	□	x	□	□	□	□	□	□	□	□	□

Figure 4. Design for a mixed species trial, showing planting positions for four species (o, +, x and □) in a 20 x 20 grid, showing three different viewpoints: the 3x3 viewpoint with 36 plots (top left), the 4x4 viewpoint with 25 plots (top right), and the 5x5 viewpoint with 16 plots (bottom left).

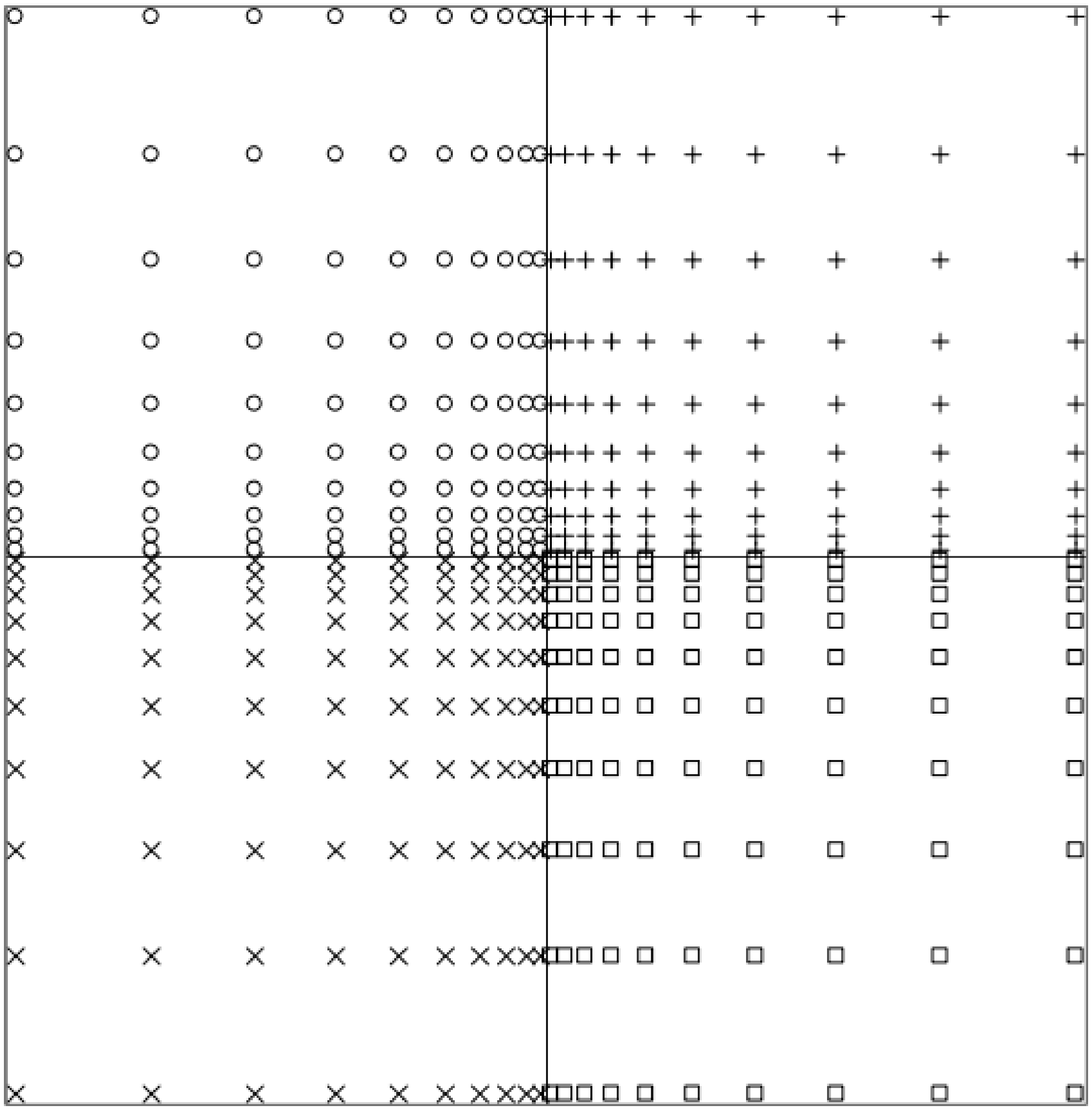
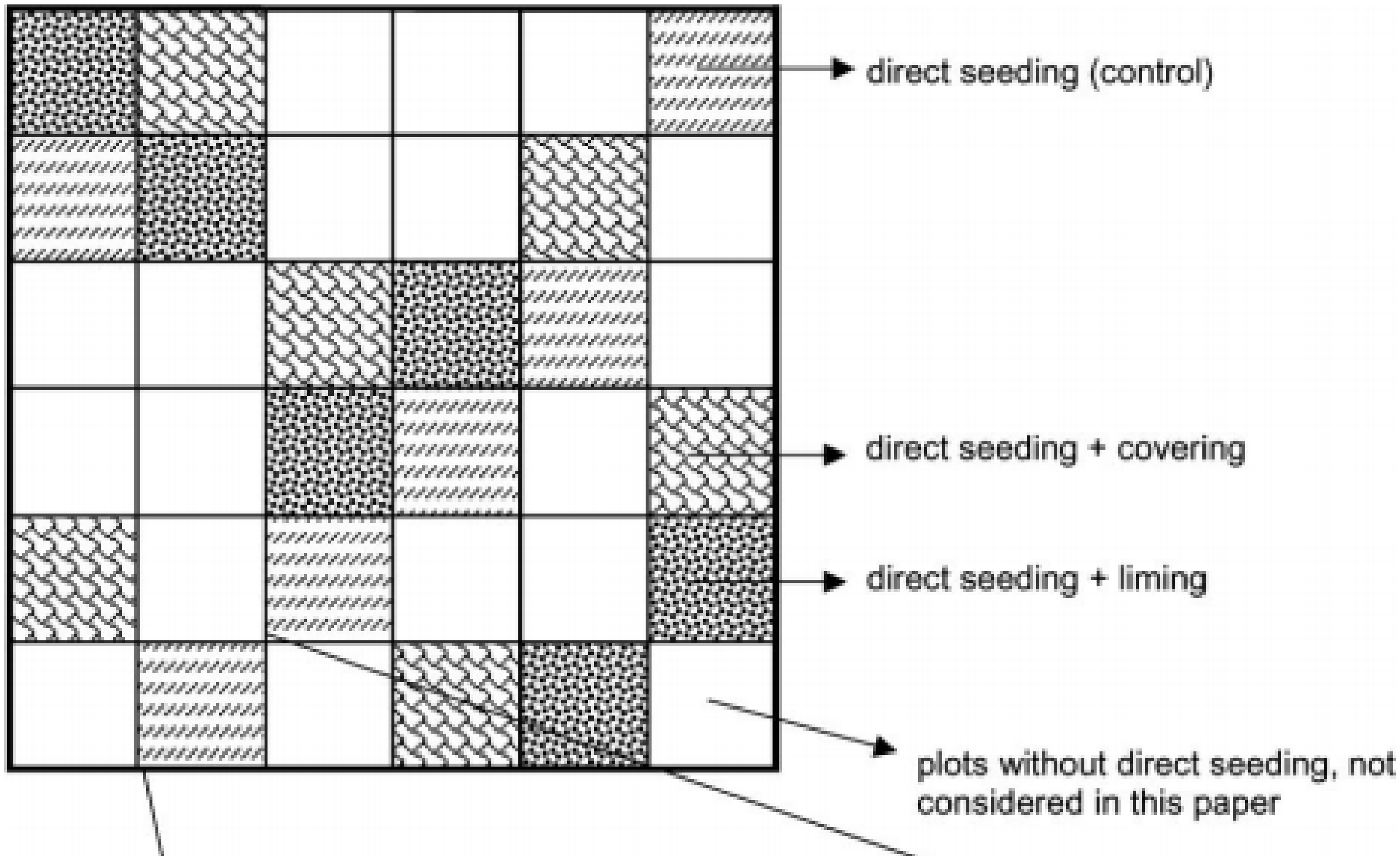
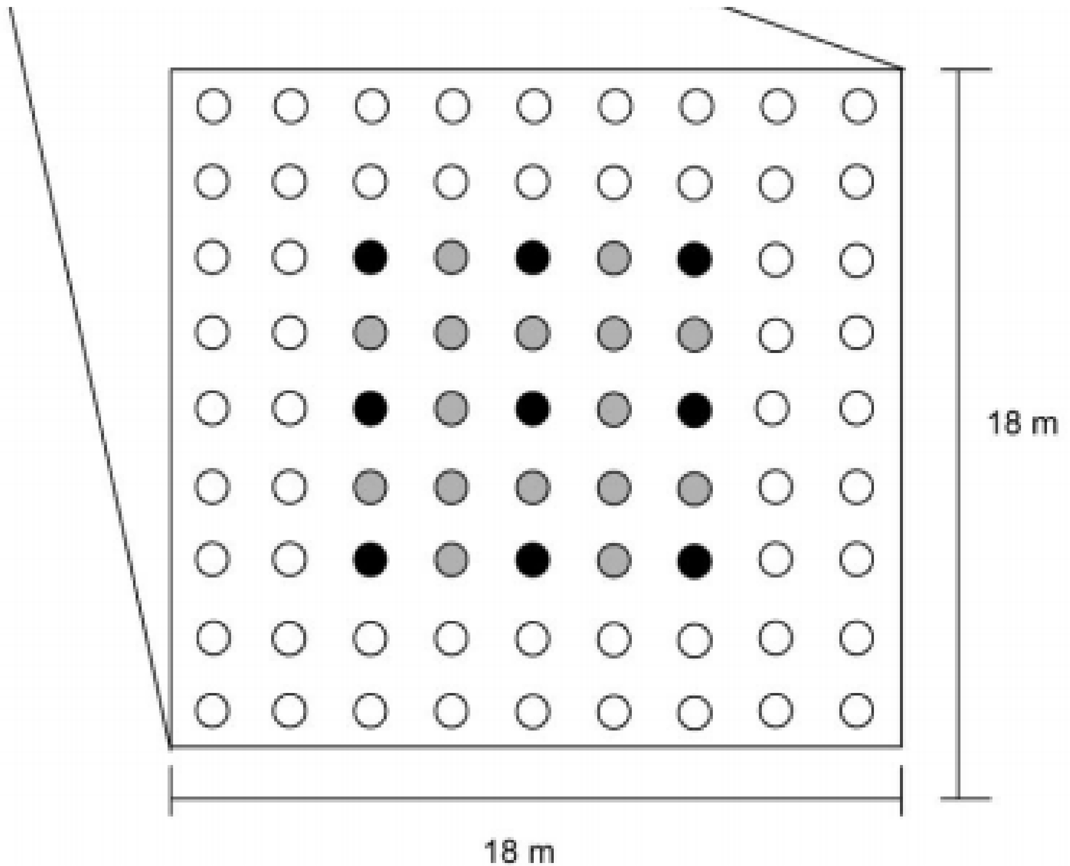


Figure 5. A possible design for a clinal planting to supplement the design in Figure 1, showing planting positions for 100 trees of each of four species (shown as o, x, + and □).

Regeneration experiment design



Experimental design



Exercise 1

- Design a form to record field measurements during the initial enumeration of a permanent plot in a forest near you. Take it to the field and try it ! Enter the data from the form into a text or spreadsheet file on a computer.
- What problems did you detect in the field and during data entry, and how would you improve your form next time?
- Would you use the same form when the plot was re-measured; if not, what changes would you make?
- How could you include some of the data from the initial measure on the remeasure form, so that field crews could cross-check these details?

Exercise 2

- Document ways that the data collected as part of Exercise 1 could be validated on the computer.
- What additional checks could you make when remeasured data become available?
- What errors might remain undetected by these procedures?
- Could these procedures be implemented on an electronic data recorder so that these checks could be made automatically in the field during plot remeasurement?

V_tot, V_ave, and Standard error

- Known: 20 ha forests grouped by age stage (young=1, mature=2, old=3), plot size: 20m*30m

	Area, ha	No. of plots, n	Volume/plot, y (m^3/ha)
Young stands	10	8	36 42 41 42 30 35 43 44
Mature stands	6	6	80 83 77 88 72 68
Old stands	4	6	122 135 140 127 136 121

- V_i = ?, V_i_ave = ?
- V_tot = ?, V_ave = ?
- Stand errors = ?

Solutions, V_i_ave

$$\hat{\bar{y}}_1 = \frac{1}{n_1}\sum_{1}^{n_1} y_{1i} = \frac{1}{8}\sum_{1}^{8} y_{1i} = \frac{36+42+41+42+30+35+43+44}{8} = 39.1\text{m}^3/\text{hm}^2$$

$$\hat{\bar{y}}_2 = \frac{1}{n_2}\sum_{1}^{n_2} y_{2i} = \frac{1}{6}\sum_{1}^{6} y_{2i} = \frac{80+83+77+88+72+68}{6} = 78.0\text{m}^3/\text{hm}^2$$

$$\hat{\bar{y}}_3 = \frac{1}{n_3}\sum_{1}^{n_3} y_{3i} = \frac{1}{6}\sum_{1}^{6} y_{3i} = \frac{122+135+140+127+136+121}{6} = 130.2\text{m}^3/\text{hm}^2$$

Variance (SD^2)

$$s_{y1}{}^2 = \frac{1}{n_1}\sum_{1}^{n_1}(y_{1i} - \hat{\bar{y}}_1)^2 = 21.1$$

$$s_{y2}{}^2 = \frac{1}{n_2}\sum_{1}^{n_2}(y_{2i} - \hat{\bar{y}}_2)^2 = 44.3$$

$$s_{y3}{}^2 = \frac{1}{n_3}\sum_{1}^{n_3}(y_{3i} - \hat{\bar{y}}_3)^2 = 52.5$$

Standard error

$$s_{\hat{\bar{y}}_1} = \sqrt{\left(1 - \frac{n_1}{N_1}\right)\frac{{s_{y1}}^2}{n_1}} = \sqrt{\left(1 - \frac{8}{10/0.06}\right)\frac{21.2}{8}} = 1.59\,\mathrm{m}^3/\mathrm{hm}^2$$

$$s_{\hat{\bar{y}}_2} = \sqrt{\left(1 - \frac{n_2}{N_2}\right)\frac{{s_{y2}}^2}{n_2}} = \sqrt{\left(1 - \frac{6}{6/0.06}\right)\frac{44.3}{6}} = 2.63\,\mathrm{m}^3/\mathrm{hm}^2$$

$$s_{\hat{\bar{y}}_3} = \sqrt{\left(1 - \frac{n_3}{N_3}\right)\frac{{s_{y3}}^2}{n_3}} = \sqrt{\left(1 - \frac{6}{4/0.06}\right)\frac{52.5}{6}} = 2.83\,\mathrm{m}^3/\mathrm{hm}^2$$

V_i, SE_i

- V_i:

$$\hat{T}_1 = A_1\hat{\bar{y}}_1 = 10 \times 39.1 = 391.0\text{m}^3$$

$$\hat{T}_2 = A_2\hat{\bar{y}}_2 = 6 \times 78.0 = 468.0\text{m}^3$$

$$\hat{T}_3 = A_3\hat{\bar{y}}_3 = 4 \times 130.2 = 520.8\text{m}^3$$

- SE_i:

$$s_{\hat{T}_1} = \sqrt{var(\hat{T}_1)} = \sqrt{A_1{}^2 var(\hat{\bar{y}}_1)} = \sqrt{10^2 \times 1.59^2} = 15.9\text{m}^3$$

$$s_{\hat{T}_2} = \sqrt{var(\hat{T}_2)} = \sqrt{A_2{}^2 var(\hat{\bar{y}}_2)} = \sqrt{6^2 \times 2.63^2} = 15.8\text{m}^3$$

$$s_{\hat{T}_3} = \sqrt{var(\hat{T}_3)} = \sqrt{A_3{}^2 var(\hat{\bar{y}}_3)} = \sqrt{4^2 \times 2.83^2} = 11.3\text{m}^3$$

V_ave, SE_ave, V_tot, SE_tot

- **V_ave：** $\hat{\bar{y}}_{tot} = \sum_{i=1}^{3} \frac{A_i}{A}\hat{\bar{y}}_i = \frac{10}{20} \times 39.1 + \frac{6}{20} \times 78.0 + \frac{4}{20} \times 130.2 = 69.0\text{m}^3/\text{hm}^2$

- **SE_ave:** $S_{\hat{\bar{y}}_{tot}} = \sqrt{\sum_{i=1}^{3}\left(\frac{A_i}{A}\right)^2 var(\hat{\bar{y}}_i)} = \sqrt{\left(\frac{10}{20}\right)^2 \times 1.59^2 + \left(\frac{6}{20}\right)^2 \times 2.63^2 + \left(\frac{4}{20}\right)^2 \times 2.83^2} = 1.25\text{m}^3/\text{hm}^2$

- **V_tot:** $T_{tot} = \hat{T}_1 + \hat{T}_2 + \hat{T}_3 = 391.0 + 468.0 + 520.8 = 1379.8\text{m}^3$

- **SE_tot：** $S_{T_{tot}} = \sqrt{\sum_{i=1}^{3} var(\hat{T}_i)} = \sqrt{s_{\hat{T}_1}{}^2 + s_{\hat{T}_2}{}^2 + s_{\hat{T}_3}{}^2} = \sqrt{15.9^2 + 15.8^2 + 11.3^2} = 25.1\text{m}^3$

Forest site evaluation

- Plantations vs. natural forests
- Forest types and forest productivity
- Site index, site form, or site class
- Dynamics of forest site quality
- Multi-data and site evaluation
- Biomass production and forest productivity

Methods for site evaluation

	Direct	**Indirect**
Phytocentric	Wood volume	Tree height
Geocentric	Soil moisture & nutrient status Photosynthetically active radiation	Climate Land form Physiography Plant indicators

Source：Vanclay（1994)

The Qinling Mountains

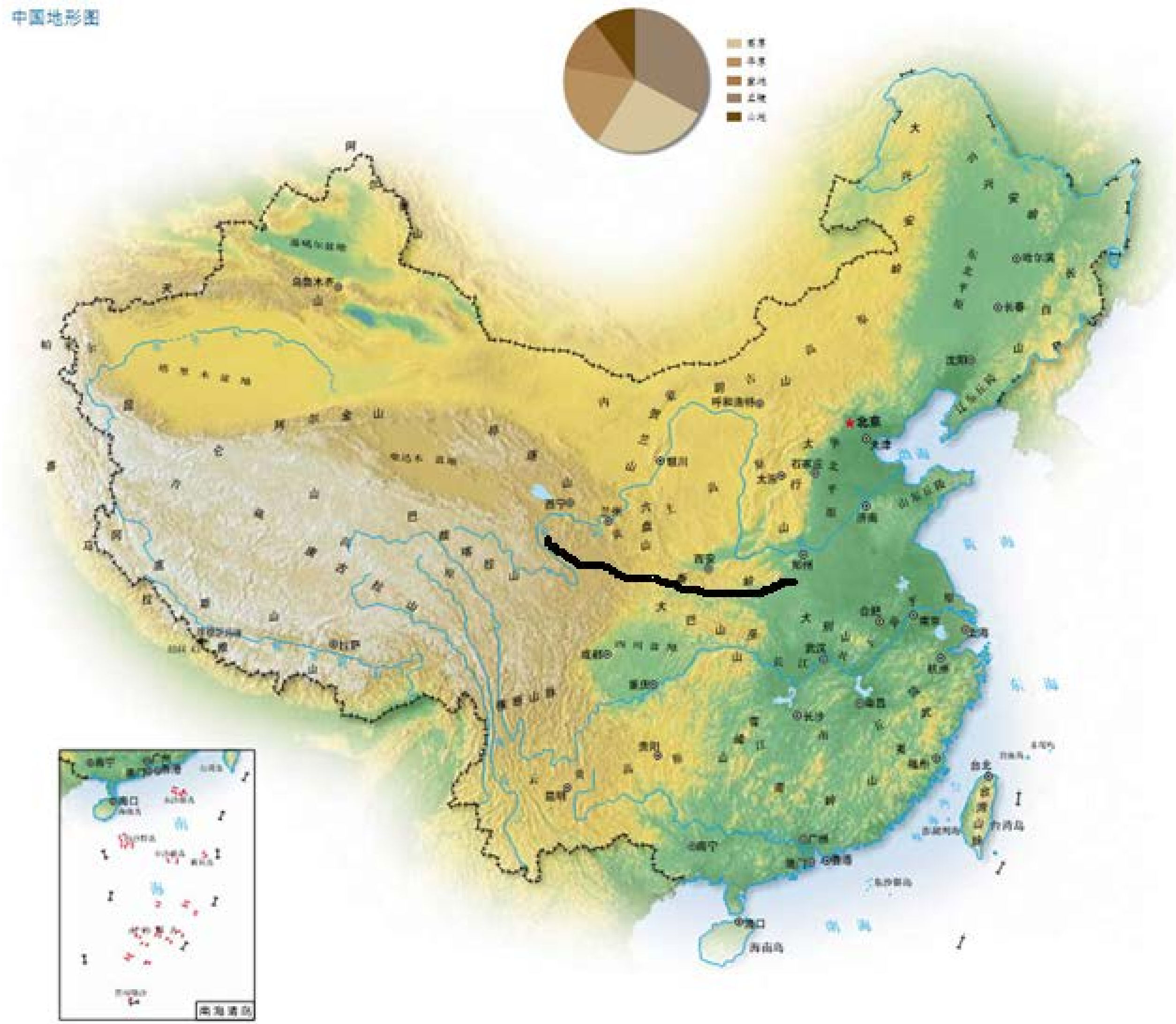
a.s.l. 3767 m

Slope 35°

Soil 50 cm

E-W 1600 km

S-N 10-300 km



Forest types in Qinling

2800-3200 m *Larix chinensis*

2400-2800 m *Abies fargesii*

2000m-2400 m *Betula albo- sinensis*, *Betula albo-sinensis* var. *septerionalis*

1700-2000 m *Pinus armandii - Betula albo- sinensis*

1400-1700 m *Pinus tabulaeformis - Quercus aliena* var. *acuteserrata*

800-1400 m *Quercus aliena, Quercus glandulifera* var. *brevipetiolata*

Forest types, con't

Classification of forest site

- Based on geo-factors
- Based on soil-factors
- Based on plant indicators
  - Cajander (1926)，forest floor vegetation
  - Daubenmire (1968)，community
- Based on climate, terrain, soil, and vegetation

Site index，or Site form？

- Site index, even-aged plantations, Hdom
- Site form, uneven-aged forests, D? （Weiskittel et al.，2011）
- Nigh（1995）, Wang（1998）, mixed forests

Site index vs. site class（Wu et al. 2015）

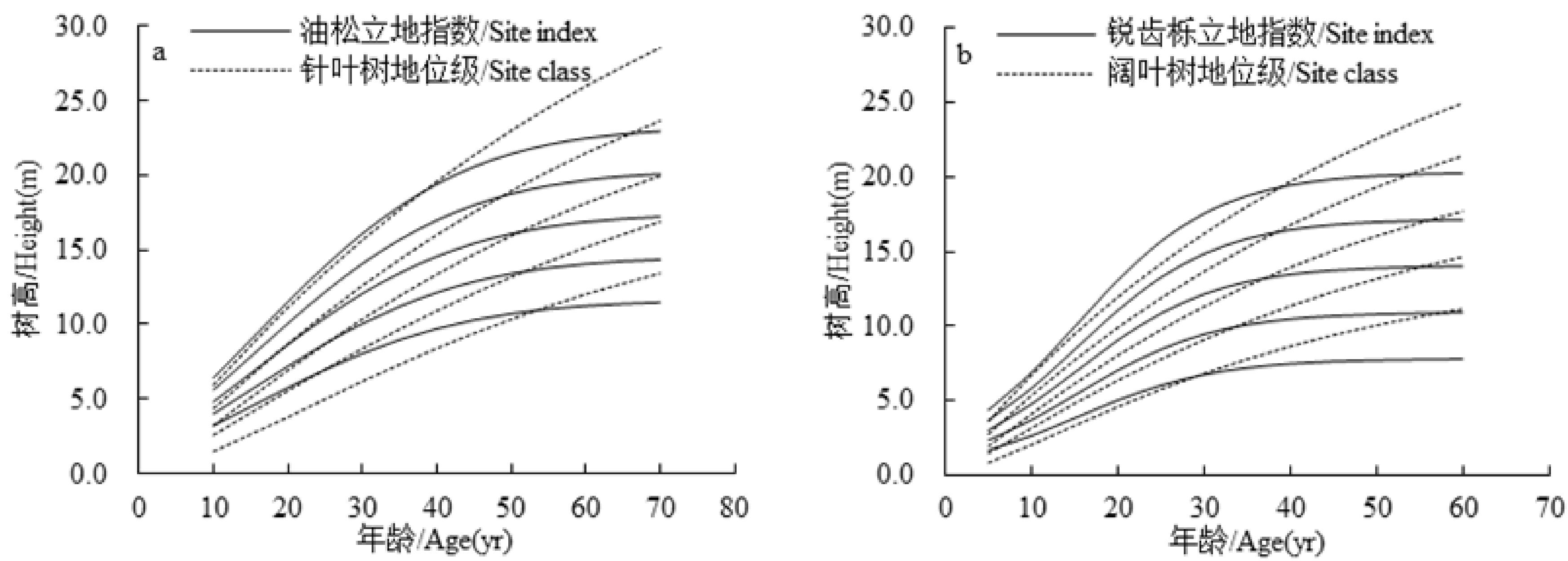


图 3-2 地位级与立地指数曲线簇

Fig.3-2 Curves of site class and site index

Site index vs. site form（Wu et al. 2015）

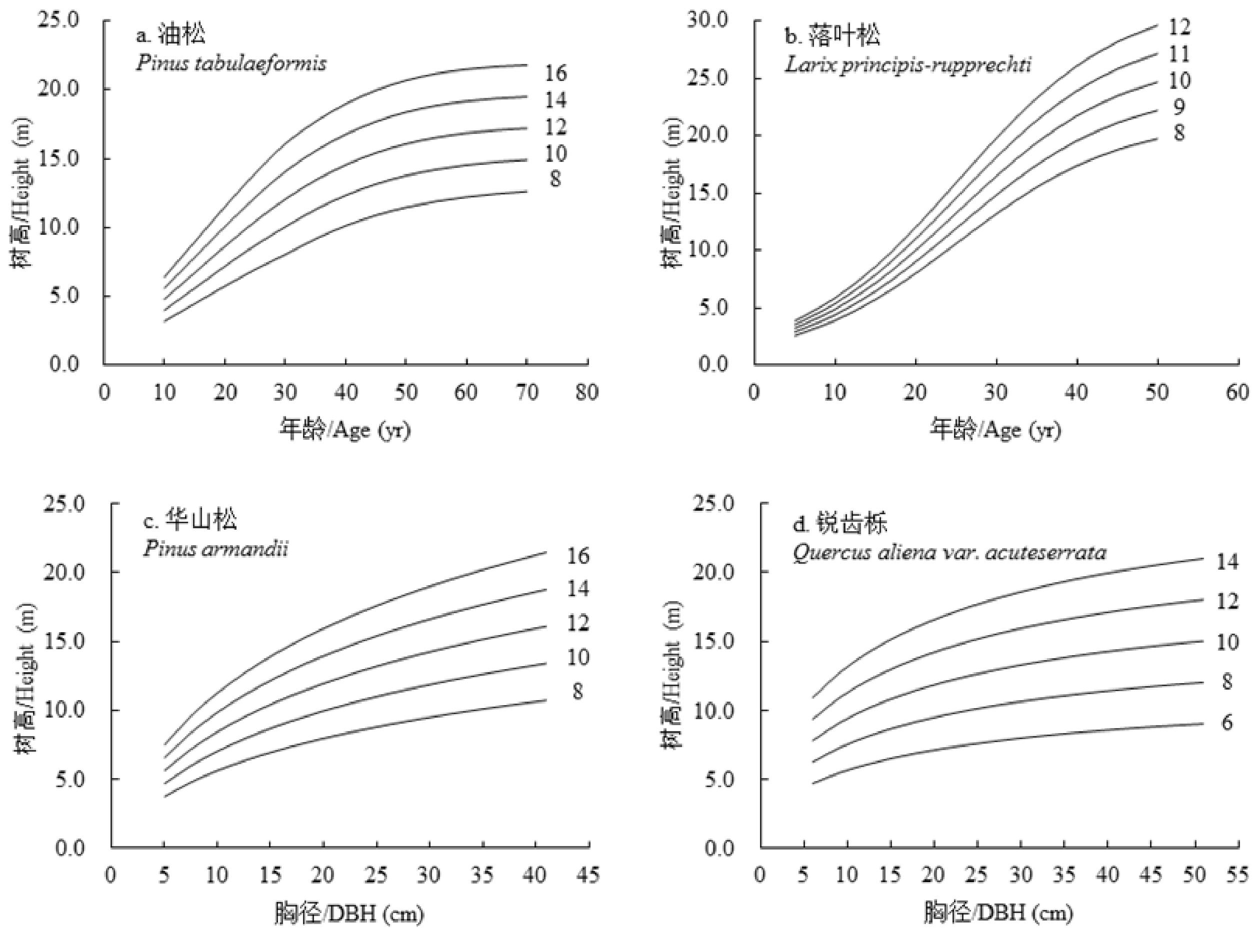


图 3-1 油松（a）落叶松（b）立地指数、华山松（c）锐齿栎（d）立地形曲线簇

Fig.3-1 Examples of site index and site form curves by species

Site form

GC，Guide Curve
ADA，Algebraic Difference Approach

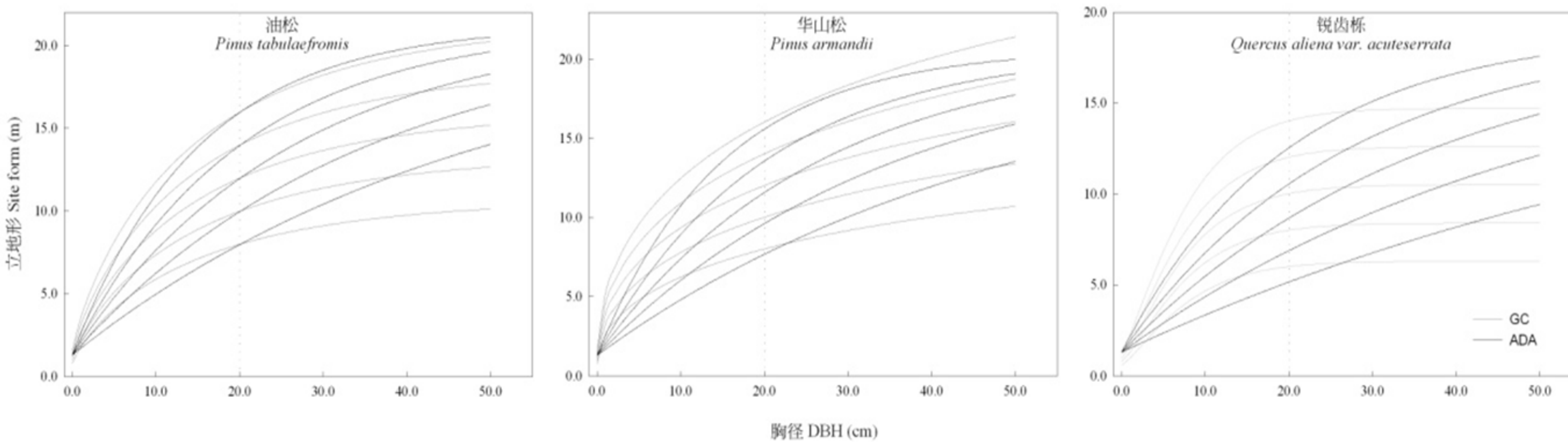


图 4-1 油松、华山松和锐齿栎立地形曲线簇

Fig.4-1 Curves of site form by species

Site form for pine-oak forest, con’t

- $SF_{PT} = a_1 SF_{PA} + e_1$
- $SF_{PA} = a_2 SF_{QA} + b\ \mathrm{Composition} + e_2$
- $SF_{QA} = a_3 SF_{PT} + c\ \mathrm{DBH} + d\ \mathrm{Density} + e_3$

Factors affecting site form

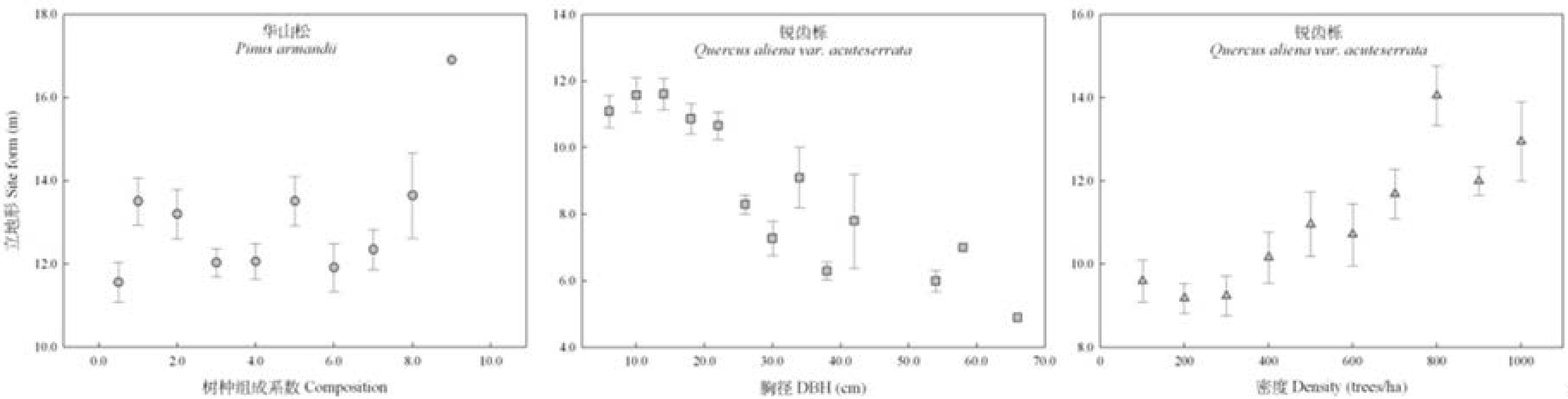


图 4-2 林分因子对立地形的影响

Fig.4-2 Influence of stand factors on site form

Changes on site class

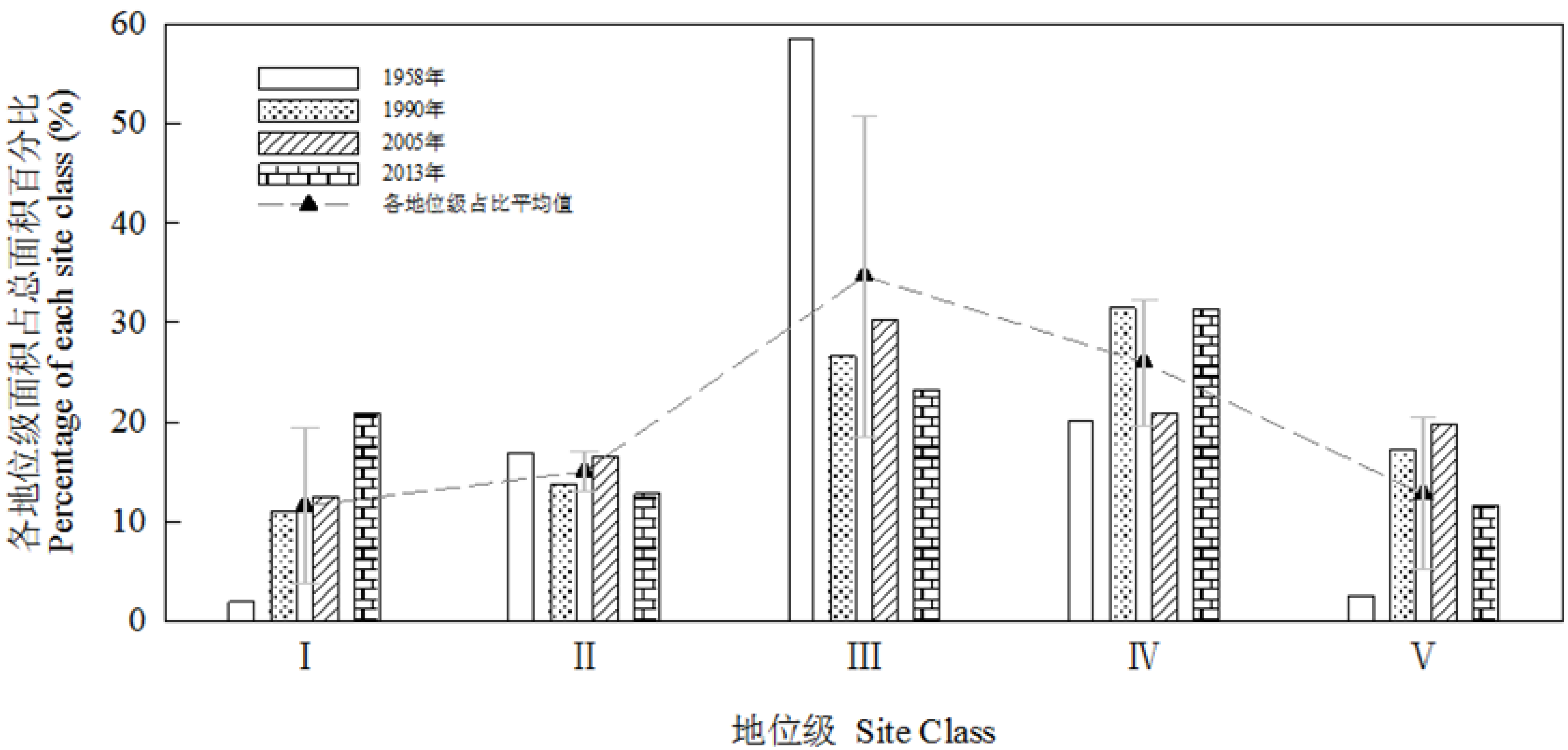
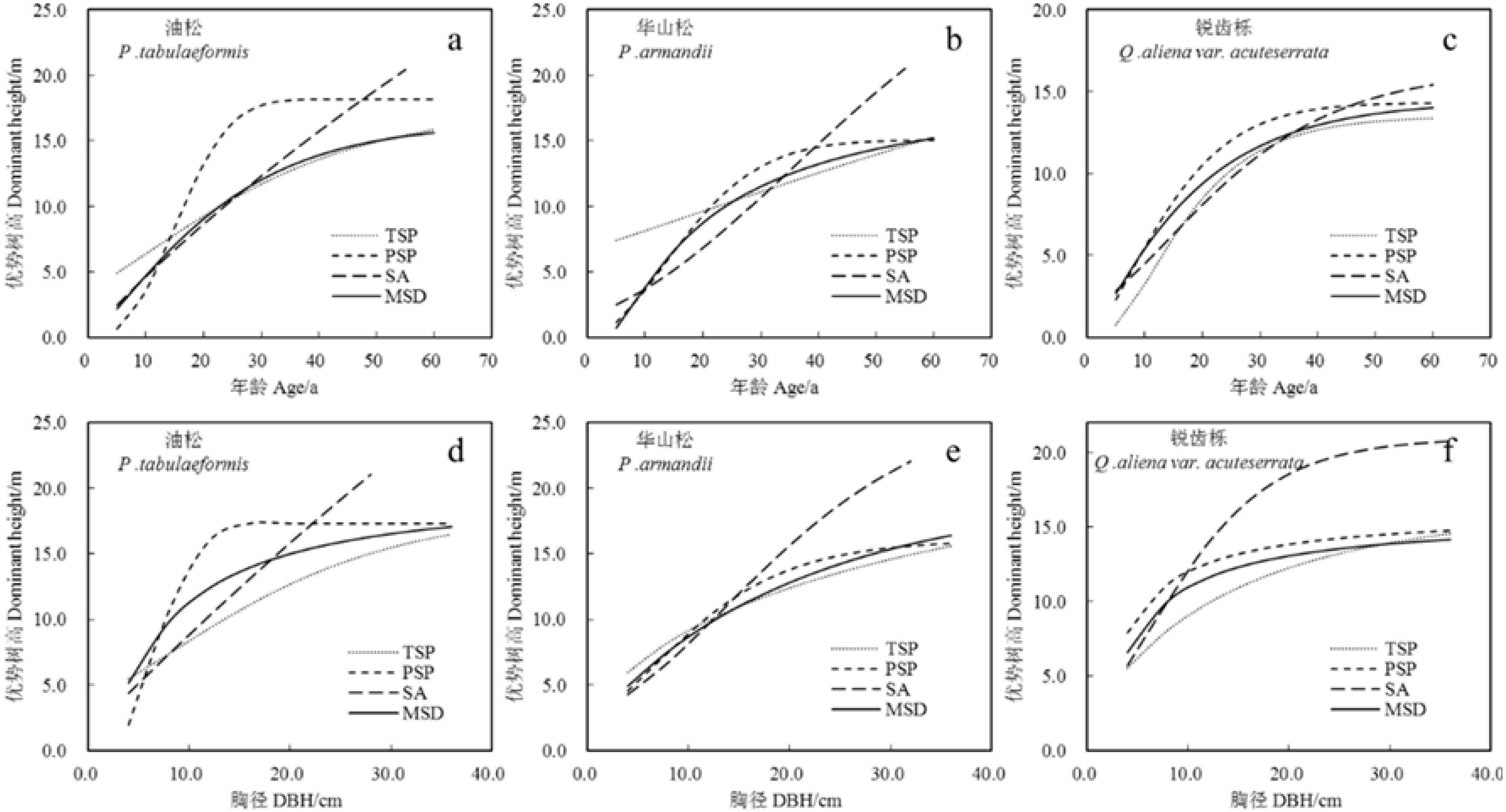


图 5-1 各地位级组成比例变化

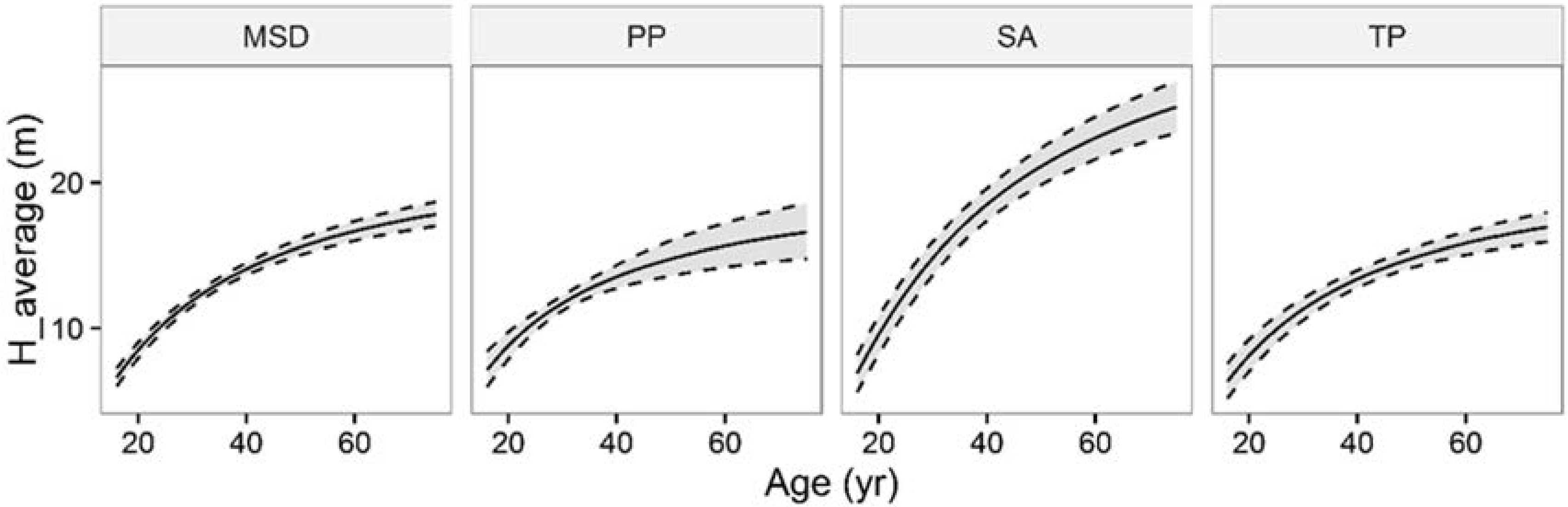
Fig.5-1 Changes of each site class percentage

Multi-data and site evaluation



Predicting average height with multi-data

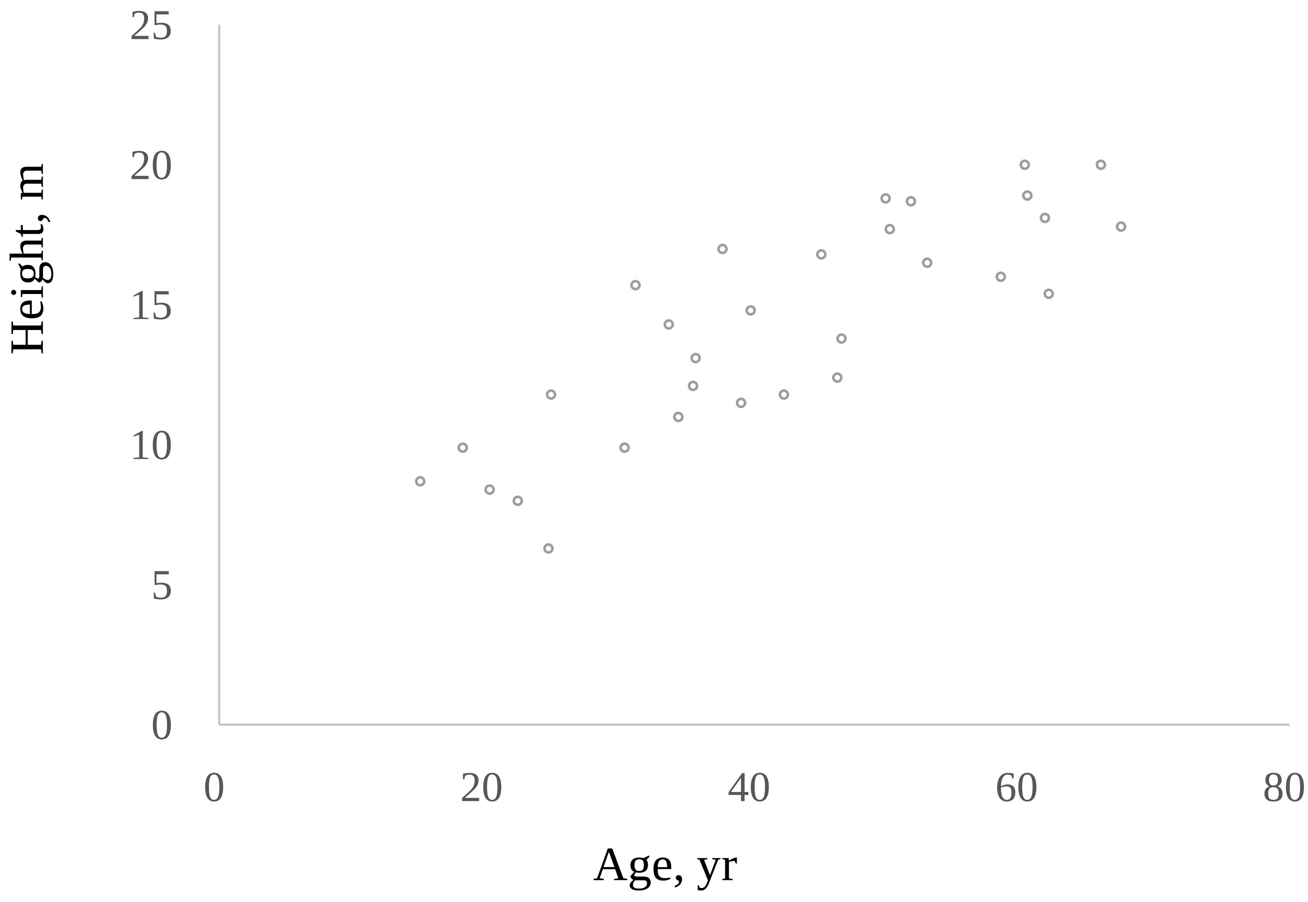
The uncertainty of predictions, 95% Bayesian credible interval, were showed in grey area.
(SCI=11, SDI=600)



Processing inventory data

Plot no.	Age, yr	H, m	Plot no.	Age, yr	H, m	Plot no.	Age, yr	H, m
1	62	18.1	11	34	11.0	21	46	12.4
2	20	8.4	12	25	6.3	22	39	11.5
3	15	8.7	13	60	20.0	23	22	8.0
4	36	13.1	14	47	13.8	24	31	15.7
5	30	9.9	15	67	17.8	25	50	17.7
6	18	9.9	16	52	18.7	26	62	15.4
7	35	12.1	17	40	14.8	27	34	14.3
8	38	17.0	18	42	11.8	28	45	16.8
9	60	18.9	19	50	18.8	29	53	16.5
10	66	20.0	20	25	11.8	30	58	16.0

Height vs. Age



Theoretical growth equations

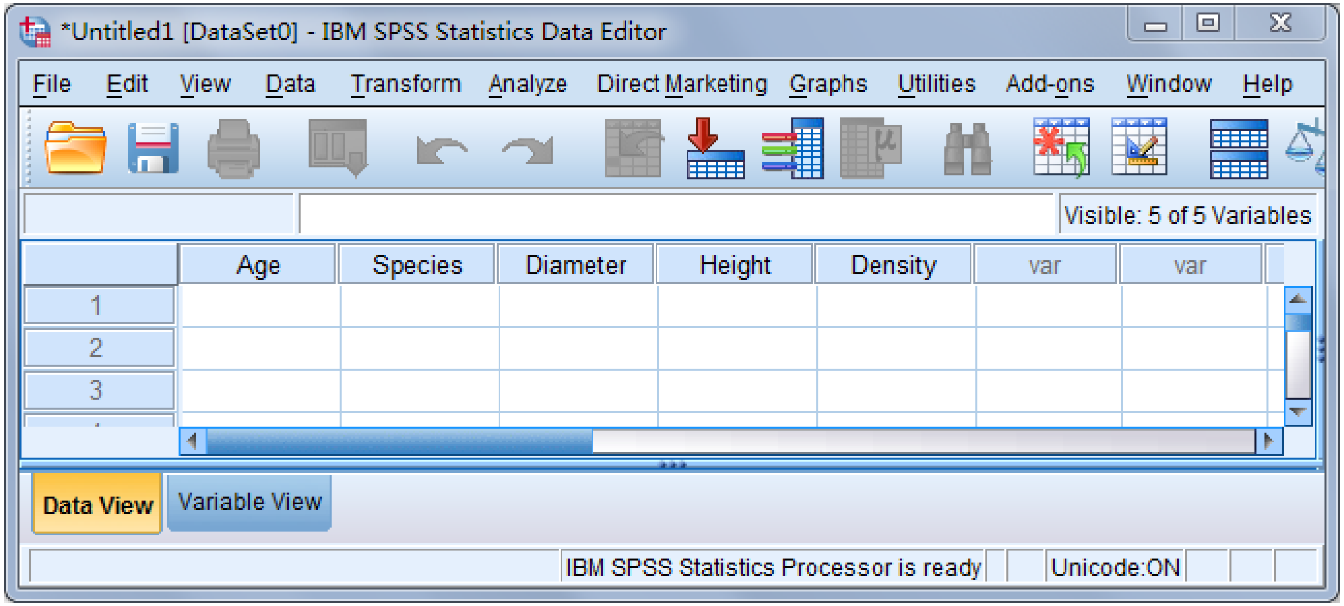
- Assume tree growth y (DBH, H, BA, or V) is a function of time, t
- y = f(t)

- Schumacher: y = a*exp(-b/t)
- Mitscherlich: y =a(1-exp(-b*t))
- Logistic: y = a/(1+c*exp(-b*t))
- Gompertz: y = a*exp(-b*exp(-c*t))
- Korf: y = a*exp(-b*t^(-c))
- Richards: y = a(1-exp(-b*t)^c

- a, b, c, parameters

Computer programs for biometrics

- Statistical Package for the Social Sciences (SPSS)
- SAS
- R
- Python



Fit-to-curve

- Step 1. select one of equations, e.g. Schumacher
- Step 2. compute initial guesses of parameters
-
- y_max = a

- Known:
- y_max = 25 m
- H = 8 m, age = 20 yrs
- 8 = 25*exp(-b/20)
- ln8 = ln25 – b/20
- b = (ln25 – ln8)*20 = 23

Iteration History

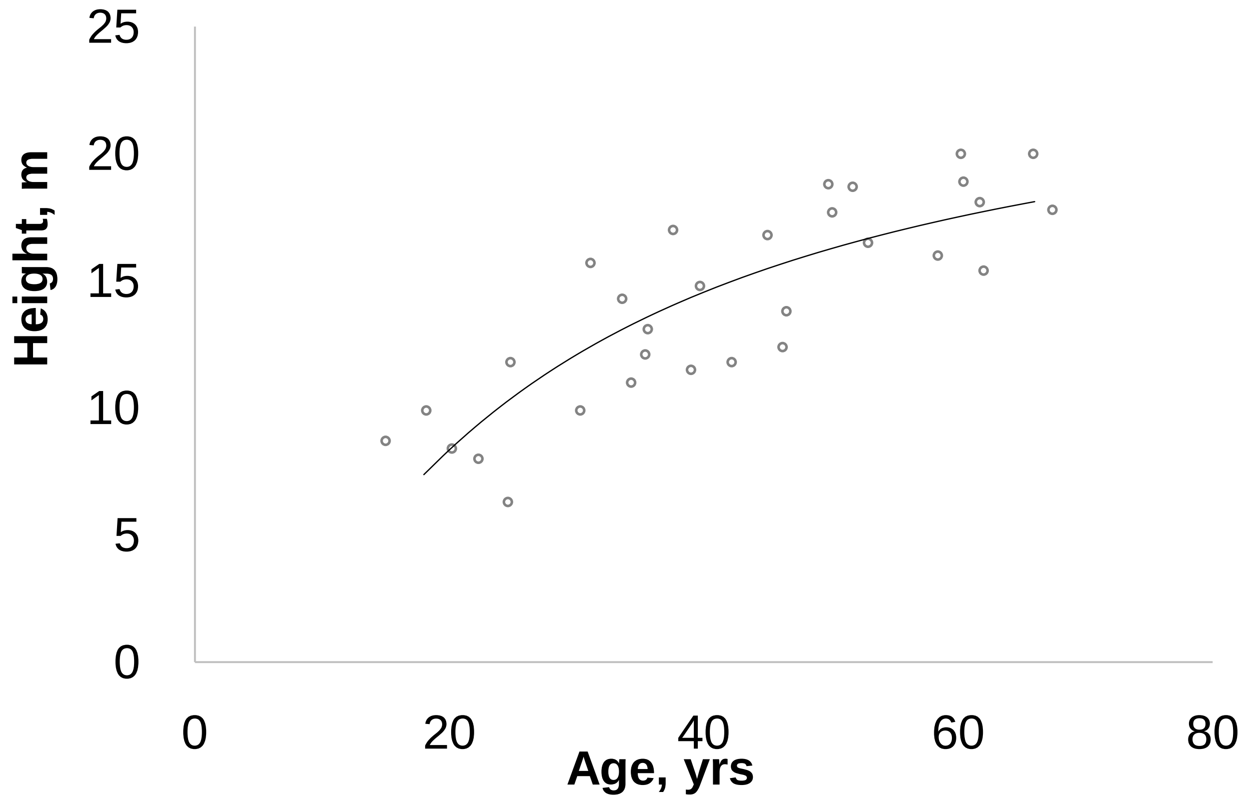
Iteration Number[a]	Residual Sum of Squares	Parameter	
		a	b
1.0	140.767	25.000	23.000
1.1	134.191	25.402	22.273
2.0	134.191	25.402	22.273
2.1	134.189	25.369	22.236
3.0	134.189	25.369	22.236
3.1	134.189	25.367	22.232
4.0	134.189	25.367	22.232
4.1	134.189	25.367	22.232

Parameter estimates

Parameter	Estimate	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
a	25.367	2.013	21.244	29.491
b	22.232	3.243	15.590	28.874

Predicting height growth

- H = 25.367*exp(-22.232/t)



Correlations of parameter estimates

	a	b
a	1.000	.938
b	.938	1.000

ANOVA analysis

Source	Sum of Squares	df	Mean Squares
Regression	6330.471	2	3165.235
Residual	134.189	28	4.792
Uncorrected Total	6464.660	30	
Corrected Total	438.159	29	

Dependent variable: H
R squared = 1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = .694.

Models for tree height growth

1	Richard	$H_T = 1.3 + b \times [1 - \exp(-a \times \mathrm{DBH})]^c$
2	Weibull	$H_T = 1.3 + b \times [1 - \exp(-a \times \mathrm{DBH}^c)]$
3	Logistic	$H_T = 1.3 + b / [1 + a \times \exp(-c \times \mathrm{DBH})]$
4	Korf	$H_T = 1.3 + b \times [\exp(-a / \mathrm{DBH}^c)]$
5	Compertz	$H_T = 1.3 + b \times \exp[-a \times \exp(-c \times \mathrm{DBH})]$
6	Quadratic	$H_T = a + b \times \mathrm{Age} + c \times \mathrm{Age}^2$
7	Inverse	$H_T = a + b / \mathrm{Age}$
8	Sigmodial	$H_T = b / \{1 + \exp[-(\mathrm{Age} - a) / c]\}$
9	Logistic	$H_T = b / [1 + (\mathrm{Age} / a)^c]$
10	Compertz	$H_T = b \times \exp\{-\exp[(a - \mathrm{Age}) / c]\}$
11	Chapman	$H_T = b \times [1 - \exp(-a \times \mathrm{Age})]^c$
12	Hill	$H_T = b \times \mathrm{Age}^a / (c^a + \mathrm{Age}^a)$
13	Hyperbola	$H_T = a - b / (1 + c \times \mathrm{Age})^{1/d}$
14	Logarithm	$H_T = c + a \times \ln\mathrm{Age} + b(\ln\mathrm{Age})^2$
15	Power	$H_T = c + a \times \mathrm{Age}^b$
16	Mitscherlich	$H_T = b \times \{1 - \exp[(c - \mathrm{Age}) / a]\}$
17	Modified Gaussian	$H_T = b \times \{1 - \exp(-[(\mathrm{Age} - a) / c]^2)\}$
18	Richard	$H_T = b \times [1 - \exp(-a \times \mathrm{Age})]^c$
19	Schumacher	$H_T = b \times \exp[-c / (\mathrm{Age} - a)]$

Fitting results

Tab.4 Fitting results of guide curves

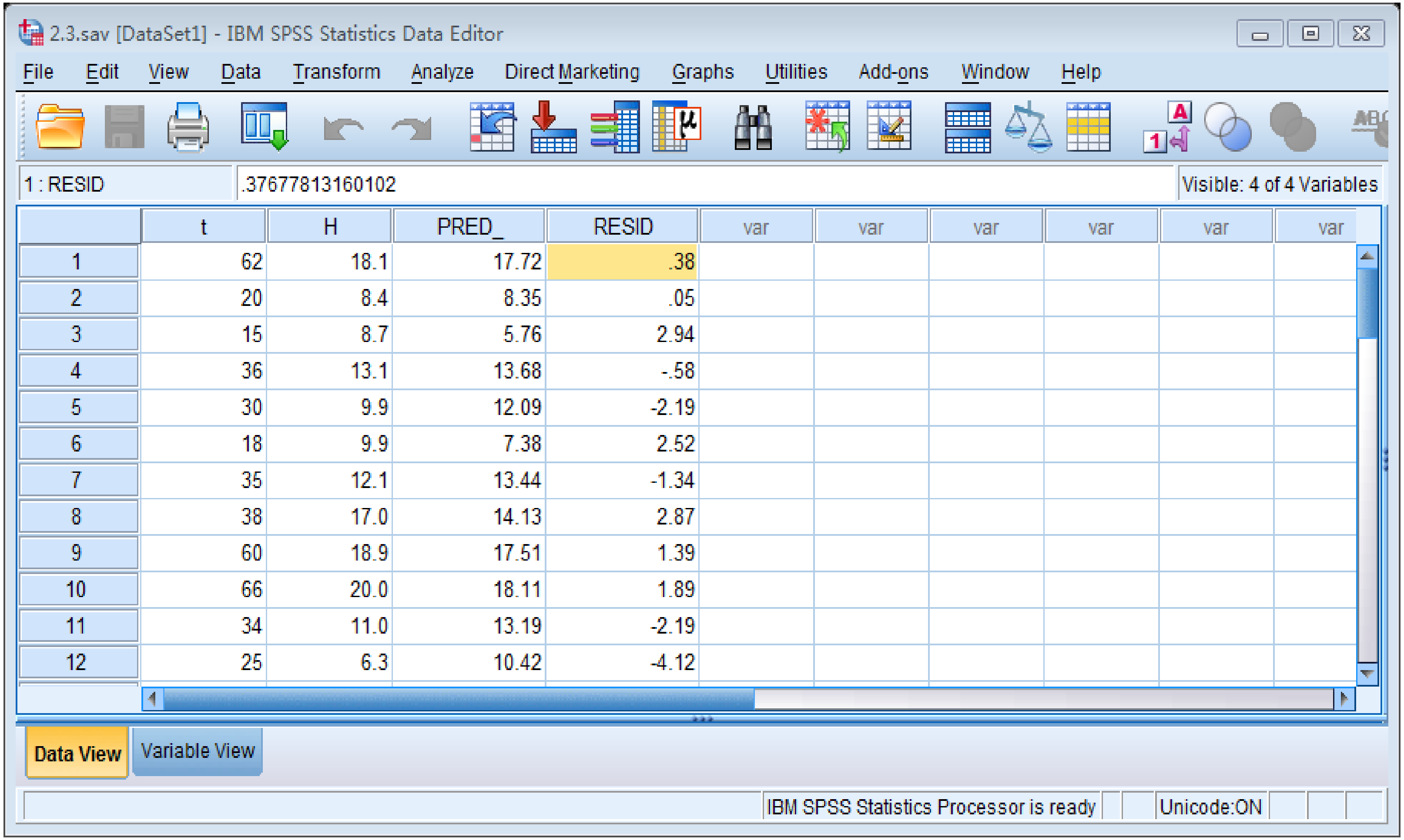
拟合结果 Fitting results	油松 *P. tabulaeformis*		华山松 *P. armandii*		落叶松 *L. principis-rupprechtii*		锐齿栎 *Q. aliena* var. *acuteserrata*	
	胸径 DBH	年龄 Age	胸径 DBH	年龄 Age	胸径 DBH	年龄 Age	胸径 DBH	年龄 Age
方程 Equation	Weibull	Gaussian	Korf	Logistic	Logistic	Sigmoidal	Weibull	Sigmoidal
n	245	245	631	631	249	249	458	458
a	0.026 0	-11.953 7	4.146 1	62.320 9	19.071 5	25.251 5	0.174 7	15.315 0
b	27.042 9	17.391 7	53.928 8	30.772 2	23.251 9	27.634 8	17.501 2	13.598 3
c	1.024 3	38.618 9	0.324 8	-0.743 1	0.209 0	10.176 7	0.586 0	7.966 1
R^2	0.60	0.50	0.41	0.40	0.86	0.89	0.36	0.39
SEE	2.84	3.16	2.34	2.40	2.71	2.38	2.26	2.20

①n: 建模样本量 Number of observations for modelling; R^2: 决定系数 Coefficient of determination; SEE: 标准估计误差 Standard error.

Site class table

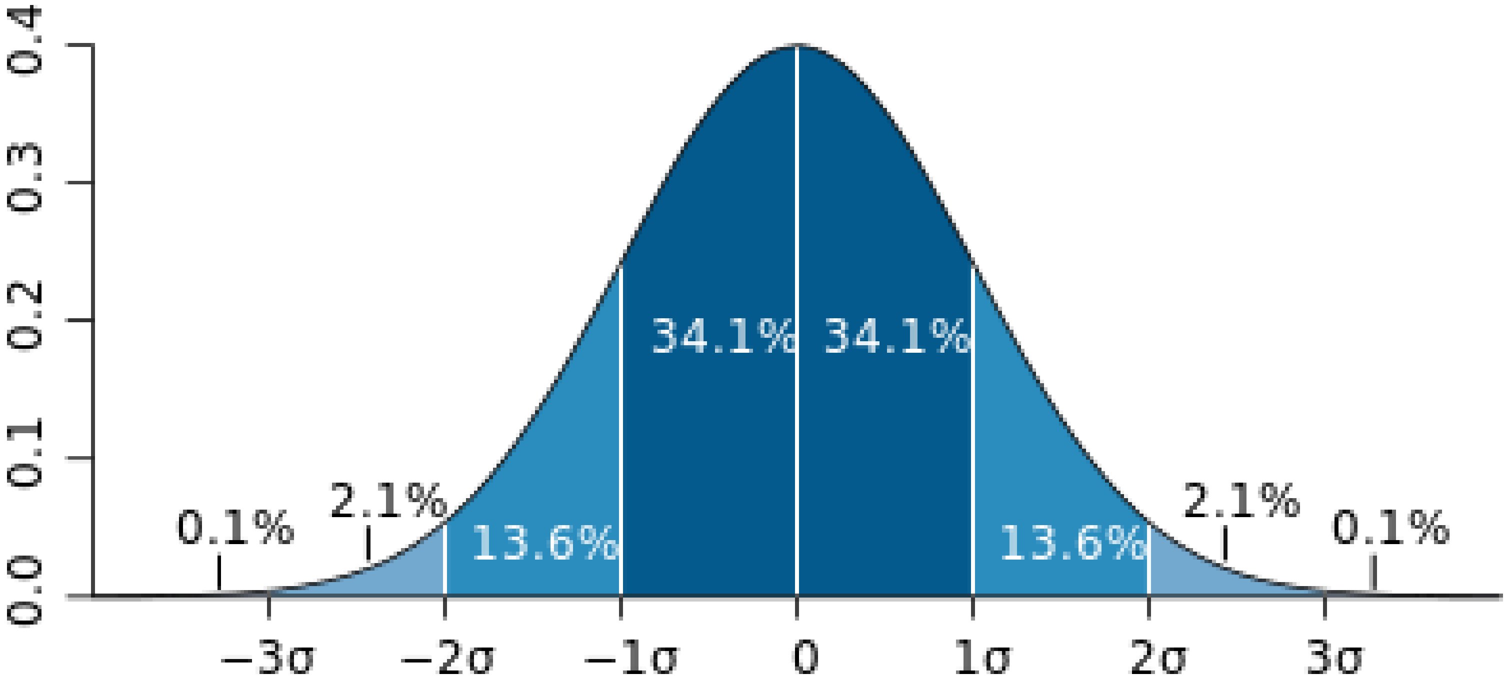
- Step 1. data processing
- Step 2. fit-to-curve using nonlinear regression
- Step 3. clean up unusual data with rule of 3*SD
- Step 4. re-fit with remaining data
  - e.g. re-fitted solution, H = 25.117*exp(-21.391/t)
- Step 5. compute SD of height residuals by age class
  - e.g. $\sigma = 0.921$
- Step 6. define upper & lower bounds based on 3*SD
- Step 7. average upper & lower bounds of site classes

Predicted value and Residuals

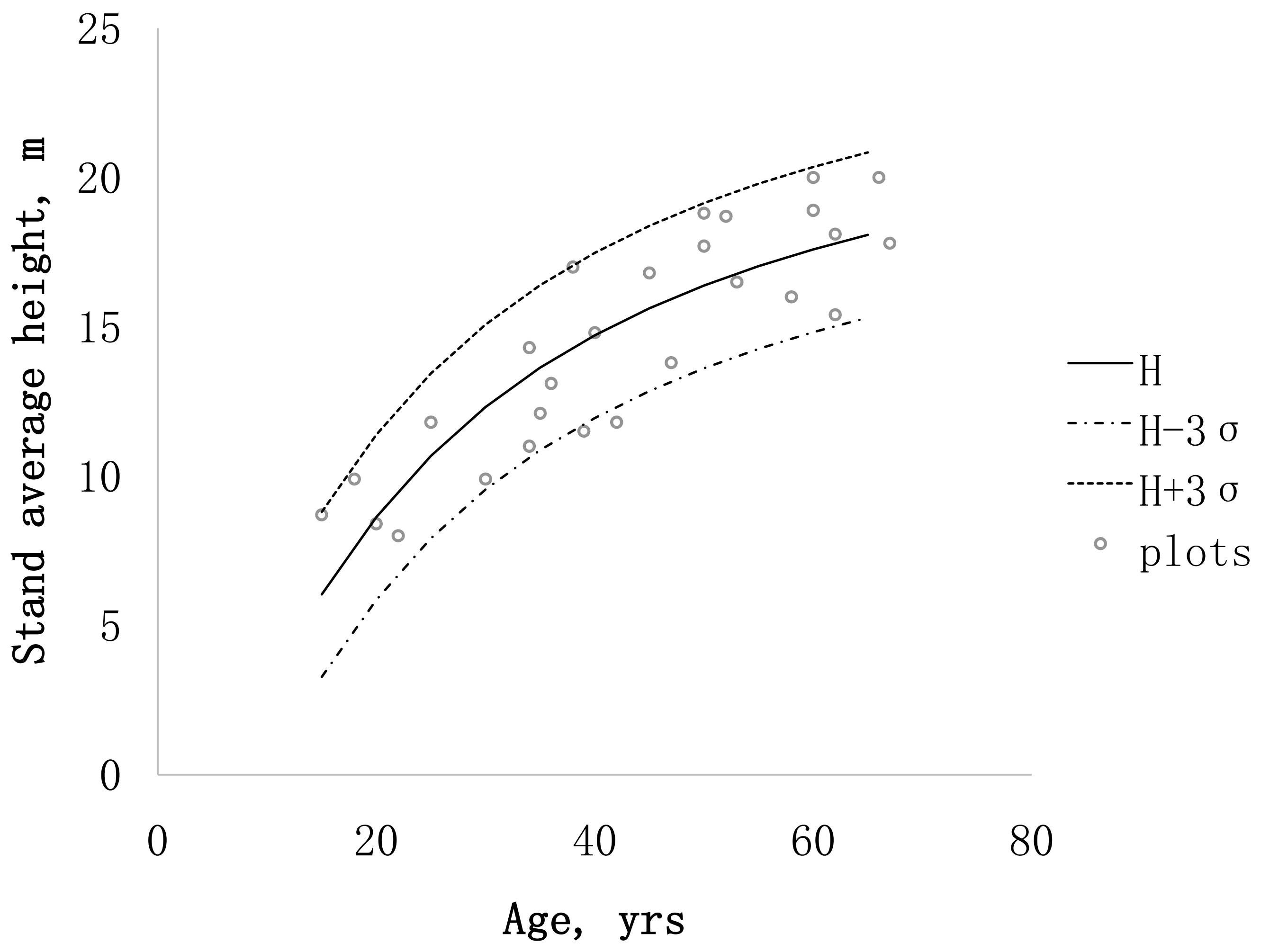


	t	H	PRED_	RESID	var	var	var	var	var	var
1	62	18.1	17.72	.38						
2	20	8.4	8.35	.05						
3	15	8.7	5.76	2.94						
4	36	13.1	13.68	-.58						
5	30	9.9	12.09	-2.19						
6	18	9.9	7.38	2.52						
7	35	12.1	13.44	-1.34						
8	38	17.0	14.13	2.87						
9	60	18.9	17.51	1.39						
10	66	20.0	18.11	1.89						
11	34	11.0	13.19	-2.19						
12	25	6.3	10.42	-4.12						

3*S.D.



Upper and lower bounds of site class



Site class table for *Pinus tabulaeformis*

Age, yrs	SC Ⅱ, m	SC Ⅲ, m	SC Ⅳ, m
20	9. 5–11. 4	7. 7–9. 4	5. 9–7. 6
30	13. 2–15. 1	11. 4–13. 1	9. 5–11. 3
40	15. 6–17. 5	13. 8–15. 5	11. 9–13. 7
50	17. 3–19. 1	15. 5–17. 2	13. 6–15. 4
60	18. 5–20. 3	16. 7–18. 4	14. 8–16. 6

Data and Models

- Like chickens and eggs, it is not obvious which comes first.
- Modeling and definition and collection of data should form an iterative process, commencing with the model formulation.
- Most modeling efforts commence with and data available
- The modeling approach often may be dictated by limitations of the data.

Models for forest ecosystem management

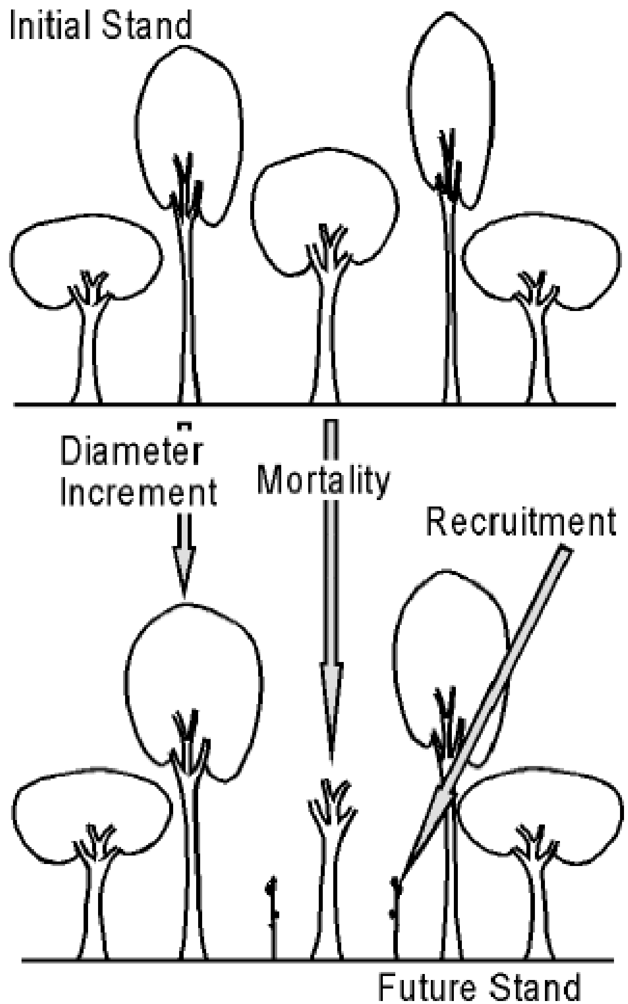
- Forest yield models (CACTOS, FPS, FVS, ORGANON, SPS)
- Ecological gap models (JABOWA, FORET)
- Ecological compartment models (CENTURY, FOREST-BGC)
- Process/mechanistic models
  (PnET, PipeStem, ECOPHYS, FOREST-BGC)
- Vegetation distribution models (Monserud et al. 1993)
- Hybrid models (PipeQual/CROBAS, Mäkelä for Finland; 3-PG, Landsberg & Waring; Ågren for Sweden; FOREST 5: Robinson)

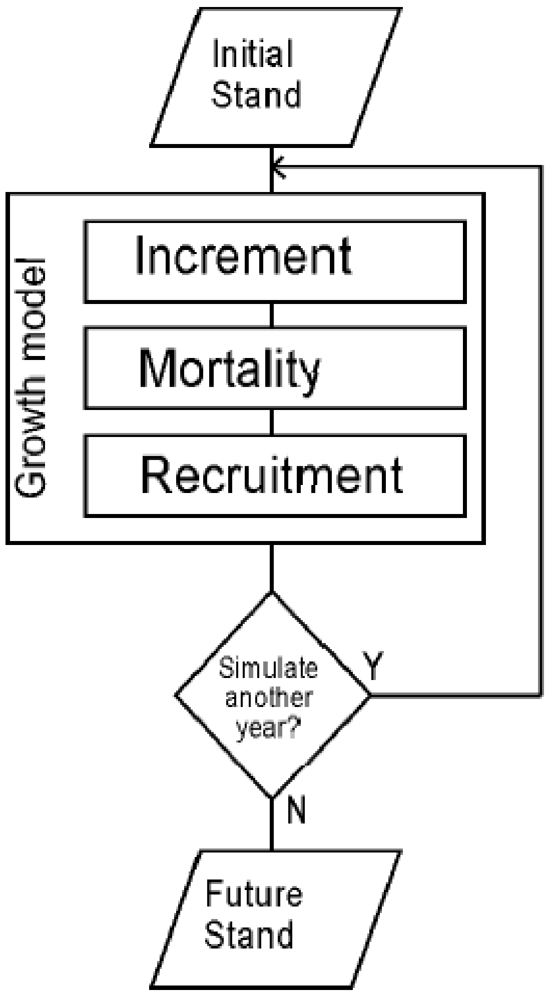
Source: Monserud (2003), Pretzsch et al. (2008)

Model classification by scales and purposes

Use	Resolution	Driving variables	Example
Empirical models			
Atmospheric studies	Global primary production	Evapo-transpiration	Lieth & Box (1972)
National forest planning	Stand variables	Age, stand basal area	Clutter (1963)
Regional planning	Individual trees	Tree species & sizes	Prognosis (Stage 1973)
Silvicultural studies	Tree crowns	Tree & branch variables	TASS (Mitchell 1975)
Silvicultural & conversion studies	Wood characteristics	Branches, ring width & density	SYLVER (Mitchell 1988)
Succession & Process models			
Ecological studies	Individual trees	Tree species & sizes	JABOWA (Botkin 1993)
Nutrient cycling	Individual trees	Trees, nutrients	FORCYTE (Kimmins 1988)
Physiological studies	Mass of foliage, branches, roots	Biomass, photosynthesis, respiration	Sievänen *et al.* (1988)

Components of forest growth





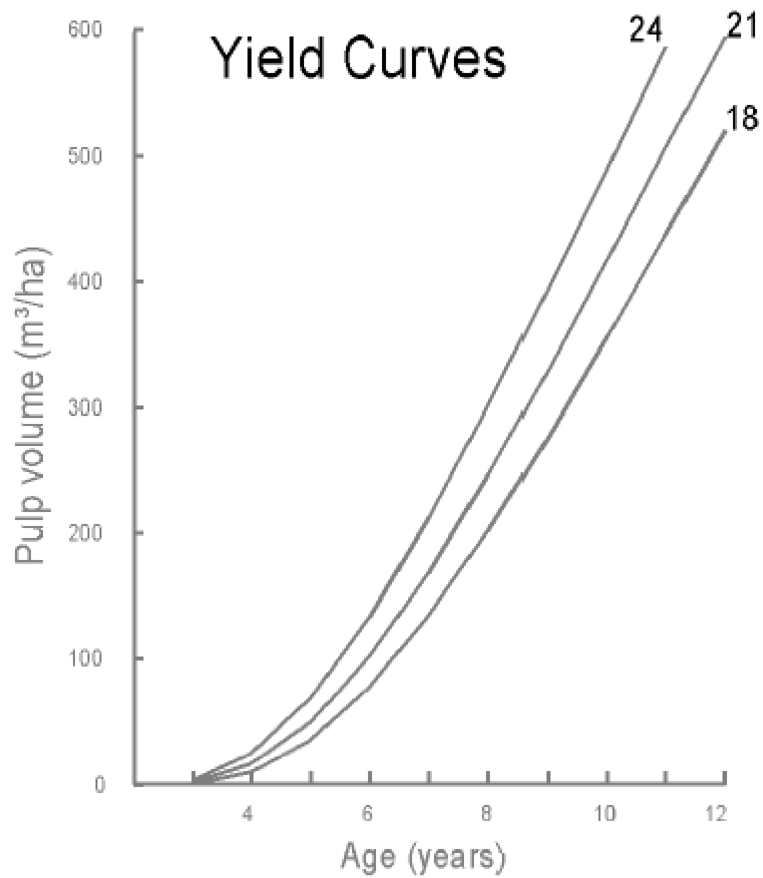
Yield tables

- A yield table presents the anticipated yields from an even-aged stand at various ages.
- One of the oldest approaches to yield estimation.
- Chinese “Lung Ch’uan codes”, some 350 yrs ago (Vuokila 1965).
- The first yield tables were published in Germany in 1787.
- Various approaches used in Europe (Vuokila 1965) and North America (Spurr 1952)

Growth and yield tables

Yield Table

Age (y)	Site Index (m) 18	21	24
3	1	2	4
4	11	17	25
5	36	50	69
6	78	102	133
7	134	169	212
8	201	246	301
9	276	330	395
10	356	418	491
11	438	507	586
12	520	594	679



Yield Equation: $\log(V+1) = 3.534 - 14.02/t + 0.2314\ S/t$

TABLE 1. *Experience table for the yield of various species for light thinning (Von Cotta, 1821, p. 34)*

Tafel V. A. Fichten.

Jahre.	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
20	269	450	632	813	994	1175	1356	1538	1719	1900
21	290	485	680	875	1071	1266	1461	1656	1851	2047
22	311	520	730	939	1149	1358	1568	1777	1987	2196
23	333	557	781	1005	1229	1453	1677	1901	2124	2349
24	355	593	832	1071	1310	1549	1788	2026	2265	2504
25	377	631	885	1139	1393	1646	1900	2154	2408	2662
26	400	669	939	1208	1477	1747	2016	2285	2555	2824
27	423	708	993	1278	1563	1848	2133	2418	2703	2989
28	447	748	1049	1350	1651	1952	2233	2554	2855	3156
29	471	788	1106	1423	1740	2057	2375	2692	3009	3327
30	495	830	1163	1497	1831	2165	2499	2832	3166	3500
31	520	871	1222	1573	1923	2274	2625	2975	3326	3677
32	546	914	1282	1649	2017	2385	2753	3120	3488	3856
33	572	957	1342	1728	2113	2498	2883	3268	3653	4039
34	598	1001	1404	1807	2210	2613	3015	3418	3821	4224
35	625	1046	1467	1887	2308	2729	3150	3571	3992	4413
36	652	1091	1530	1969	2408	2848	3287	3726	4165	4604
37	679	1137	1595	2053	2510	2968	3426	3883	4341	4799
38	707	1183	1660	2137	2613	3089	3566	4042	4519	4995
39	735	1231	1726	2222	2717	3213	3709	4205	4701	5197
40	764	1279	1793	2308	2822	3338	3853	4369	4884	5400
41	794	1328	1861	2395	2928	3464	4000	4534	5070	5606
42	823	1377	1929	2481	3035	3590	4145	4701	5256	5812
43	853	1426	1998	2570	3143	3718	4295	4870	5445	6020
44	882	1475	2067	2660	3252	3847	4443	5038	5633	6229
45	912	1525	2137	2750	3362	3977	4593	5208	5824	6438
46	942	1575	2207	2840	3472	4107	4743	5378	6013	6649

Whole stand models

- Whole stand models are those growth and yield models in which the basic units of modelling are stand parameters such as basal area, stocking, stand volume and parameters characterizing the diameter distribution.

- They require relatively little information to simulate the growth of a stand, but consequently yield rather general information about the future stand.

Example, Growth and yield table

- 15 years Chinese fir stand, H_ave 12.0 m，D_ave 12 cm，V=150 m3/ha，
- Try to predict its H_ave, D_ave, V at age 30.

- H_ave = 17.0 * (12/12.4) = 16.5 m
- D_ave = 23.0 cm * (12/14) = 19.78 cm
- V = 351 * (150/165) = 319.4 m3/ha

Stand yield table, Chinese fir, SC II

Age, yr	H, m	D, cm	N, n/ha	V, m3/ha	V_ave, m3/ha/yr	ΔV, m3/ha/yr	V%, %
5	4.0	5.0	2400				
10	8.8	10.0	2100	75	7.5	15	
15	12.4	14.0	2100	165	11.0	18	15.0
20	14.5	18.0	1500	240	12.0	15	7.41
25	16.0	21.0	1500	300	12.0	12	4.4
30	17.0	23.0	1500	351	11.7	10.2	3.13

Growth tables and percentages

- Yield tables => require data: stand age,
  => uneven-aged ?
- Growth tables => volume, density, height, average diameter, and crown class, other than age
- Growth percentages => expected growth expressed as a percentage rather than in absolute terms.
- Short-term or long-term?
- Other stands?

Growth and yield equations

- Delta_Vn = V2 – V1 + Vc
- ln(Vt) = 1.34 + 0.394lnG0 + 0.346lnt + 0.00275SC(t^-1)
- Delta_v = beta_0 + beta_1(d) + beta_2(d)^2
- Delta_G = beta_0 + beta_1*G(t^-1)+(beta_2+beta_3(t^-1)+beta_4*SI)*G^2
- ln(delta_G) = -3.071+1.094lnG+0.007402G*SF-0.2258G

Lab exercise, whole stand model

- Suppose H, D, G growth is a function of t,
- Select the theoretical equation, Schumacher formula, for regression analysis.
- Constructing H-t, D-t, G-t regression model.

Pinus tabulaeformis, SC II

No.	t	H	D	G	No.	t	H	D	G
1	60	19.2	29.0	26.0	21	58	16.5	22.5	23.8
2	36	11.6	21.4	20.6	22	34	12.8	19.1	24.4
3	37	13.9	14.1	26.2	23	25	11.1	14.0	20.0
4	59	16.4	21.8	23.3	24	46	16.1	19.6	26.1
5	62	15.0	21.1	27.4	25	45	15.3	18.9	21.6
6	20	11.4	14.1	18.2	26	56	16.9	23.9	28.7
7	63	17.3	27.8	27.9	27	57	16.1	21.6	27.5
8	48	14.1	22.9	21.1	28	20	9.3	11.3	12.9
9	46	15.6	22.0	25.5	29	46	14.9	24.1	24.0
10	67	18.5	23.8	22.5	30	53	13.8	20.6	27.2
11	72	19.1	32.5	30.8	31	25	12.2	19.2	15.0
12	42	15.2	21.6	20.5	32	33	12.6	18.0	17.4
13	26	9.9	12.2	16.6	33	43	15.1	17.5	21.6
14	38	12.4	15.2	23.4	34	35	12.7	21.6	27.3
15	24	8.0	8.2	16.7	35	43	15.3	23.9	23.0
16	54	16.8	23.8	22.1	36	66	17.2	23.2	23.1
17	45	13.4	19.7	21.9	37	31	12.2	14.4	19.2
18	58	16.5	26.4	27.9	38	54	13.7	19.9	26.8
19	41	13.8	18.3	22.9	39	64	16.7	23.3	26.9
20	40	13.0	16.6	27.4	40	31	12.7	16.3	23.3

The whole stand model

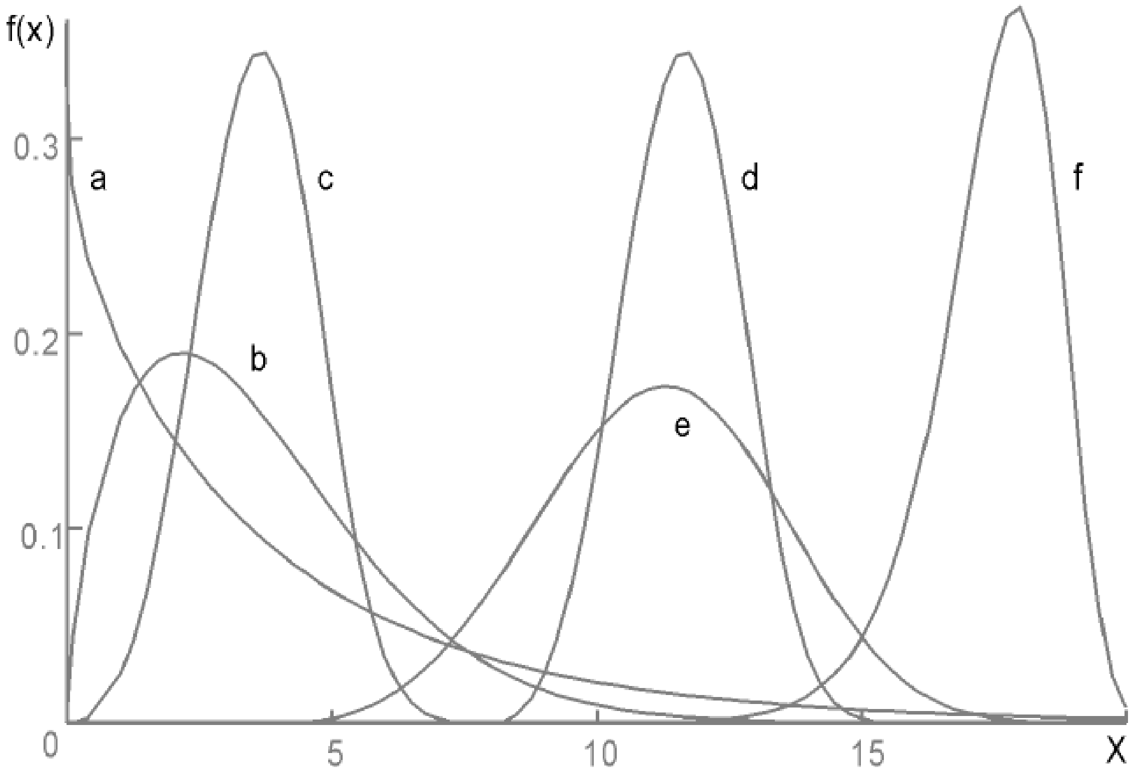
- H = 22.841*exp(-19.284/t)
- D = 36.149*exp(-24.507/t)
- G = 33.767*exp(-15.443/t)

- N = 40000*(G/(pi*D^2))
- V_pinus_t = 0.33123*(D^2)*H+0.00805*D*H-0.00274*D^2+0.00002
- V_pinus_t = (H+3)G*f_pinus_t, f_pinus_t = 0.41

The yield table

年龄 (a)	树高 (m)	平均胸径 (cm)	林分密度 (株/ha)	林分断面积 (m^2/ha)	单木材积 (m^3)	林分蓄积 (m^3)
15	6. 3	7. 1	3086	12. 1	0. 0140	43. 2
20	8. 7	10. 6	1764	15. 6	0. 0399	70. 4
25	10. 6	13. 6	1261	18. 2	0. 0759	95. 6
30	12. 0	16. 0	1008	20. 2	0. 1169	117. 8
35	13. 2	17. 9	859	21. 7	0. 1594	136. 9
40	14. 1	19. 6	762	23. 0	0. 2014	153. 5
45	14. 9	21. 0	694	24. 0	0. 2417	167. 8
50	15. 5	22. 1	644	24. 8	0. 2798	180. 3
55	16. 1	23. 2	606	25. 5	0. 3154	191. 2
60	16. 6	24. 0	576	26. 1	0. 3486	200. 8
65	17. 0	24. 8	552	26. 6	0. 3794	209. 4

Whole-stand distribution models



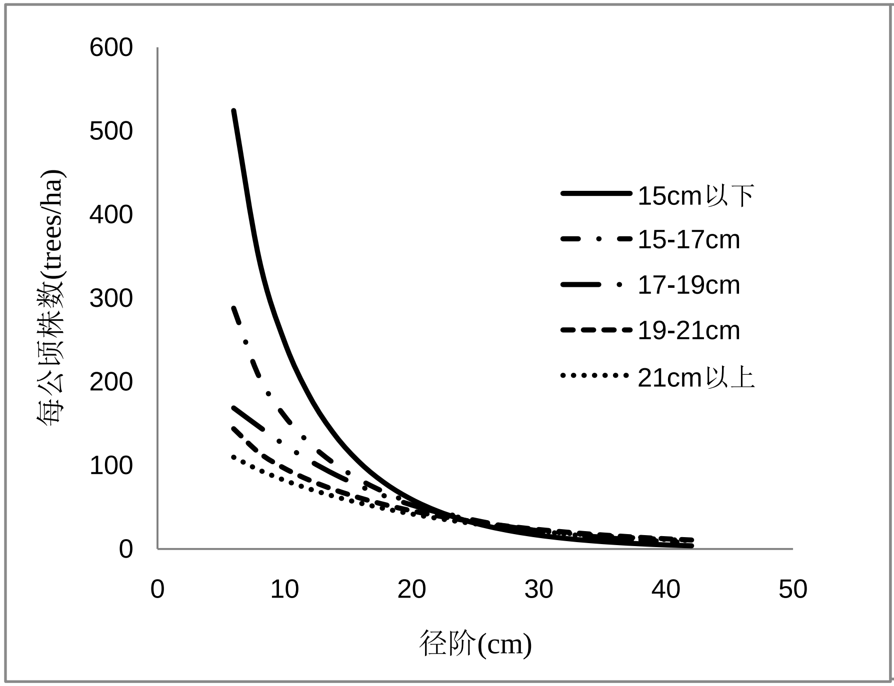
$$f(x) = \frac{\beta}{\alpha}\left[\frac{x-\gamma}{\alpha}\right]^{\beta-1} \exp\left[-\left(\frac{x-\gamma}{\alpha}\right)^{\beta}\right]$$

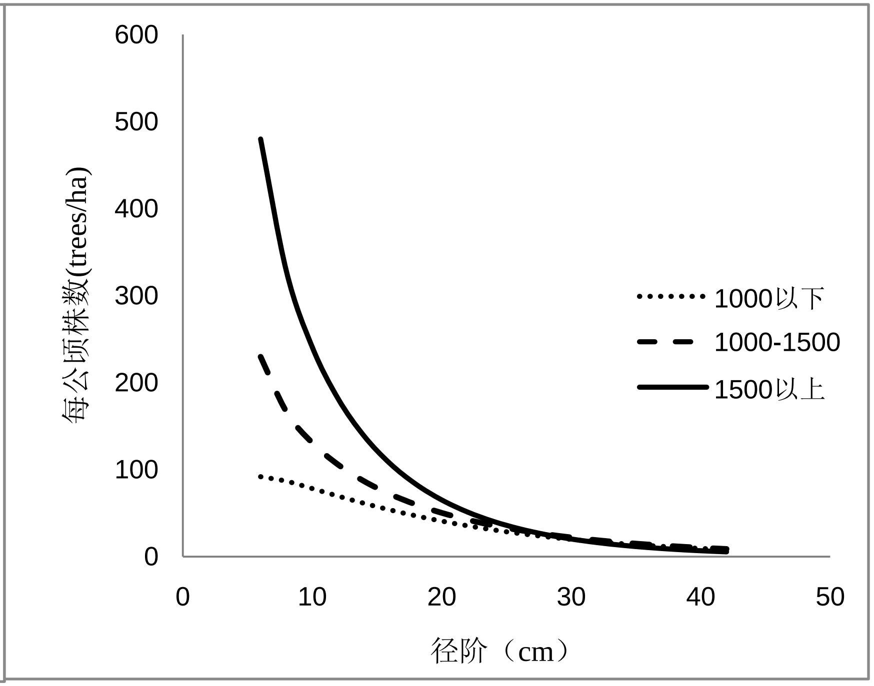
Fig. 2.3. Possible diameter distributions generated by the Weibull p.d.f. (Eqn 2.9), showing the influence of each parameter (Table 2.1) on the shape of the distribution.

Weibull distributions

Example	alpha	beta	gamma
a	4	0.95	0
b	4	1.6	0
c	4	3.6	0
d	4	3.6	8
e	8	3.6	4
f	18	18.0	0

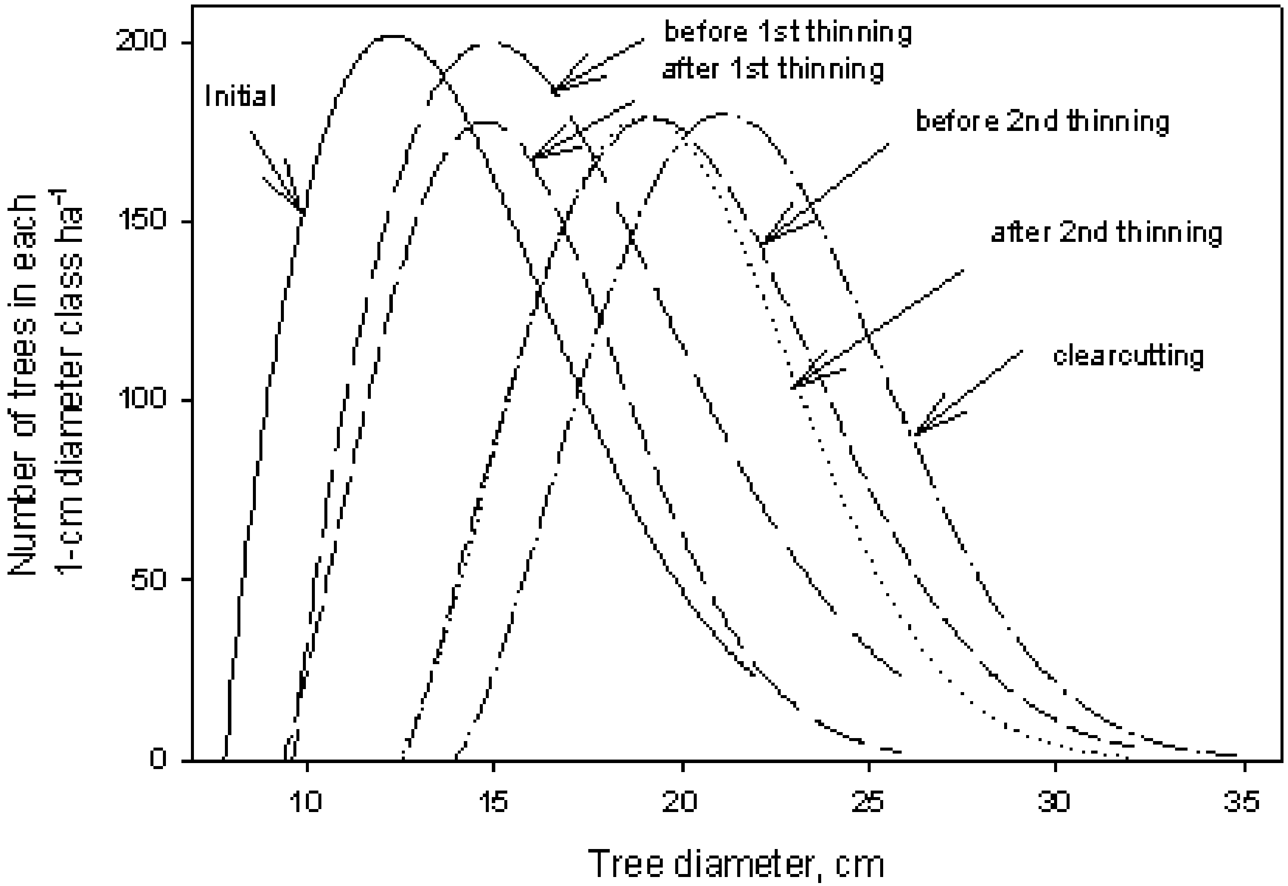
Diameter distribution by D_ave and #tree at stand level





Stand structure development (Cao, 2003)

- Optimal solutions of plot 61 at 3% rate of interest, fit to 4-parameter Weibull distributions.



Optimization of Weibull parameters

Objective function:

$$\min Z = \frac{\sum(D_i - \widehat{D}_i)^2}{\widehat{\sigma}_D^2} + \frac{\sum(B_i - \widehat{B}_i)^2}{\widehat{\sigma}_B^2} + \frac{\sum(N_i - \widehat{N}_i)^2}{\widehat{\sigma}_N^2} \quad (3)$$

where D is diameter, B is basal area, N is density, $\hat{\sigma}$ is estimated value of the standard deviation.

- stand attributes then can be predicted by m tree species and n diameter classes with stand-level data

  s.t. E_Dg <= 0.25

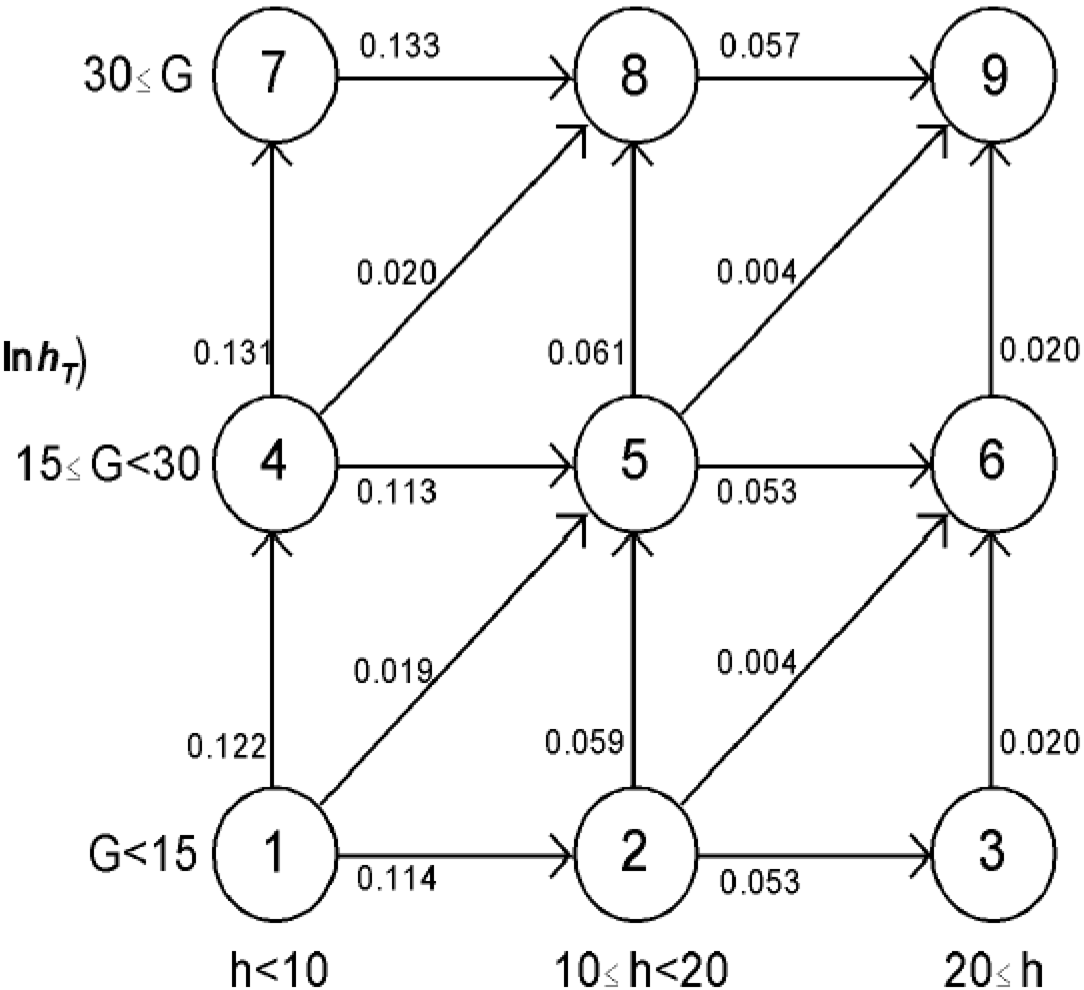
  E_BA_i <= 0.1, for all i = 1, ..., m

  d_ij >= 0, for all i =1, ..., m; j = 1, ..., n

Whole-stand transition matrices

$$dh_T/dt = 0.0752\, h_T (3.59 - \ln h_T)^2$$

$$dG/dt = 0.0752\, G\, (4.08 - \ln G)\, (3.59 - \ln h_T)$$



Stand table projection

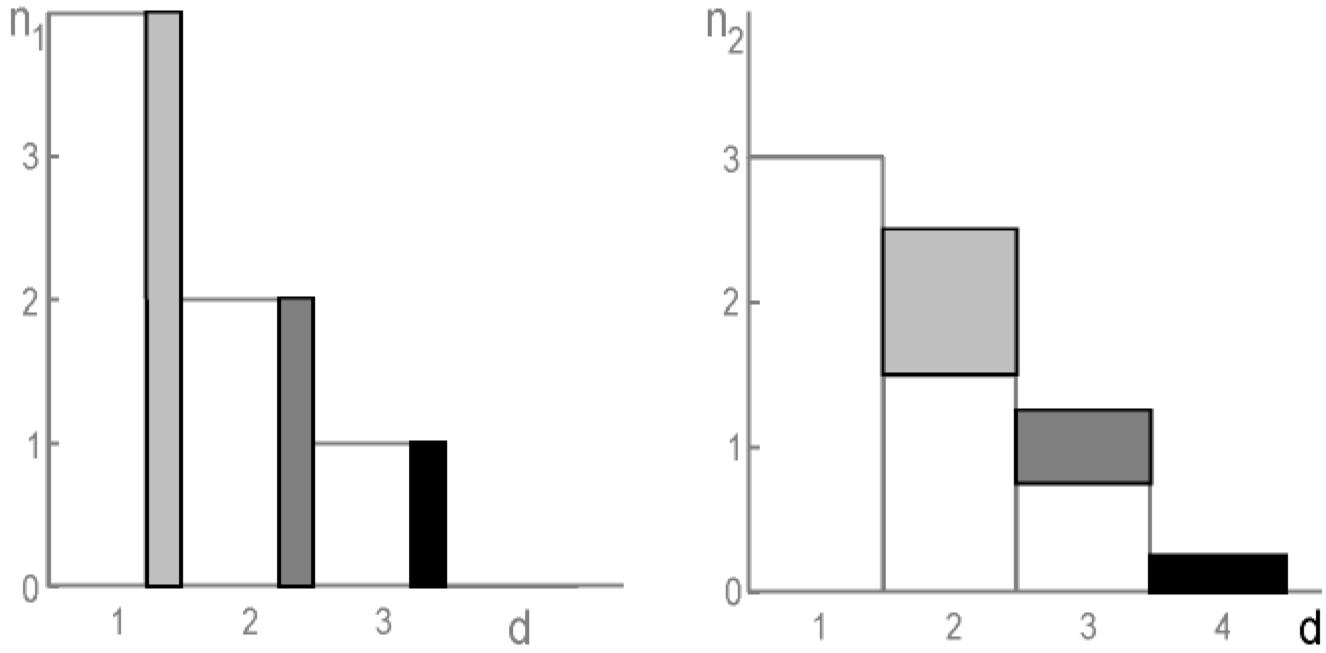


Fig. 3.1. Stand table projection with movement ratio 0.25, so that 25% of each class moves up to the next class.

Transition Matrices

- Consider a hypothetical system S, with n distinct states $S_1, S_2, \ldots, S_n$. If the system starts in state S_i, then in a single time interval, it has probability P_{ij} of moving to state S_j.

- Provided that these P_{ij} depend only on the current state S_i and not on any historic events,

- These probabilities can be expressed in a square matrix, termed the transition probability matrix or stationary Markov chain.

Size class model: diameter distribution

size class	DBH, cm	N, trees/ha	d_ave, cm	ba_ave, m2	BA, m2/ha
1	[10,20)	840	15	0.02	14.8
2	[20,35)	234	27	0.06	13.4
3	[35,)	14	40	0.13	1.8
tot		1088			30

Transition matrix

- $y_{1,t+1} = a_1 * y_{1t} + R_t$
- $y_{2,t+1} = b_1 * y_{1t} + a_2 * y_{2t}$
- $y_{3,t+1} = b_2 * y_{2t} + a_3 * y_{3t}$

- y_{it}, number of trees per ha in size class i at time t
- a_i, ratio of trees remains in class i during time t to t+1
- b_i, ratio of trees moves to class i+1 during time t to t+1
- $1-a_i-b_i$, mortality of trees (%) in class i during time t to t+1

Size class transition and ingrowth

- Known:
  - $R_t = 109 - 9.7G_t + 0.3N_t$
  - $N_t = y_{1t} + y_{2t} + y_{3t}$
  - $G_t = 0.02y_{1t} + 0.06y_{2t} + 0.13y_{3t}$
- Solutions:
  - R_t, recruits during time t to t+1
  - $R_t = 109 - 9.7(0.02y_{1t} + 0.06y_{2t} + 0.13y_{3t}) + 0.3(y_{1t} + y_{2t} + y_{3t})$
  - $R_t = 109+0.106y_{1t}-0.282y_{2t}-0.961y_{3t}$
  - $y_{1,t+1} = 109+0.906y_{1t}-0.282y_{2t}-0.961y_{3t}$
  - $y_{2,t+1} = 0.04y_{1t} + 0.9y_{2t}$
  - $y_{3,t+1} = 0.02y_{2t} + 0.9y_{3t}$

Size class model: transition probability

size class	a_i	b_i	1-a_i-b_i		
1	0.8	0.04	0.16		0.96
2	0.9	0.02	0.08		0.98
3	0.9	0	0.1		1

	size class				
Year	i_1	i_2	i_3	BA	V
0	840	234	14	32.66	299
5	790.598	244.2	17.28	32.71036	297.4996
10	739.8113	251.4039	20.436	32.53714	294.1002
15	688.7341	255.856	23.42048	32.1707	289.0953
20	638.3347	257.8197	26.19555	31.6413	282.7724
25	589.4521	257.5712	28.73239	30.97852	275.4084
30	542.7967	255.3921	31.01057	30.21084	267.266
35	498.9521	251.5648	33.01736	29.36519	258.5902
40	458.3796	246.3664	34.74692	28.46668	249.606
45	421.4248	240.0649	36.19956	27.53834	240.517
50	388.3248	232.9154	37.3809	26.60094	231.5036
55	359.2171	225.1569	38.30112	25.6729	222.723
60	334.1491	217.0099	38.97414	24.77021	214.3089
65	313.0881	208.6749	39.41693	23.90645	206.372
70	295.9319	200.3309	39.64873	23.09283	199.0006
75	282.5185	192.1351	39.69048	22.33824	192.2617
80	272.6371	184.2223	39.56413	21.64942	186.2027
85	266.0374	176.7056	39.29216	21.03106	180.8524
90	262.4392	169.6765	38.89706	20.48599	176.2231
95	261.541	163.2064	38.40088	20.01532	172.3123
100	263.0287	157.3474	37.82492	19.61866	169.1044

Individual-tree models

- delta(DBH)=f(DBH,BA,SI,...)
- delta(HT)=f(HT,BA,SI,...)
- P_s=f(DBH,HT,BA,SI,...)
- R=f(BA,SI,...)

- Tree Records

- Spp, DBH, HT, n

$\downarrow$

- Spp,DBH+delta(DBH),HT+delta(HT), p_s*n

Tree list models

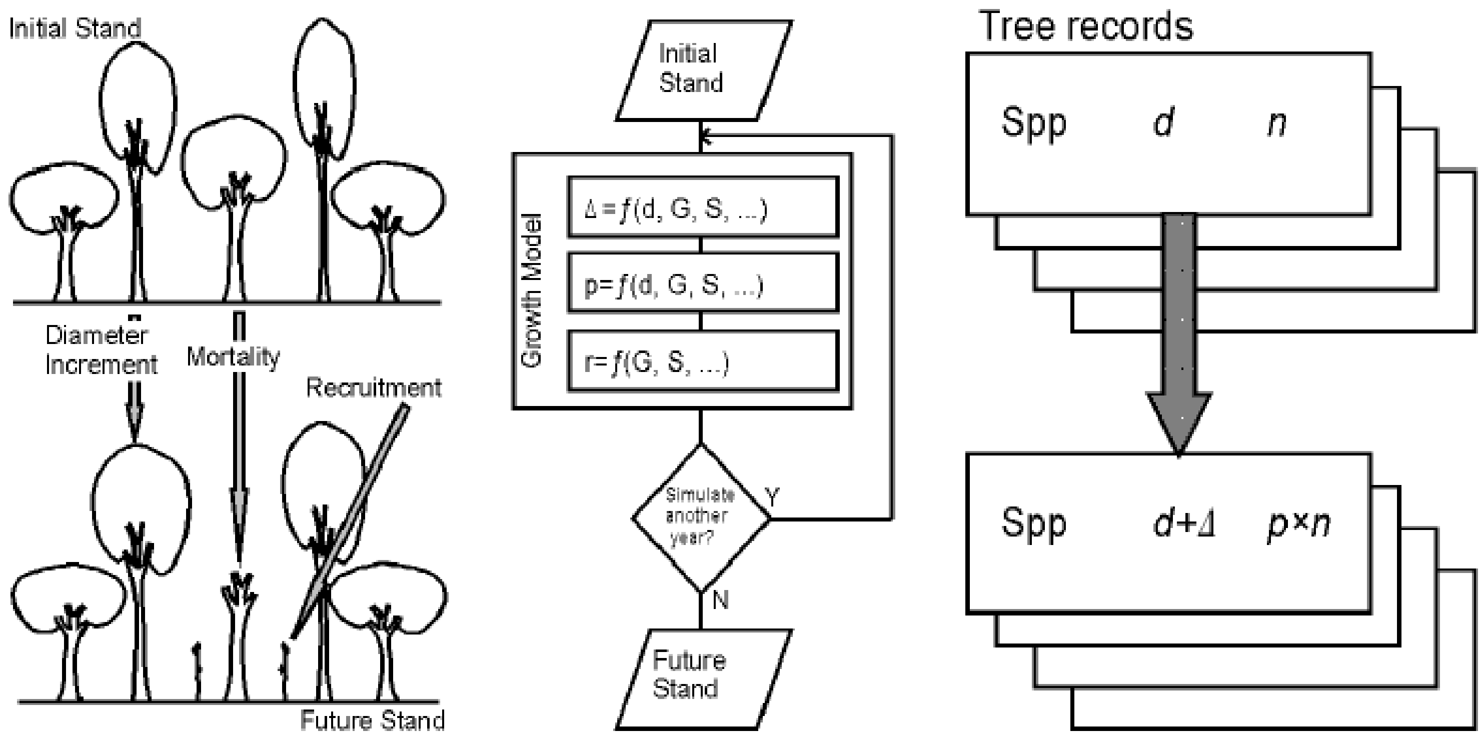


Fig. 4.3. Tree records representing a forest stand. Growth is modelled by incrementing the diameters in each record ($d+\Delta$) and mortality is accommodated by reducing expansion factors ($p \times n$).

Basal area increment model

- $\ln(BAI) = a + b*SIZE + c*COMP + s*SITE$

- $b*SIZE = b\_1*\ln(DBH) + b\_2*DBH^2 + b\_3*\ln(CR)$
- $c*COMP = c\_1*BAL + c\_2*CCF$
- $s*SITE = d*SITE\_1 + e*SITE\_2 + f*SITE\_3$

- $d*SITE\_1 = d\_1(ELEV-d\_2)^2 + d\_3*SL^2 + d\_4*SL*\sin(AZ) + d\_5*SL*\cos(AZ)$
- $e*SITE\_2 = e\_1*HF + e\_2*HH$
- $f*SITE\_3 = f\_1*DP + f\_2*M + f\_3*P + sigma(f\_4i*S\_i) + sigma(f\_5i*V\_i) + sigma(f\_6i*GD\_i)$

Competition

- Plants modify their environment as they grow, reducing the resources available for other plants.
- The primary mechanism of competition is spatial interaction.
- Plant death due to competition is a delayed reaction to the growth reduction following resource depletion.
- Plants adjust to environmental change, responding to competition and altering the nature of the competition.
- There are species differences in the competition process.

Competition index

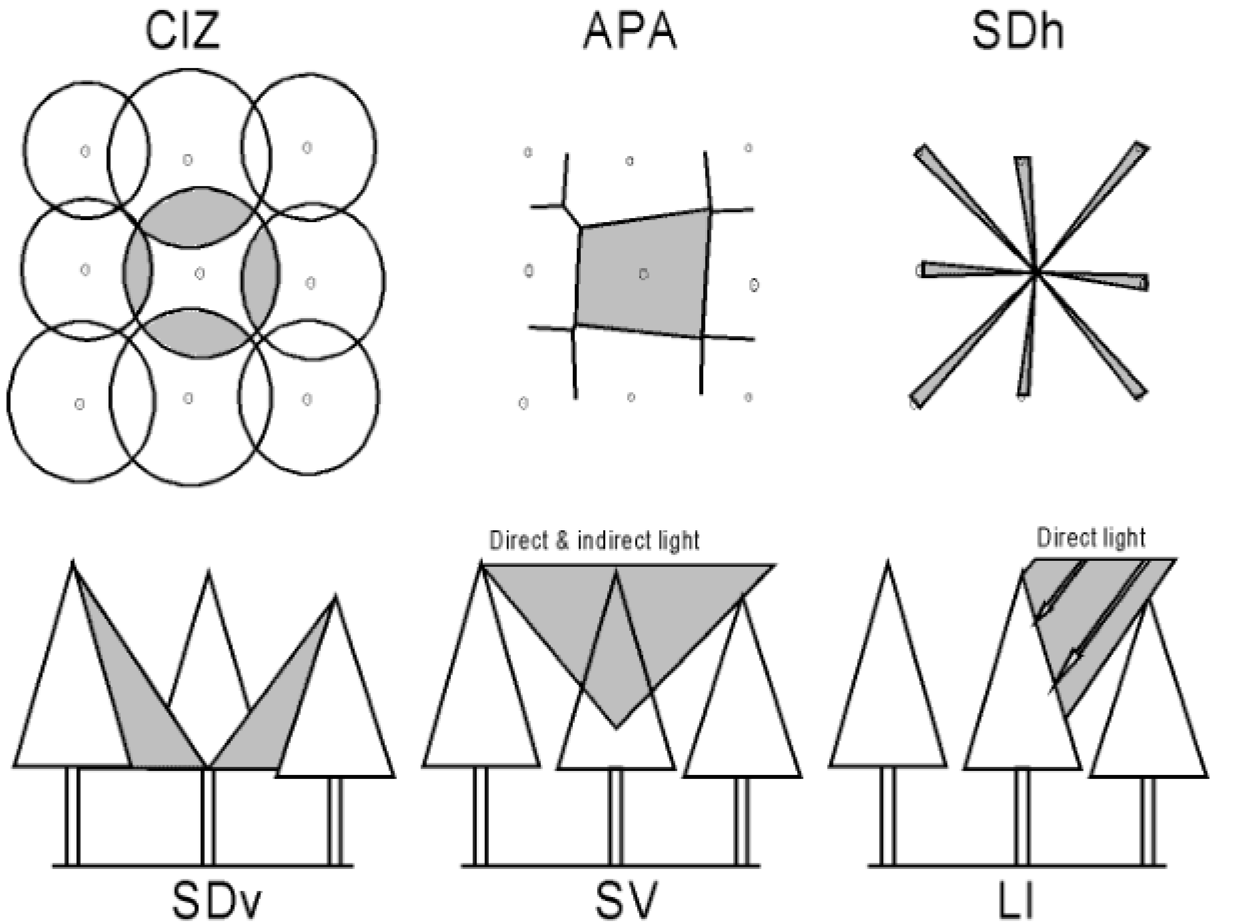


Fig. 4.1. Competition indices include the competitive influence zone (CIZ), area potentially available (APA), horizontal or vertical size–distance (SDh & SDv), sky view (SV) and light interception (LI) approaches.

Process-based models

- Light interception

  $LI_N=LI_0*\exp(1-(k*LAI/N))$

- Photosynthesis = f(carboxylation, ribulose) = f(CO2, leaf nutrition, leaf temperature)
- Stomatal conductance

  $g_s = g_{max}*f_1(APAR)*f_2(T)*f_3(VPD)*f_4(C_i)*f_5(\theta)$

- Respiration
- Carbon allocation
- Soil water and nutrients

A carbon balance model CROBAS

$$G = \sum_i G_i = Y^{-1}(P-R)$$

$$P = P_0(1-e^{-kl})/N$$

$$R_m = r_1(W_f + W_r) + r_2(W_s + W_b + W_t)$$

Source: Mäkelä (1997)

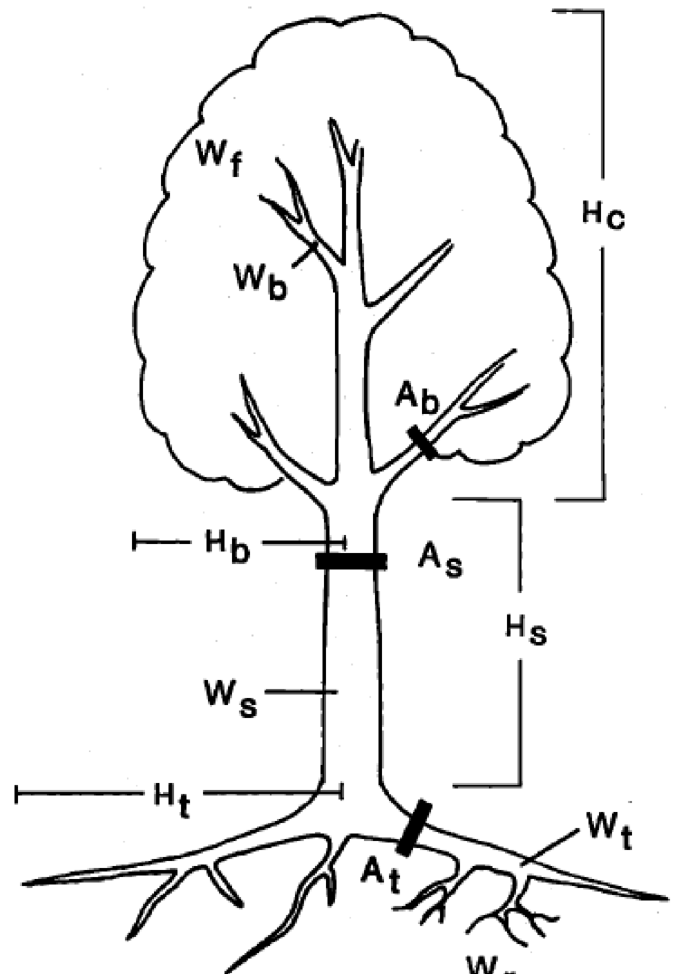
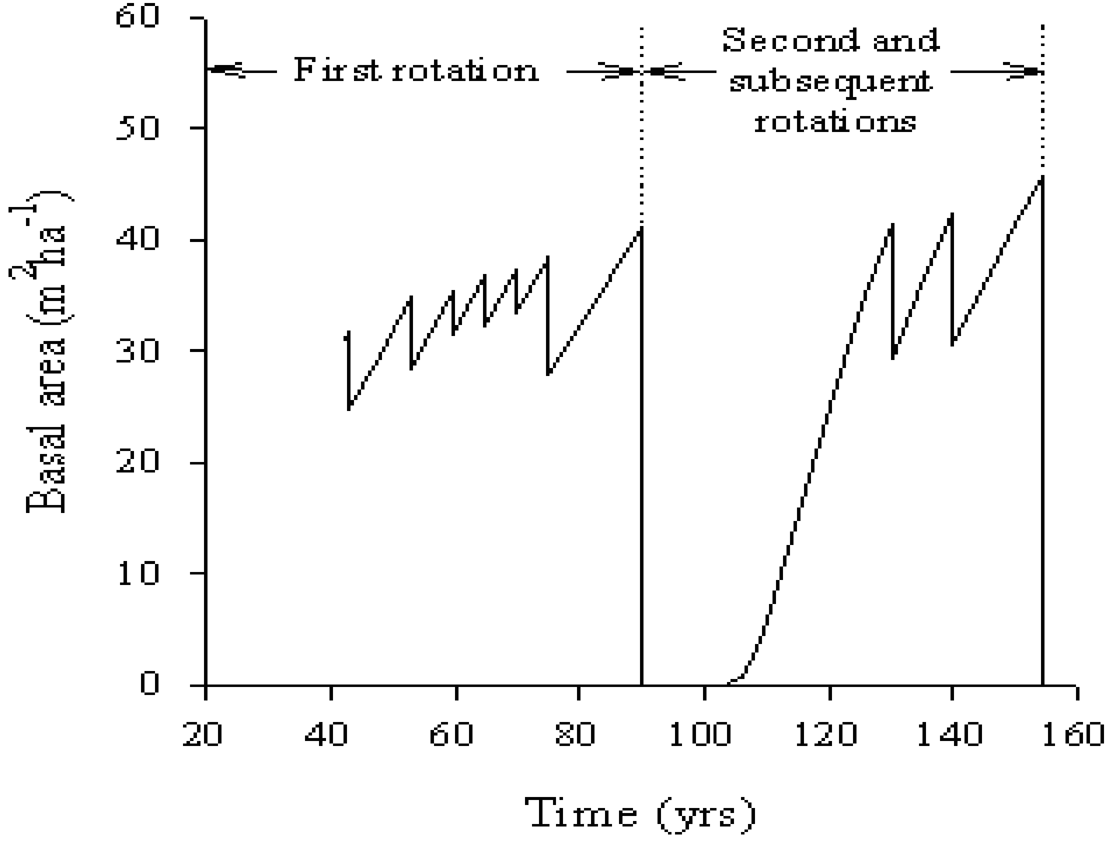


Figure 1. Schematic representation of tree structure as applied in the model.

Applications: even-aged stands

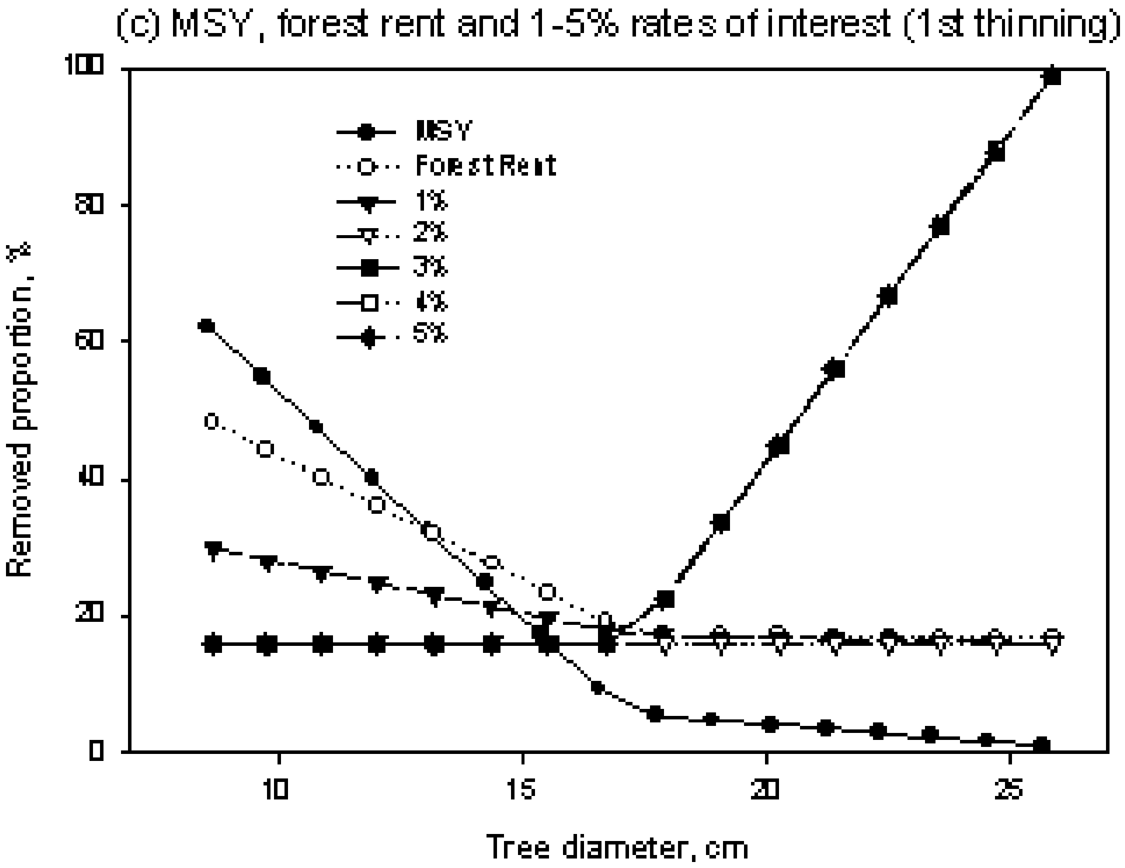
- Characteristics of even-aged stands
- Predicting growth and yield for even-aged
- Assessing forest resources for even-aged stands
- Assessing forested land for even-aged stands

Even-aged management



Source: Cao et al. (2006), Fig. 1

Type of thinning (Cao, 2003)



- Thinning from below was more profitable with interest rates less than 2%, whereas thinning from above was superior with interest rates of 3-5%.

Even-aged forest stand

- Regeneration
  - Natural vs. artificial regenerated
- Plantations
  - Sawlog or pulpwood
  - Energy wood
- Tree specises
  - Conifer vs. broadleaf
  - Ever-green vs. deciduous
- Stand structure
  - Diameter distribution
  - Spatial distribution

Predicting even-aged stand growth

- Empirical vs. process-based
- Whole stand vs. individual-tree
- Site quality evaluation
- Competition
- Tree height growth
- Diameter growth
- Basal area growth
- Mortality and self-thinning
- Ingrowth
- Thinning response

Pine-oak forests

- Forests dominated by *Pinus tabuliformis*, *Pinus armandii* and *Quercus aliena* occupy the major area of Qinling Mountains,

- Accurate prediction of the growth and yield for these species has been a problem for many years, due to the diversity in composition and structure of pine-oak forests.

- Generally, forest growth modeling needs a large number of continuous observed data, which is unavailable in Qinling.

A three-stage modeling approach

- The purpose is to develop and evaluate diameter increment models for pine-oak forests in Qinling Mountains with different approaches

- based on temporary plots,
- combining the analysis of increment cores
- and the exploration of diameter structure dynamics.

Diameter increment model

- Age-independent
  - Uneven-aged forest
  - Mixed forest

- Distance-independent
  - Practicability
  - Simplicity
  - Accuracy

Diameter increment model

Model form:

$$\Delta d_i = f\left(D_i, V_i, C_i, S_i\right) \qquad (1)$$

where Δd_i is the 5-year diameter increment of tree i; D_i, V_i, C_i, S_i represents tree size factor, historical vigor factor, competition factor and site productivity factor, respectively.

➢Tree size	DBH, DBH^2, ln(DBH), 1/DBH, etc.
➢Historical vigor	Crown ratio, canopy closure, LAI, etc.
➢Competition	BA, BAL, CCF, etc.
➢Site productivity	Site index, Forest type, Elevation, etc.

Stand diameter distribution

3-parameter Weibull distribution：

$$f(x) = \left(\frac{c}{b}\right)\left(\frac{x-a}{b}\right)^{c-1} \exp\left[-\left(\frac{x-a}{b}\right)^{c}\right]; \quad x \geq a, b > 0, c > 0$$

where *x* is tree diameter; *a* is the location parameter; *b* is the scale parameter; *c* is the shape parameter.

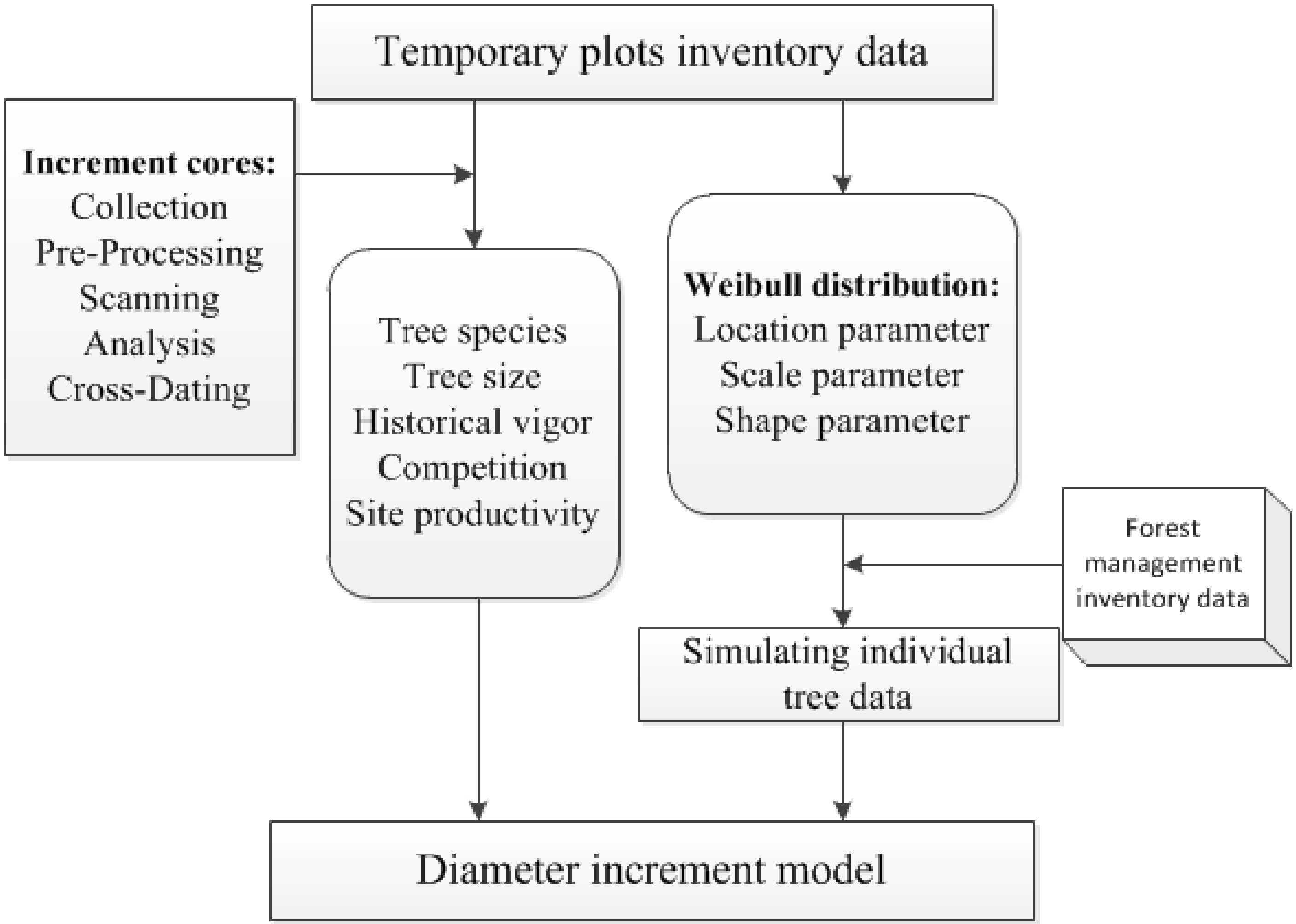
Optimizing diameter distribution

Objective function:

$$\min Z = \frac{\sum(D_i - \hat{D}_i)^2}{\hat{\sigma}_D^2} + \frac{\sum(B_i - \hat{B}_i)^2}{\hat{\sigma}_B^2} + \frac{\sum(N_i - \hat{N}_i)^2}{\hat{\sigma}_N^2} \qquad (3)$$

where D is diameter, B is basal area, N is density, $\hat{\sigma}$ is estimated value of the standard deviation.

Simulated data => computational experiments



Even-aged stand management

- Alternative tree specises
- Planting density
- Silvicultural treatments
- Thinning and rotation
- Timber production
- Biodiversity
- Carbon sequestration
- Landscape
- Risk management

Characteristics of inventory data

Date types and problems for modeling:

1. Temporary plots (some with increment cores)

    Lacking tree-level information

2. Permanent plots

    Most permanent plots only contain one-time observations

3. Stem analysis

    Accurate but few

Different type of measurement error comparing with plots data

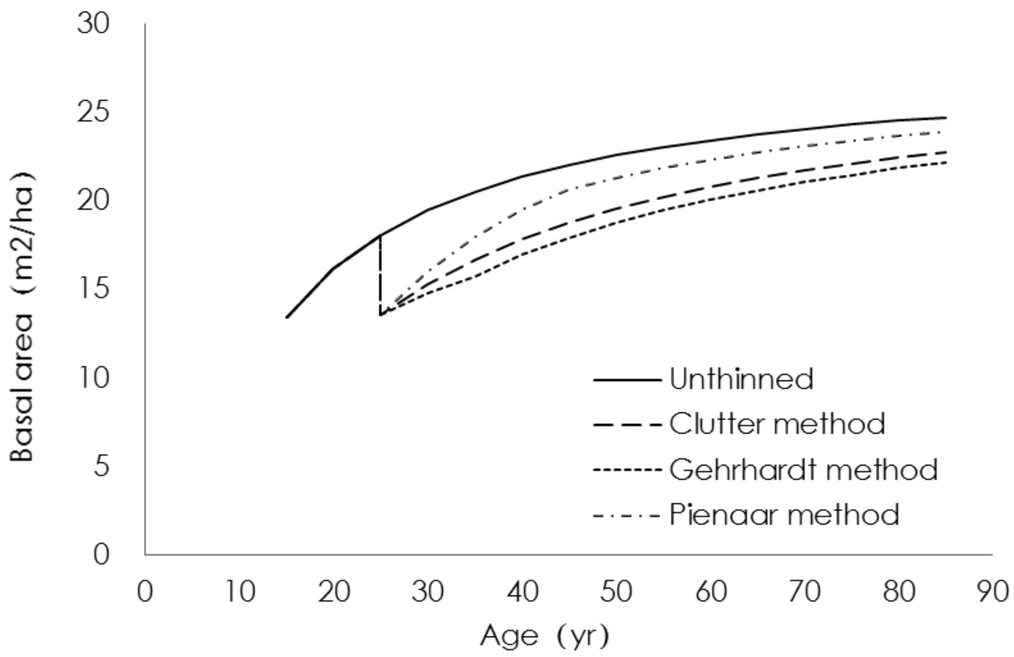
Using Bayesian Calibration

1. Data requirements: small amount of data is acceptable, bridging the gap between models and data.

2. Previous investigation data is good prior information for next experiment, considering that forest inventory is continuous.

3. Bayesian calibration provides a likelihood framework for different types of measurement error.

Bayesian Model Averaging



Each type of model has its advantages and disadvantages on certain situation, especially for thinning modifier. There are different theoretical hypothesis for thinning effects.

Bayesian model averaging is developed as an approach to combine inferences and forecasts from multiple competing modify models. BMA can clearly show the information update process and can also combine predictive distributions from different sources.

A Bayesian framework

1. Uncertainty quantification (UQ)

Predictive uncertainty can be quantified for each model before any parameter calibration has been carried out (prior UQ), and after calibration (posterior UQ).

2. Bayesian calibration (BC)

To estimate the posterior distributions, we use a Markov Chain Monte Carlo (MCMC) algorithm.

3. Bayesian model comparison (BMC)

BMC evaluates models not at one single parameter vector value but takes into account parameter uncertainty.

4. Bayesian model averaging (BMA)

Bayesian model averaging uses the different model probabilities, derived in preceding BMC, to calculate a weighted probability distribution for model outputs.

Tools for Bayesian Calibration

To estimate the posterior distributions, we use a Markov Chain Monte Carlo (MCMC) algorithm.

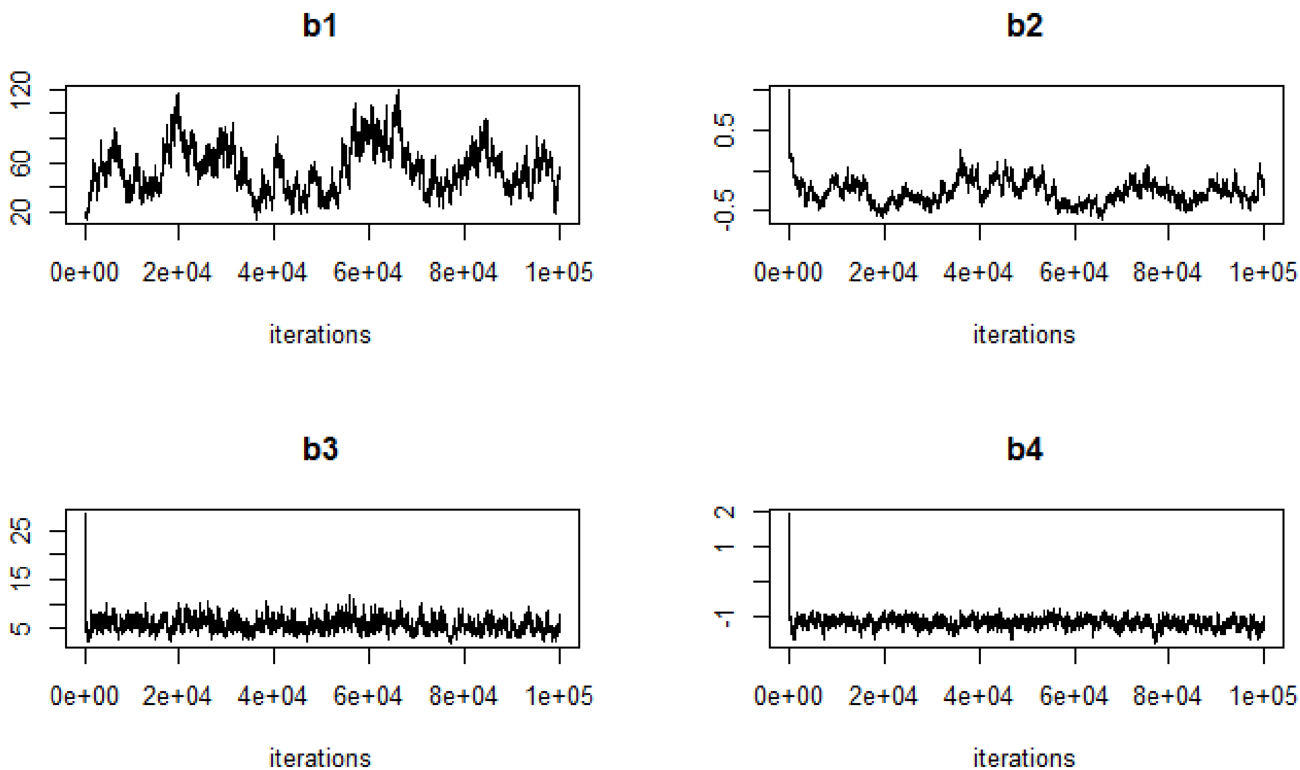
1. WinBUGS
   (Bayesian inference Using Gibbs Sampling)

2. R2WinBUGS
   Package linking R with WinBUGS

3. R programming
   （For example, Gibbs within Metropolis）

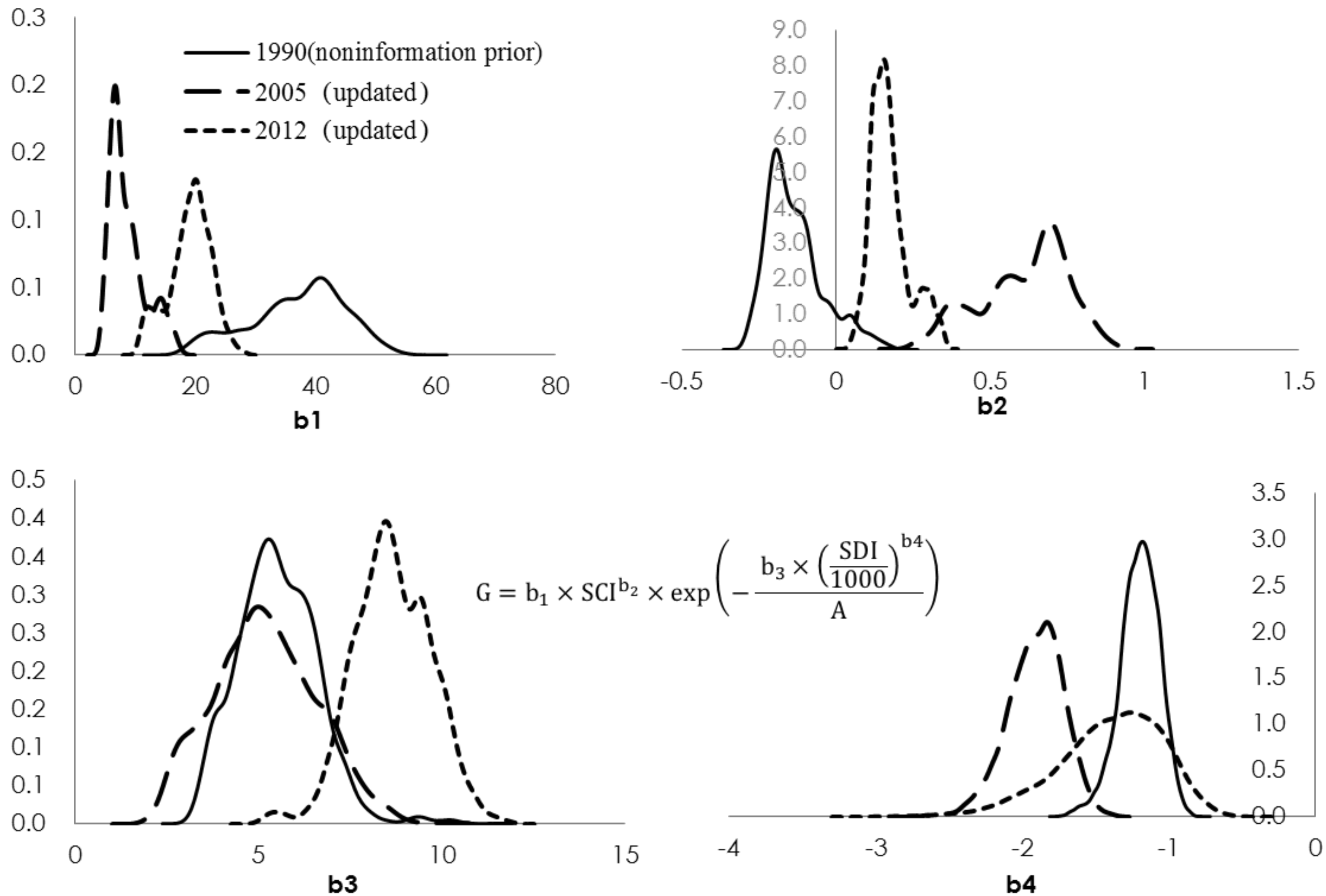
More R packages see CRAN Task View: Bayesian inference (https://cran.r-project.org/web/views/Bayesian.html)

$$G = b_1 \times SCI^{b_2} \times \exp\left(-\frac{b_3 \times \left(\frac{SDI}{1000}\right)^{b4}}{A}\right)$$



To some extent, the parameters were failed to convergence. Maybe that is because the model form is not good enough to describe the data pattern.

New inventory data updated



$$G = b_1 \times SCI^{b_2} \times \exp\left(-\frac{b_3 \times \left(\frac{SDI}{1000}\right)^{b4}}{A}\right)$$

The default is a bandwidth computed from the variance of x, specifically the ‘oversmoothed bandwidth selector’ of Wand and Jones (1995, page 61)

Model linkages

- Biomass equations
- Taper curve equations
- Timber grading module
- Tracheid properties
- Wind throw
- Forest fire
- Forest biotic damages

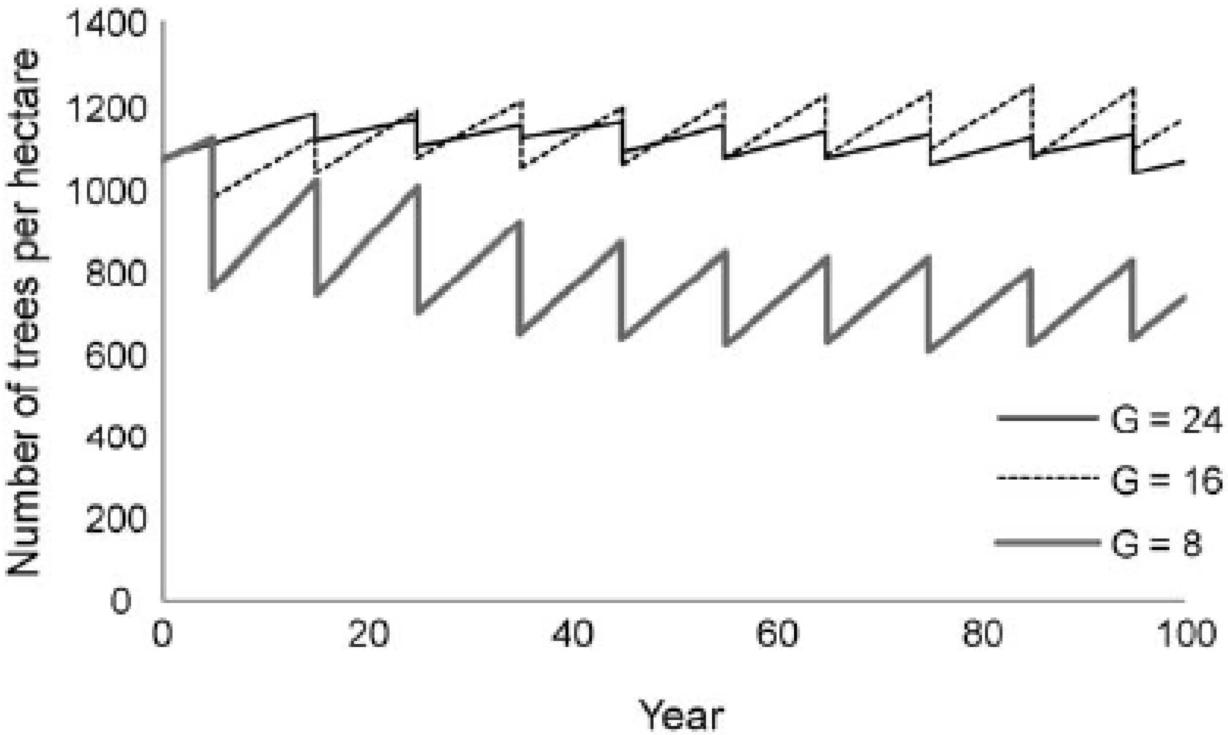
Applications: uneven-aged stands

- Characteristics of uneven-aged stands
- Predicting growth and yield for uneven-aged stands
- Assessing forest resources for uneven-aged stands
- Assessing forested land for even-aged stands

Uneven-aged management

- Uneven-aged
- Mixed forest
  - Boreal forest (conifer dominant)
  - Temperate forest (conifer & broadleaf)
  - Tropical forest (broadleaf dominant)
- Diameter distributiion: reverse "J" sharp
- Regeneration: natural vs. artificial
- Succession: shade tolerant vs. light-demanding

Uneven-aged management



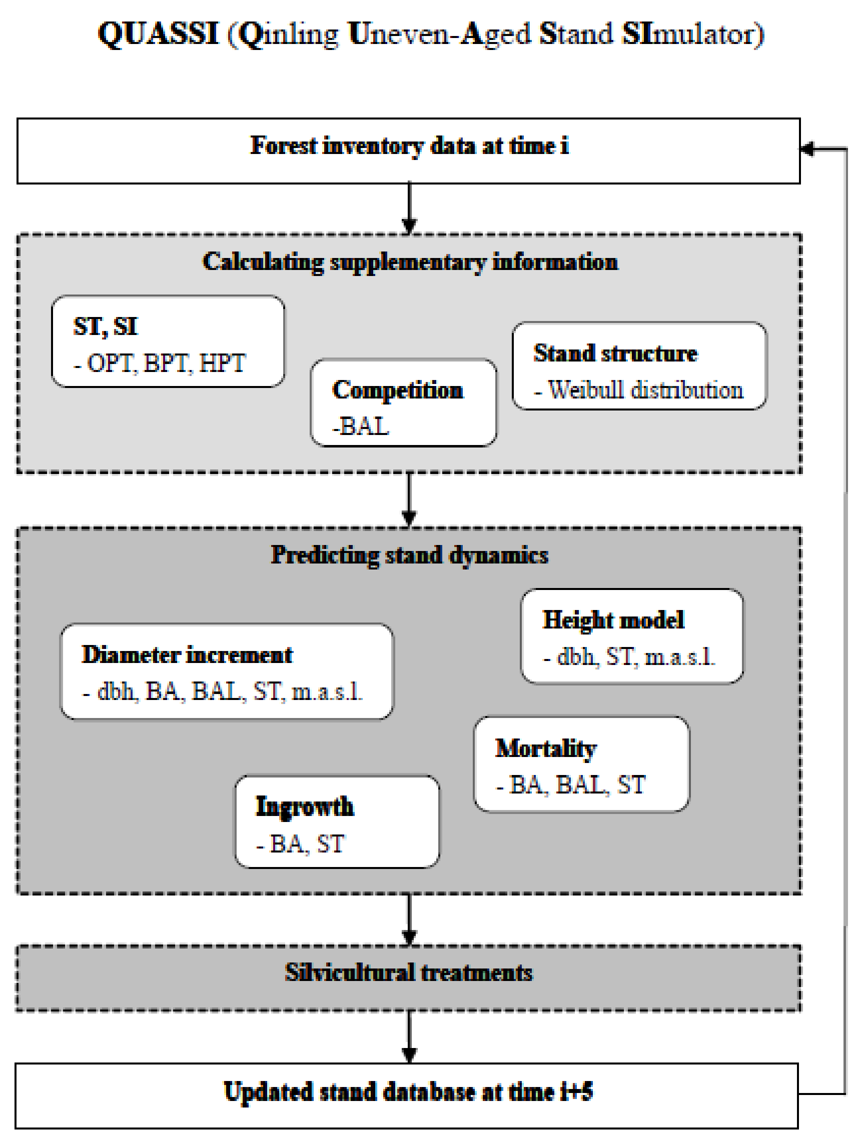
Source: Pukkala et al. (2009), Fig. 8

Predicting uneven-aged stand growth

- Whole stand models: few
- Transition matrix models: e.g., Buongiorno & Michie (1980)
- Tree list models: e.g., Prognosis (Stage, 1972)
- Individual-tree growth models
  - Spatial models (e.g., Ek and Monserud, 1974)
  - Non-spatial models
    - Stochastic, e.g. JABOWA (Botkin et al., 1972)
    - Pukkala et al. (2009)

QUASSI

- A hybrid modelling approach
- Cooperation
  - UH
  - SCU
  - LSU



Modeling uncertainty and risks

- Stochastic vs. deterministic

- Stochastic variables
- Stochastic parameters

- Stochastic simulation
- Stochastic optimzation

Risk and uncertainty in forestry

- Economic risk and uncertainty
  - Timber prices and Interest rate
- Management risk and uncertainty
  - Human interventions
  - Natural disasters
  - Forest health

(Kangas and Kangas 2004, Lohmander 2007)

- Ecological stochastic processes
  - Competition and succession
  - Climate change

Competition and succession

- Methods for modeling mortality
- Methods for modeling recruitment

Mortality

Methods for modeling mortality

- Reineke (1933), Yoda et al. (1963), Curtis (1982)
  - Stand density index, self-thinning line, relative density
- Mäkelä and Hari (1986), Hawkes (2000)
  - Negative carbon balance
- Haenauer et al. (2001)
  - Neural networks, LOGIT model
- Flewelling and Monserud (2002)
  - Logit Model for Proportions, Least squares, Walker-Duncan algorithm, Weighted Least Squares, Maximum Likelihood

Recruitment





Methods for modeling recruitment

- Buongiorno and Michie (1980), Liang (2010)
  - Assume observed reflect long-term ave.
  - Negatively correlated with stand density or BA

- Shifley et al. (1993)
  - Recruitment = f(CCF, diameter threshold)

- Fergusen et al. (1986), Vanclay (1992)
  - 1) Logistic function, 2) conditional function

- Hasenauer and Kindermann (2002)
  - Neural networks
  - Juvenile height growth

Markov process

- Mortality is not a Markov process, however, survival is.

- Survival = 1 – Mortality

  $P_s = 1 - P_m$
- 1-year: $P_{s1} = 1 - P_{m1}$
- n-year: $P_{sn} = (P_{s1})^n = (1 - P_{m1})^n$

  (The Markov property)
- 1-year: $P_{m1} = 1 - (P_{sn})^{(1/n)} = 1 - (1 - P_{mn})^{(1/n)}$
- $P_m = \exp(b'X)/(1 + \exp(b'X))$

(Flewelling and Monserud 2002)

Neural networks

Advantages:

- Neural networks are a viable alternative to the conventional LOGIT approach for estimating tree mortality.
- Artificial neural networks to be more effective at predicting regeneration establishment than regression equations.
- Artificial neural networks were effective predictors when regeneration data were not available.

(Hasenauer et al. 2001, Hasenauer and Merkl 2001, Hasenauer and Kindermann 2002)

Bayesian calibration

- Many parameters
- Many output variables
- Relatively few measured data available

Quantifying uncertainty rather than maximizing fit

- p(theta|D) = cp(D|theta)p(theta)
- p(D|theta) = p(E = D – M(theta))
- theta’ = theta_t + epsilon
- beta = p(theta’|D) / p(theta_t|D)
  = p(D|theta’)p(theta’) / p(D|theta_t)p(theta_t)

(Van Oijen et al. 2005)

Markov Chain Monte Carlo

- Bayesian calibration cannot be performed analytically
- The posterior parameter distribution must be approximated in the form of a representative sample of parameter values

↓

- MCMC does not require advance knowledge of the shape of the posterior distribution
- MCMC: Metropolis-Hastings sampling

(Van Oijen et al. 2005)

Modeling forest regeneration

- Uncertainty of forest regeneration
- Juvenile tree height growth model
- Forest recruitment simulation

Simulation and uncertainty analysis

■ **Methods:**

Prior distribution of parameter can be guessed which come from others research. The data which come from sample plots investigation will be used to update the prior information by the computation of the posterior distribution.

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{\sum_{j=1}^{k} P(A_j)P(B|A_j)} \quad i = 1, 2, \cdots, k$$

$P(A_i)$: Prior distribution

$P(B|A_i)$: likelihood function

$P(A_i|B)$: Posterior distribution

Environment and competition factors

- Environment factors
  - elevation ,
  - slope ,
  - aspect ,
  - the interaction of slope and aspect

- Competition factors
  - stand density ,
  - stand basal area ,
  - stand average diameter (DBH),
  - stand canopy closure

Height growth of juvenile trees

- Explanatory variables:

Tree size: height

Competition factors: stand density , stand basal area , stand average diameter (DBH), larger trees basal area (BAL)

Site factors: site index , slope, the interaction of slope and aspect

- The height growth model of juvenile trees:

$$HTG = \beta_0 + \beta_1 \times SL \times COS(ASP) - \beta_2 \times SL \times SIN(ASP) - \beta_3 \times SL$$
$$+ \beta_4 \times LN(HT) + \beta_5 \times CCF + \beta_6 \times (\frac{BAL}{100}) + \varepsilon_2$$

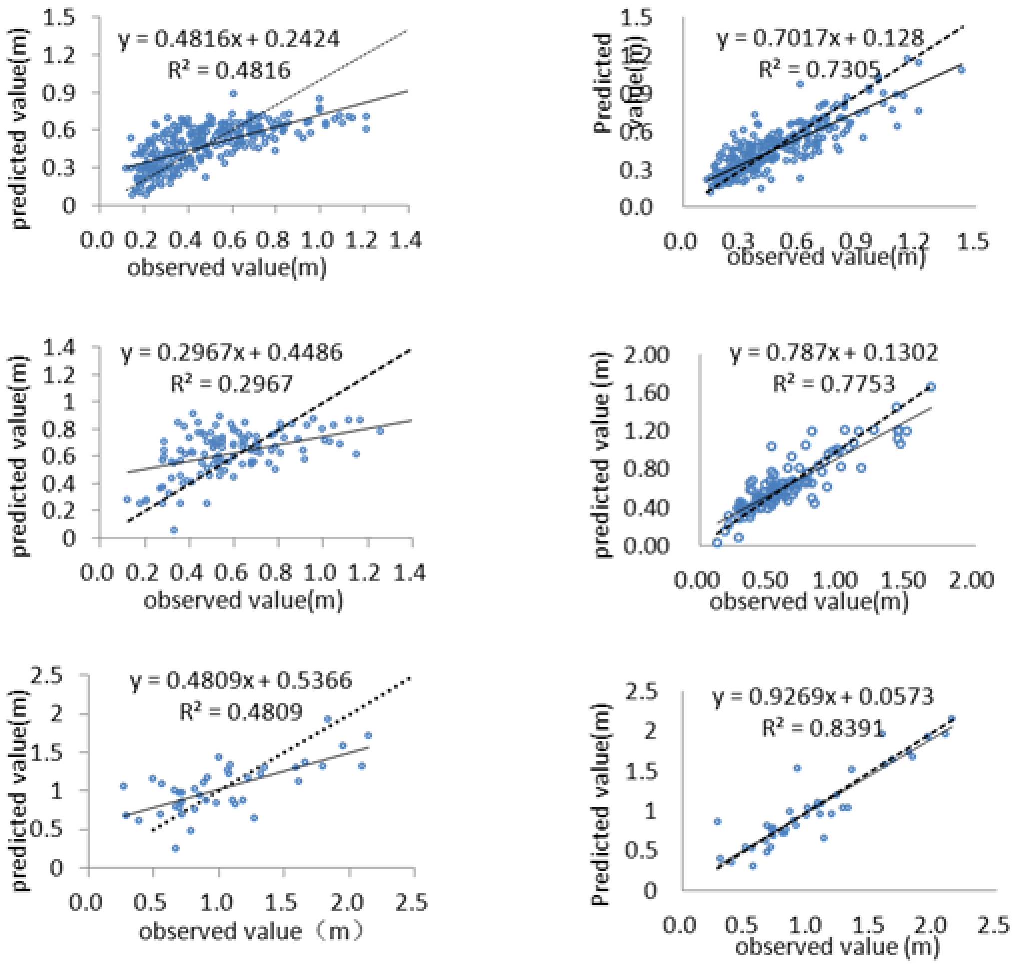
Recruitment models

- Recruitment models predict tree reaching a specified threshold size , usually based on height (breast height) or diameter (5, 7, 9, or 10 cm), a threshold diameter of 5 cm was selected in this study.
- Forest stand recruitment is a complicated stochastic process influenced by several stand characteristics, climatic, geographical factors at a range of spatial and temporal scales.

Recruitment models, con't

- Likelihood function
  - Estimating number of recruitment trees by 5-year juvenile tree height model and 5-year diameter increment model.
- Posterior distribution
  - To update the prior information by the likelihood function, and to obtain Posterior distribution of recruitment model parameter.

ANN vs. Bayesian, 5-yr height growth



Pinus armandii

Pinus tabulaeformis

Pinus tabulaeformis

Bayesian method

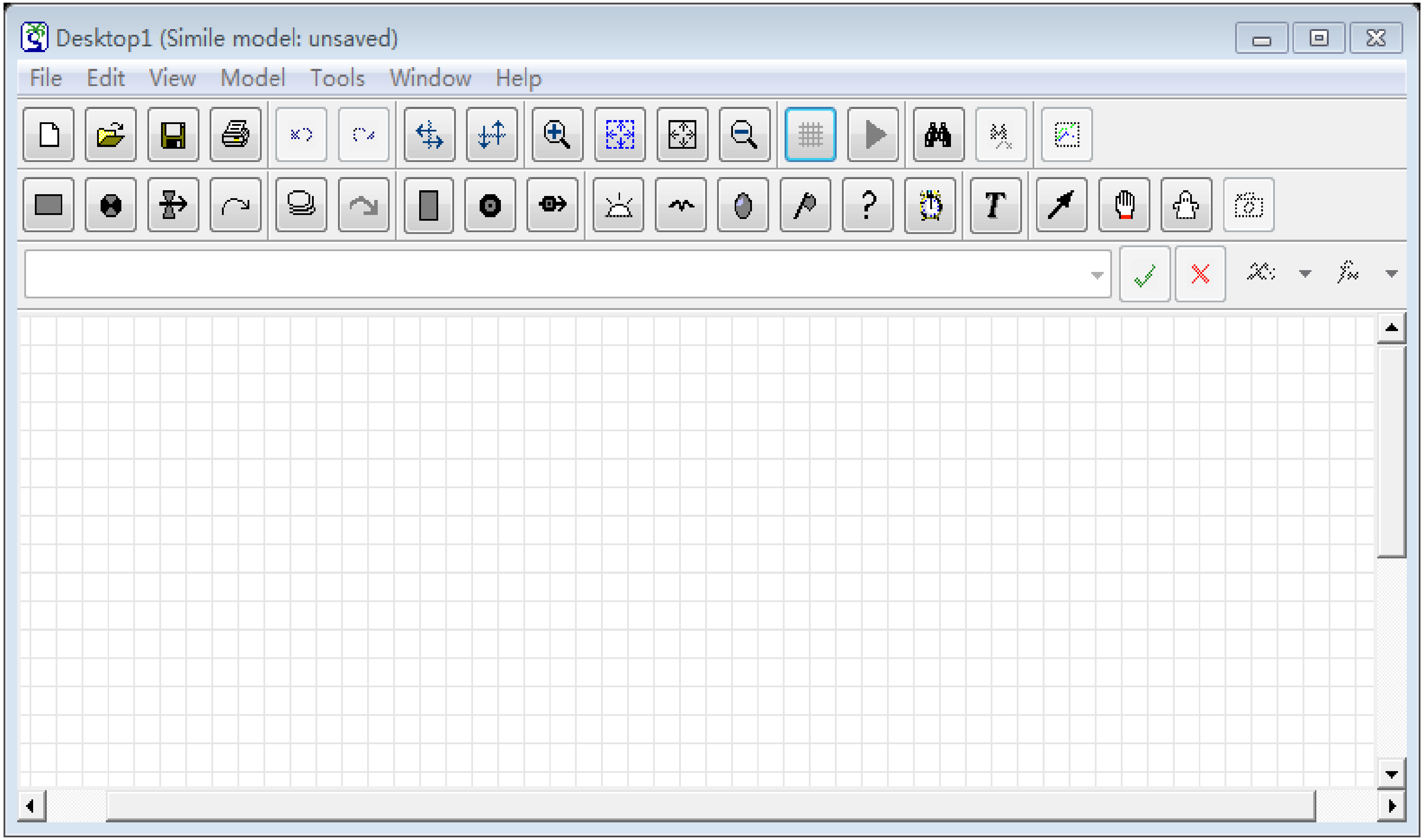
ANN

Model linkages with Simile

- http://www.simulistics.com/

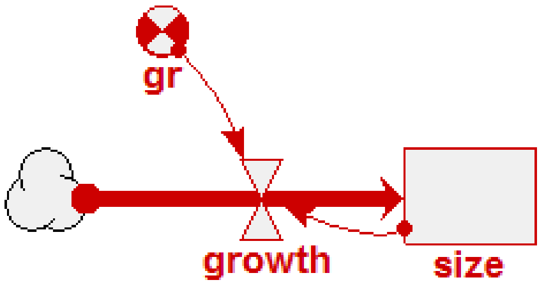
- Stage 1: Modelling the growth of a single tree
- Stage 2: Extending the model to represent a population of trees
- Stage 3: Calculating aggregate information
- Stage 4: Visualising the trees in space

Interface of Simile



Simile step-by-step

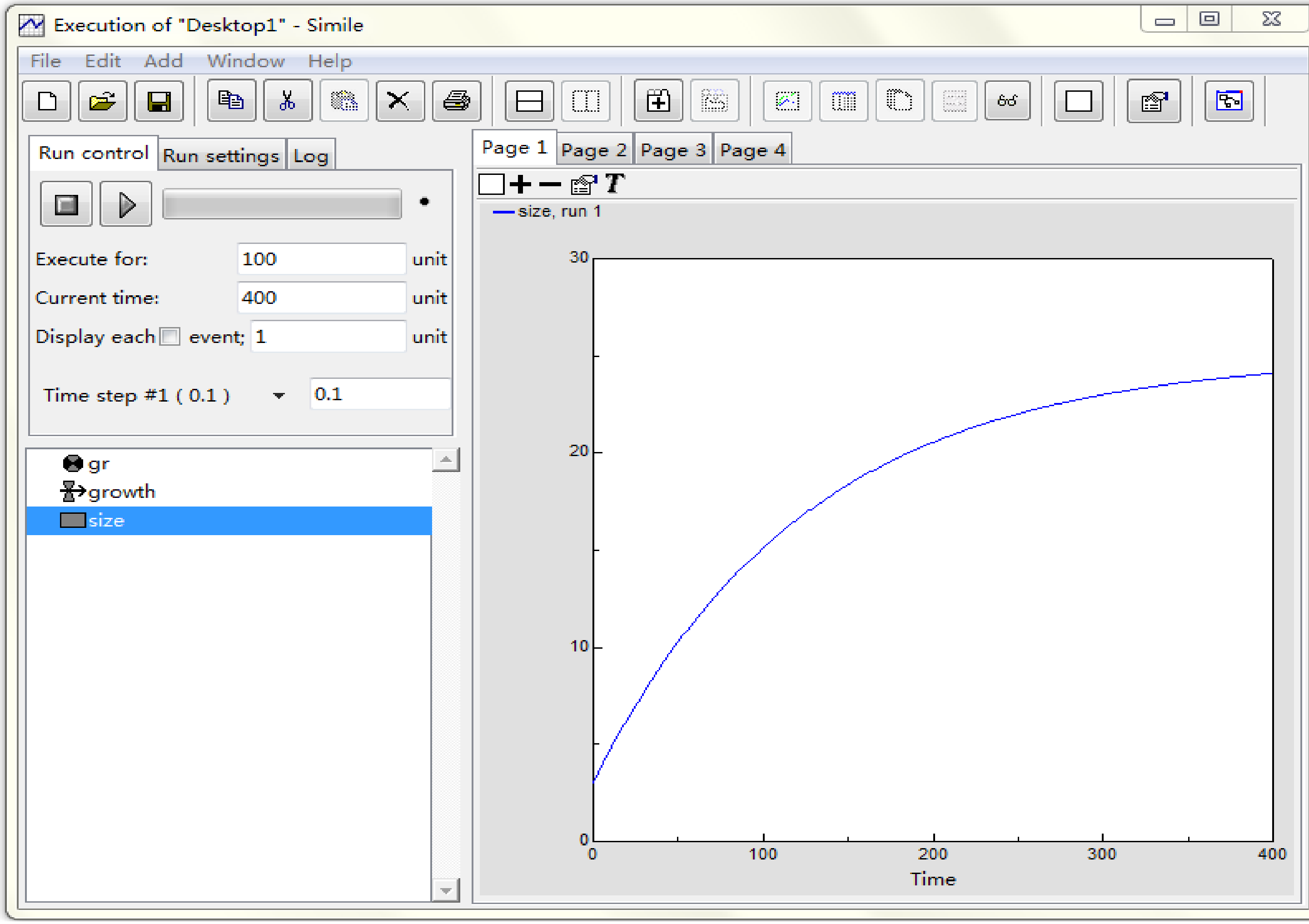
- Step 1 Add a compartment to the desktop, and rename it size.
- Step 2 Draw a flow into the size compartment, and rename it growth.
- Step 3 Add a variable to the desktop, above the growth flow, and rename it gr.
- The variable gr represents the maximum rate of growth of the tree.
- Step 4 Draw an influence arrow from both the size compartment and the gr to the growth flow.



Simile step-by-step, con’t

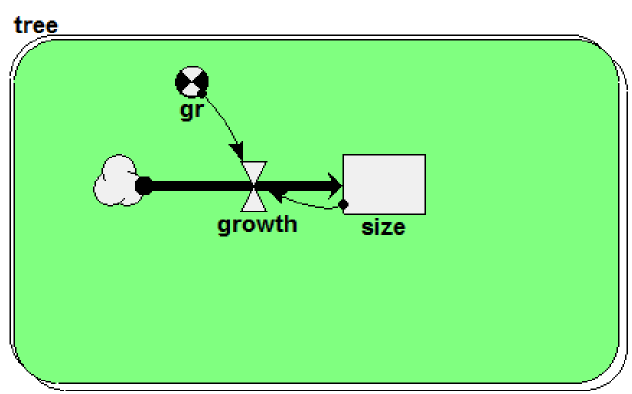
- Step 5 Enter the following expression for the growth flow: gr*(1-size/25)
- Step 6 Set the initial value for the size compartment to 3.
- Step 7 Set the value for the variable gr to 0.2.
- Step 8 Prepare the model for running.
- Step 9 Set up a plotter display helper for the size compartment
- Step 10 Run the model

Size growth of single tree



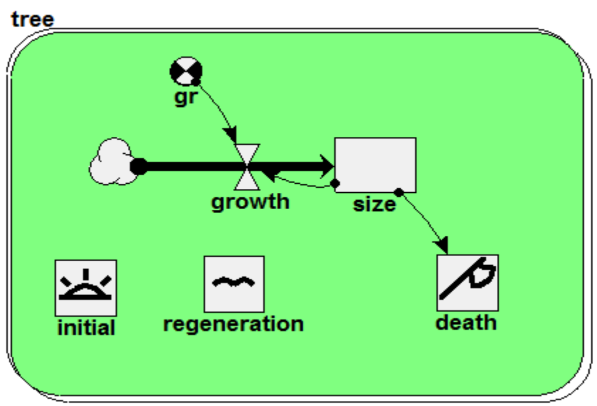
Extending the model to a stand

- Step 1 Draw a submodel box to completely enclose your model diagram.
- Step 2 Rename the submodel tree.
- Step 3 Open up the submodel Properties dialogue window.
- Step 4 Click the radio button labelled “Using population symbols”.
- Step 5 Choose a nice background colour for the submodel.
- Step 6 Close the submodel properties dialogue window.
- Step 7 Change the expression for gr from its current value (0.2) to rand_const(0.1,0.3).



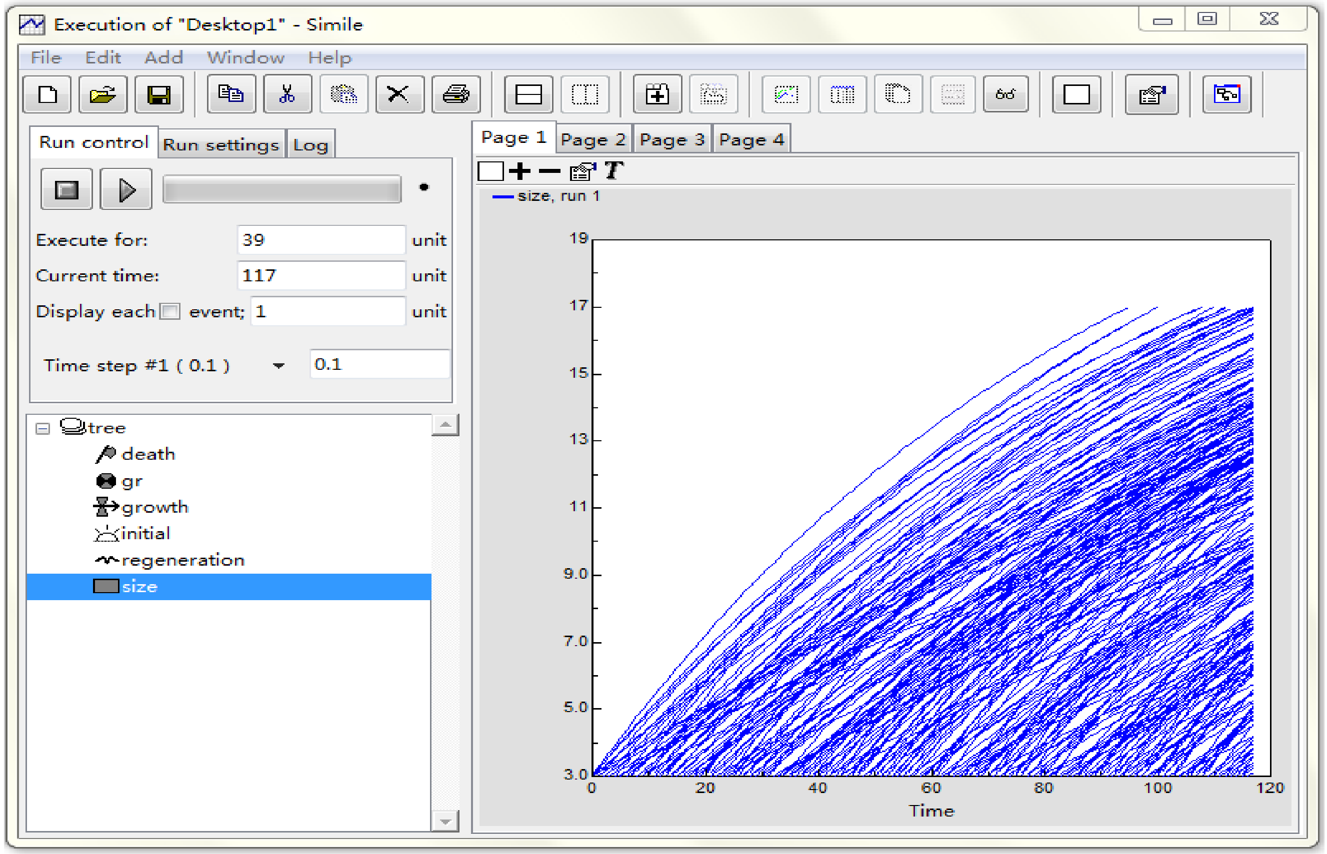
Extending the model, con't

- Step 8 Add a creation symbol to the model diagram, and give it a value of 5.
- Step 9 Add an immigration symbol to the model diagram, and give it a value of 2.
- Step 10 Add a loss symbol to the model diagram.
- Step 11 Rename the loss symbol death.
- Step 12 Draw an influence arrow from the compartment size to this symbol.



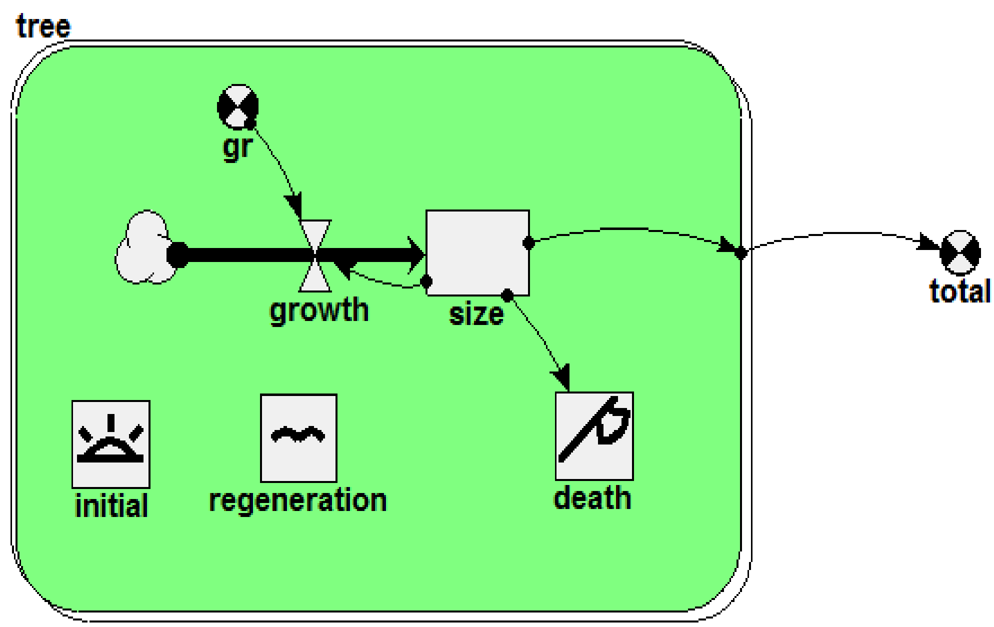
Rebuild the model

- Step 13 Enter the expression size>17 for the symbol labelled death.
- Step 14 Rebuild the model, and run it again.



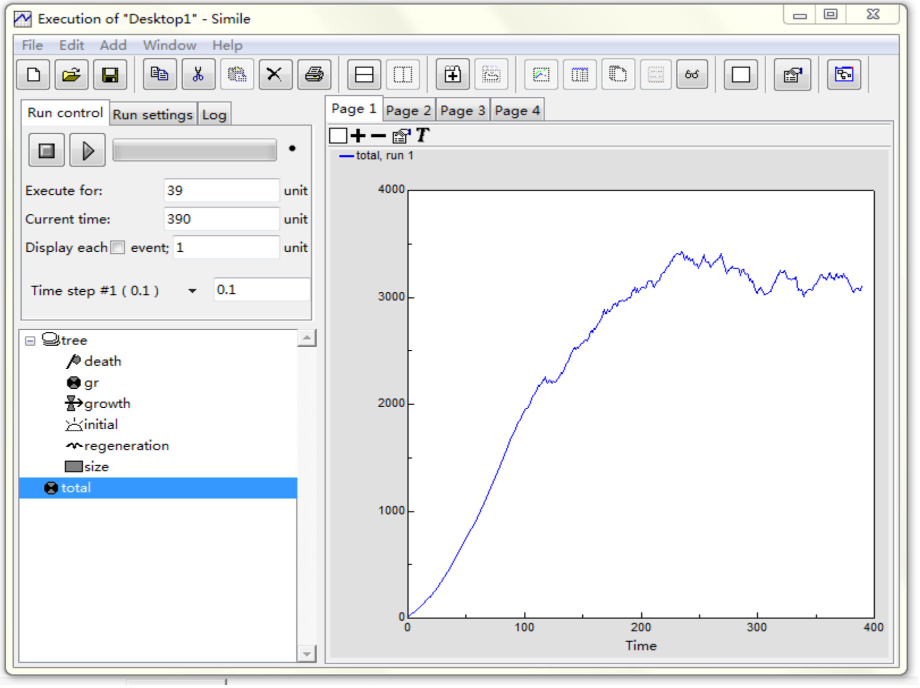
Calculating aggregate information

- Step 1 Add a variable outside the tree submodel, and rename it total.
- Step 2 Draw an influence arrow from the compartment size inside the tree submodel to the variable total.



Rerun the model, con’t

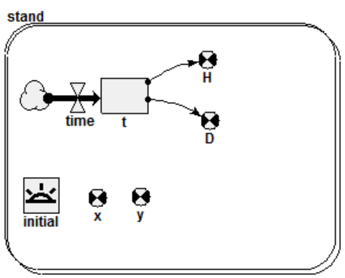
- Step 3 Enter the expression sum({size}) for the variable total.
- Step 4 Re-build the model.
- Step 5 Call up a plotter display for the variable total.
- Step 6 Run the model again.

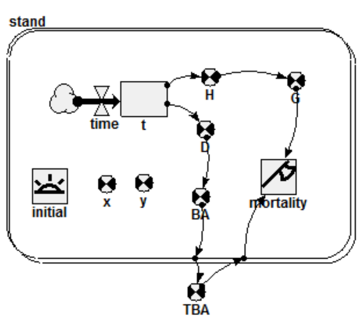


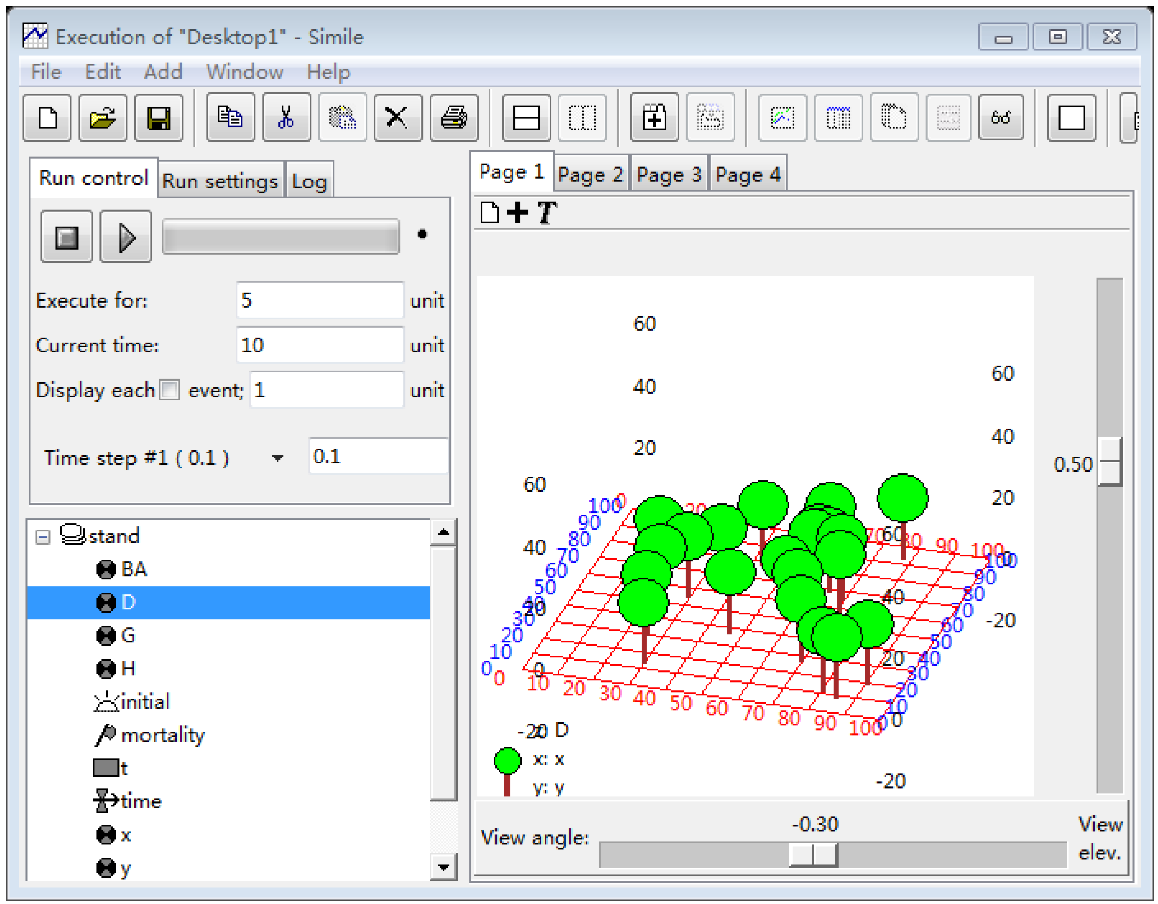
Visualising the trees

- Step 1 Add two variables inside the tree submodel, and rename them x and y.
- Step 2 Enter the following expression for the variable x: rand_const(0,50)
- Step 3 Enter the following expression for the variable y: rand_const(0,100).
- Step 4 Re-build the model.
- Step 5 Call up the lollipop display. When it prompts you for the three variables required to set up the display, click on the variable x, the variable y, and the variable size respectively, in that order.
- Step 6 Run the model again.

Visualization, con't







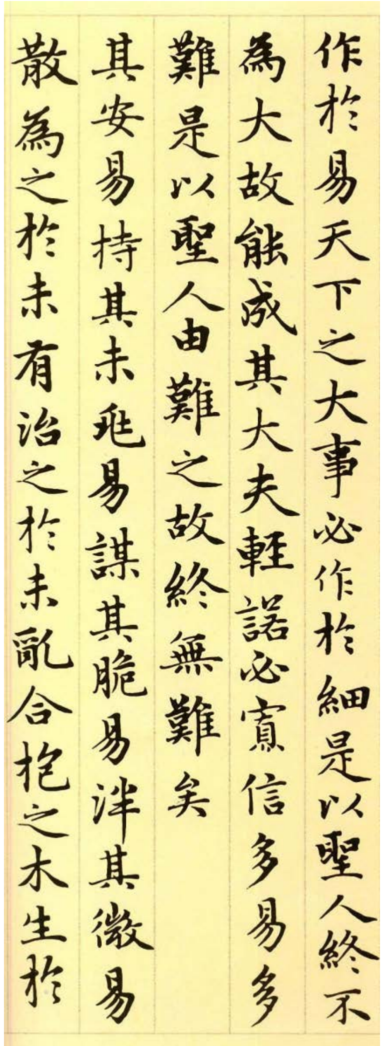
Part I, the final report

- 1. Field form
- 2. Data processing
- 3. Site conditions
- 4. Diameter distribution
- 5. Stand growth

Part II: Forest Planning

DSSs and Applications

Philosophy of modeling


作於易天下之大事必作於細是以聖人終不
為大故能成其大夫輕諾必寡信多易多
難是以聖人由難之故終無難矣
其安易持其未兆易謀其脆易泮其微易
散為之於未有治之於未亂合抱之木生於

What lies still is easy to grasp; What lies far off is easy to anticipate; What is brittle is easy to shatter; What is small is easy to disperse.

Laozi (Taoteching, chapter 64)

Interdisciplinary study

- Forest sciences
  - Silviculture
  - Forest ecology
  - Forest biometrics

- Applied mathematics
  - Operations research
  - Nonlinear programming
  - Artificial intelligence

Contents

- Model linkages and applications
- Forest management objectives
- Optimization modeling techniques
- Forest management planning
- Simulation optimization systems

Forest management objectives

- Timber production
- Forest biodiversity
- Carbon sinks
- Forest bioenergy
- Climate mitigation
- Multi-functional services

Model linkages and applications

- Growth model + dynamic carbon model
- Growth model + wood quality module
- Growth model + climate-sensitive module
- Growth model + energy wood module
- Growth model + biodiversity index

Optimization modeling techniques

- Linear programming
- Goal programming
- Integer programming
- Dynamic programming
- Nonlinear programming
- Artificial intelligence

Forest management planning

- Even-aged management
- Uneven-aged management
- Regeneration methods
- Silvicultural operations
- Logging methods

Decision support systems

- Inventory DSS
- Simulation DSS
- Two-level DSS
- Theoretical optimization DSS
- Simulation optimization DSS

Model linkages

All models are false,

but some models are useful.

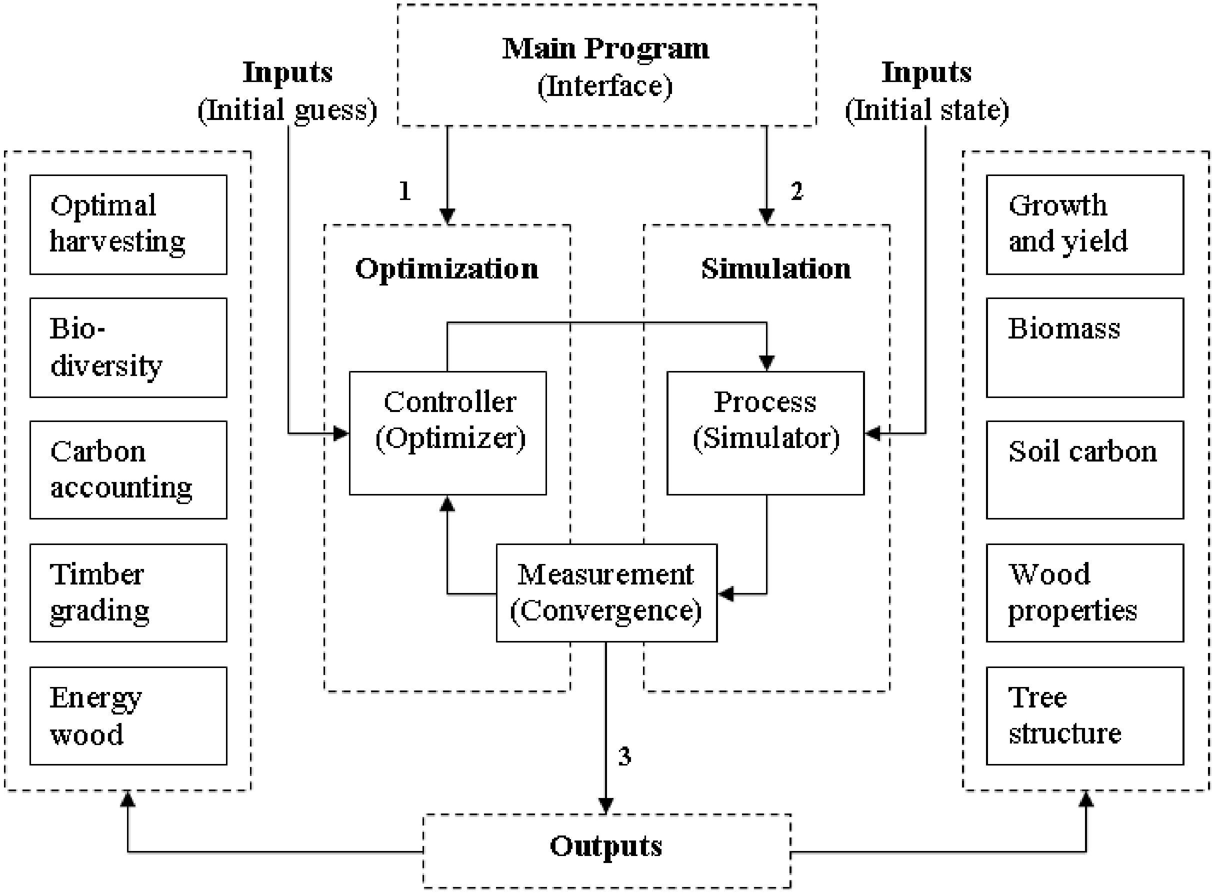
Model linkages

- Forest growth and yield models
- Wood quality models
- Dynamic carbon models
- Nutrient cycling models
- Water balance models
- Logging models

Forest growth models

- Tree level vs. size class level
- stand level vs. forest level
- Even-aged vs. uneven-aged
- Empirical vs. mechanistic
- Deterministic vs. stochastic

OPTIFOR 1.0 (Cao, 2010)



Simulation models in OPTIFOR 1.0

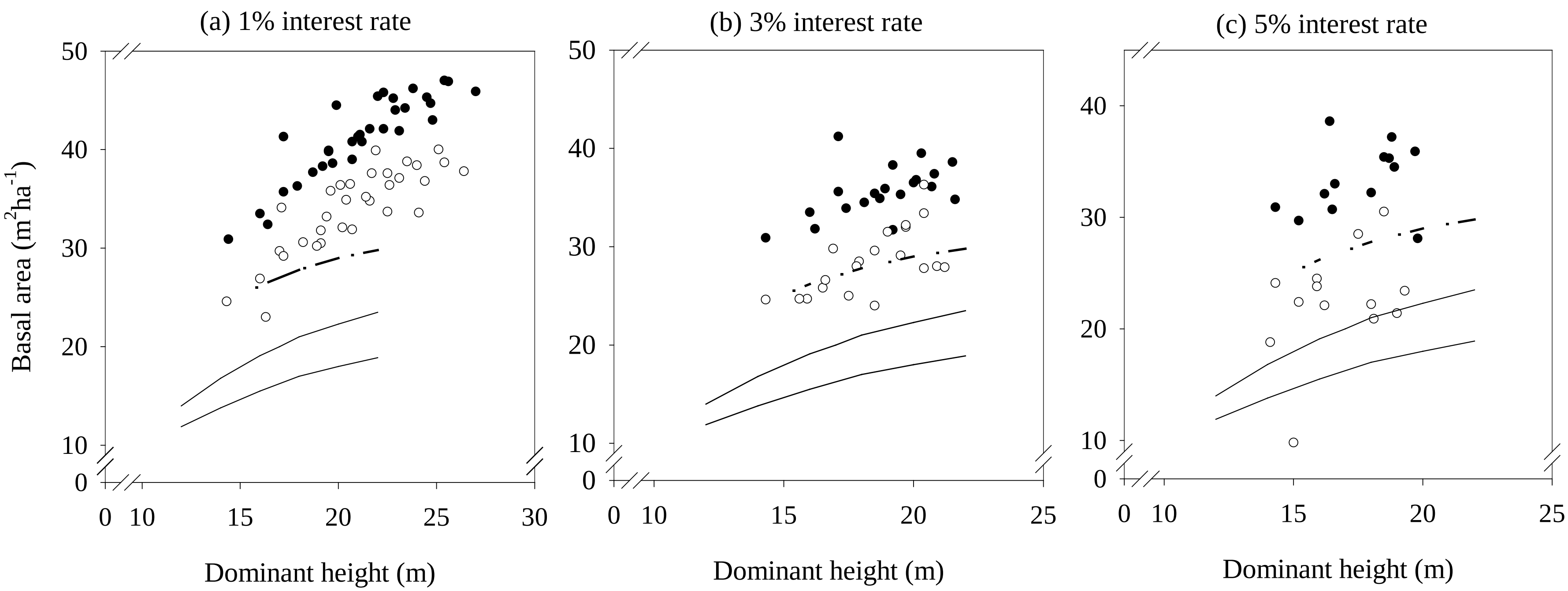
- MOTTI (Hynynen et al. 2002, 2005)
- Crobas/PipeQual (Mäkelä 1997, Mäkelä and Mäkinen 2003)
- Yasso (Liski et al. 2005)
- Wood quality (Mäkinen et al. 2007)

Applications of OPTIFOR 1.0

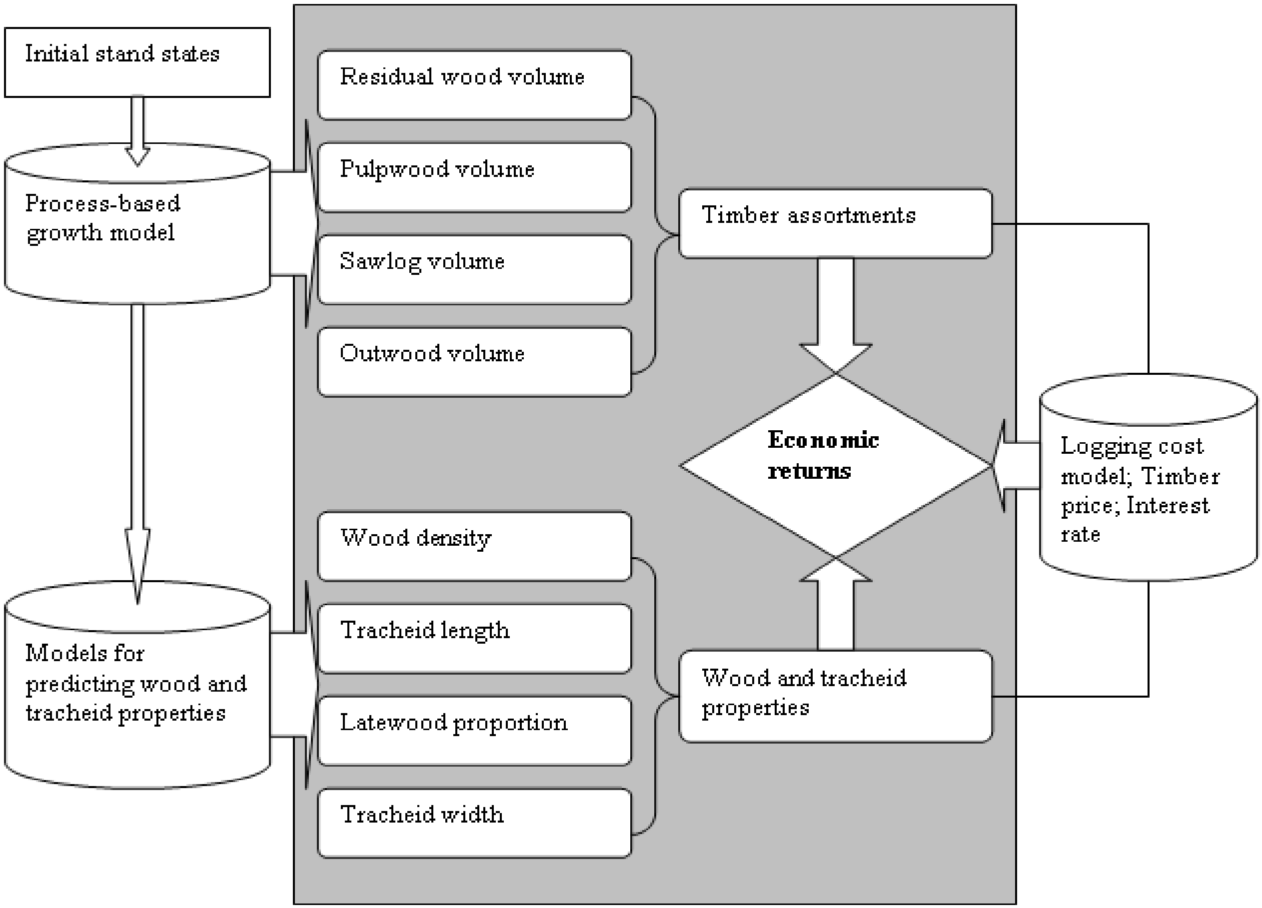
- 优化间伐 Optimal thinning (Cao et al., 2006)
- 木材质量 OPTIFOR Wood (Cao et al., 2008)
- 森林碳汇 OPTIFOR Carbon (Cao et al., 2010)
- 气候变化 OPTIFOR Climate (Nikinmaa et al. 2011)
- 生物能源 OPTIFOR Energy (Cao et al., 2015)

蓄积优化调控 （Cao et al., 2006）

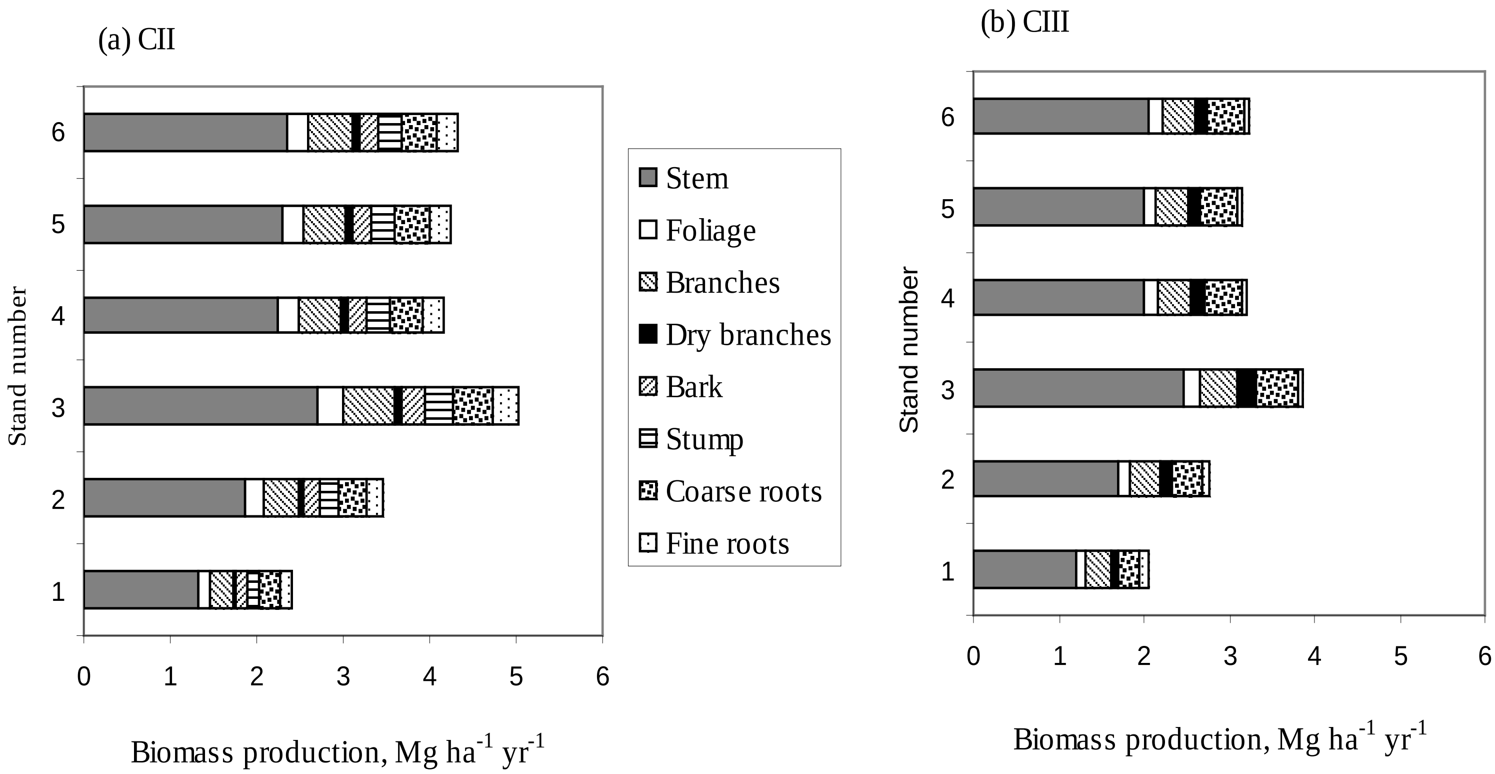
■ **Optimal stocking control**



OPTIFOR Wood (Cao et al., 2008)



OPTIFOR Carbon (Cao et al. 2010)



OPTIFOR Energy（Cao et al. 2015）

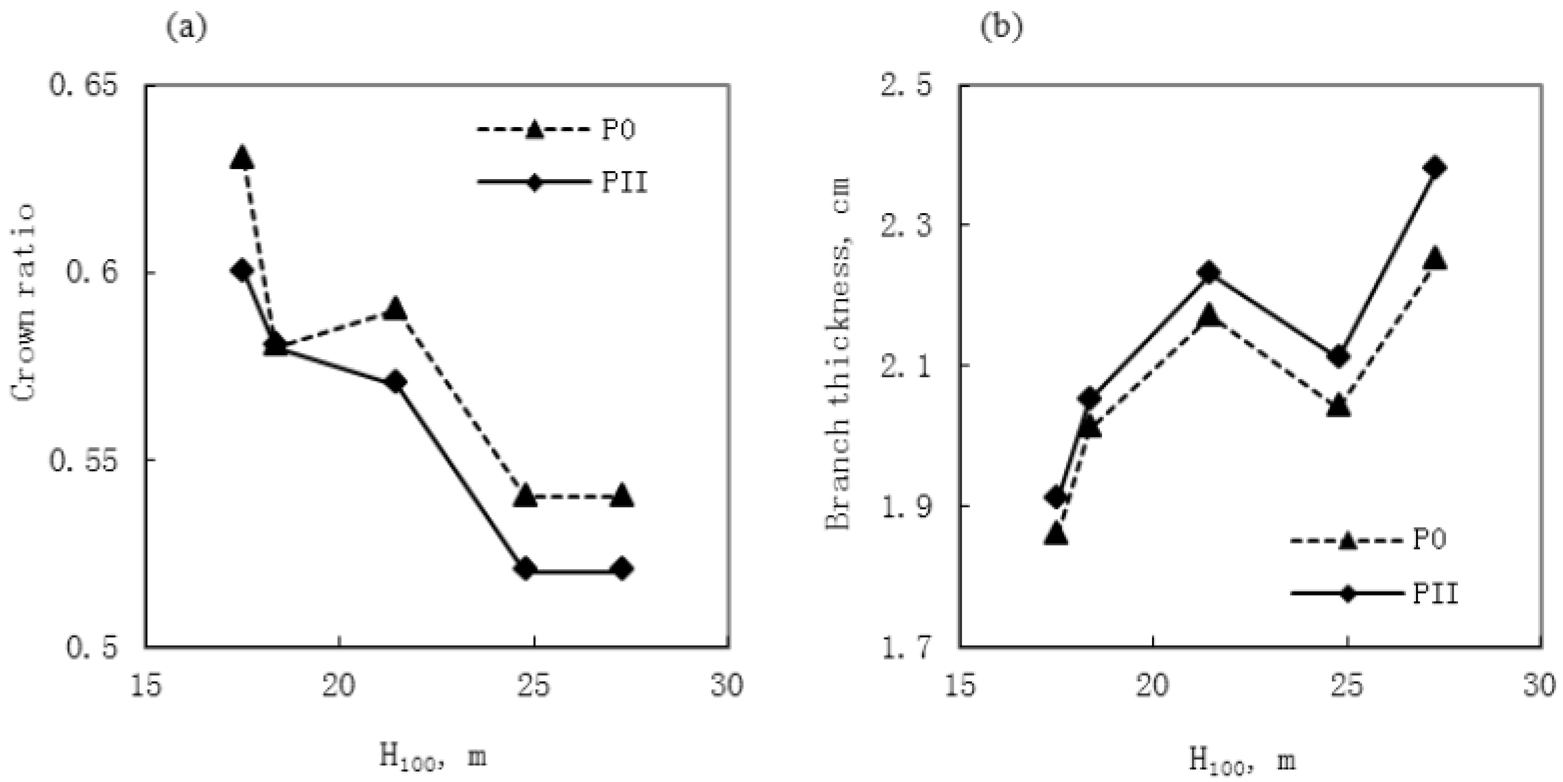


Figure 4. Crown ratio of dominant trees and branch thickness (cm) at the second thinning with Policy 0 and Policy II for five Scots pine stands (initial density 3000 trees ha^{-1}) by H_{100} (dominant height at 100 yrs, m), energy wood price 15 € m^{-3}, interest rate 3%.

Simulation models in OPTIFOR 2.0

- Yasso 07 (Tuomi et al. 2009)
- Wood quality (Mäkinen and Hynynen 2012)
- PreLES (Mäkelä et al. 2008, Peltoniemi et al. 2012)
- ROMUL (Chertov et al. 2001)
- QUASSI (Liao et al. 2017)

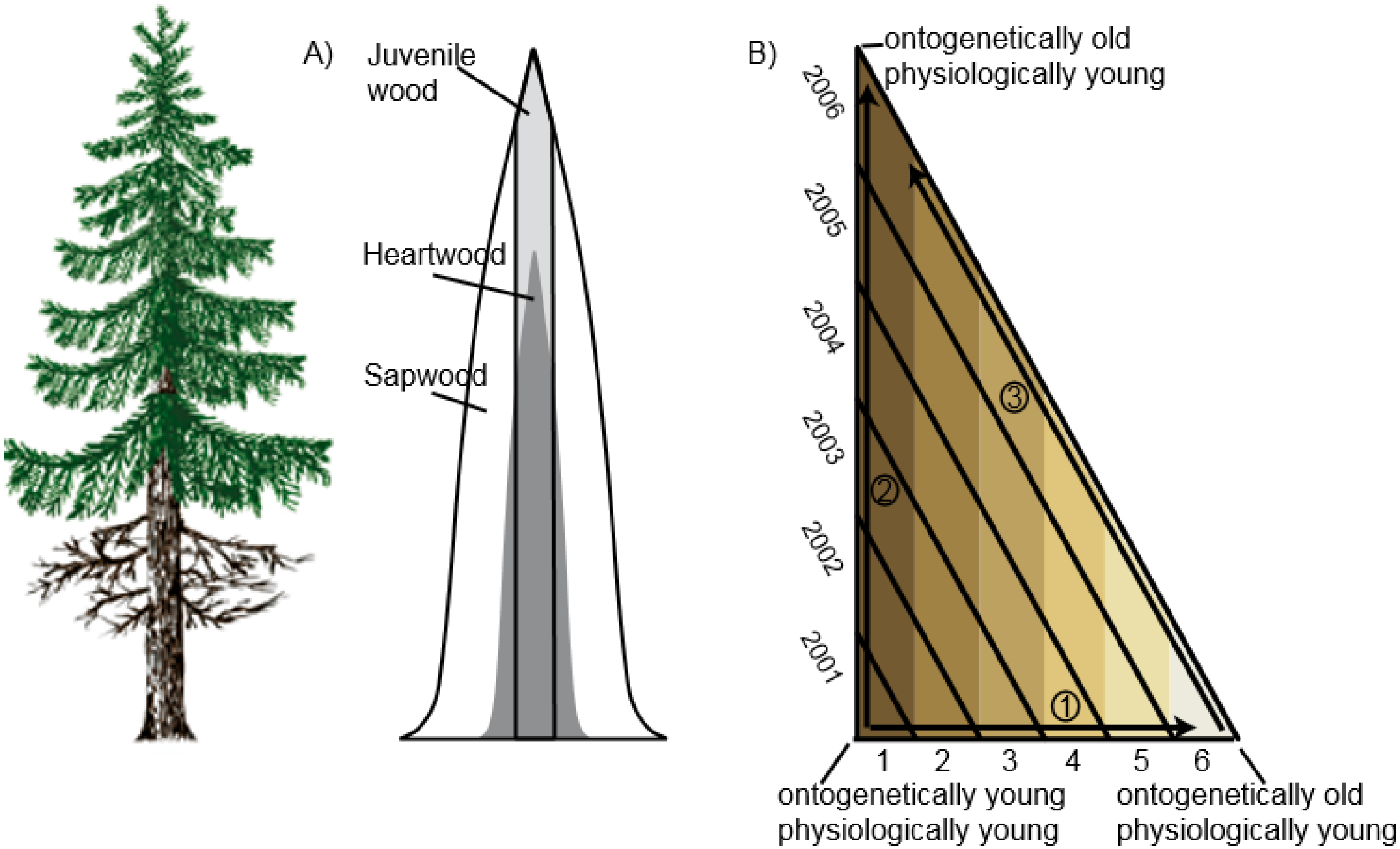


Figure 2. Juvenile wood, heartwood and sapwood in Norway spruce stem (A). Ageing of the xylem along the three major axes of the stem (B): radially from the pith to the bark (1), vertically from the stem base to the stem apex in a given annual ring from the pith (2), and concentrically around the given annual ring from the bark (3) (redrawn and modified from Duff and Nolan 1953, Schweingruber et al. 2006).

CROBAS: a process-based model

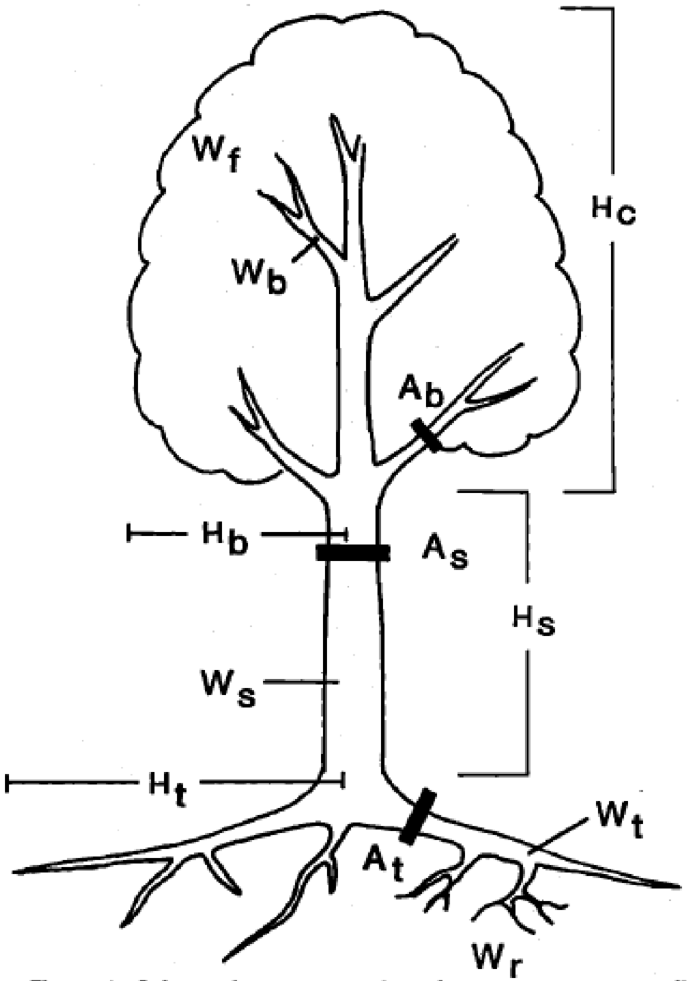
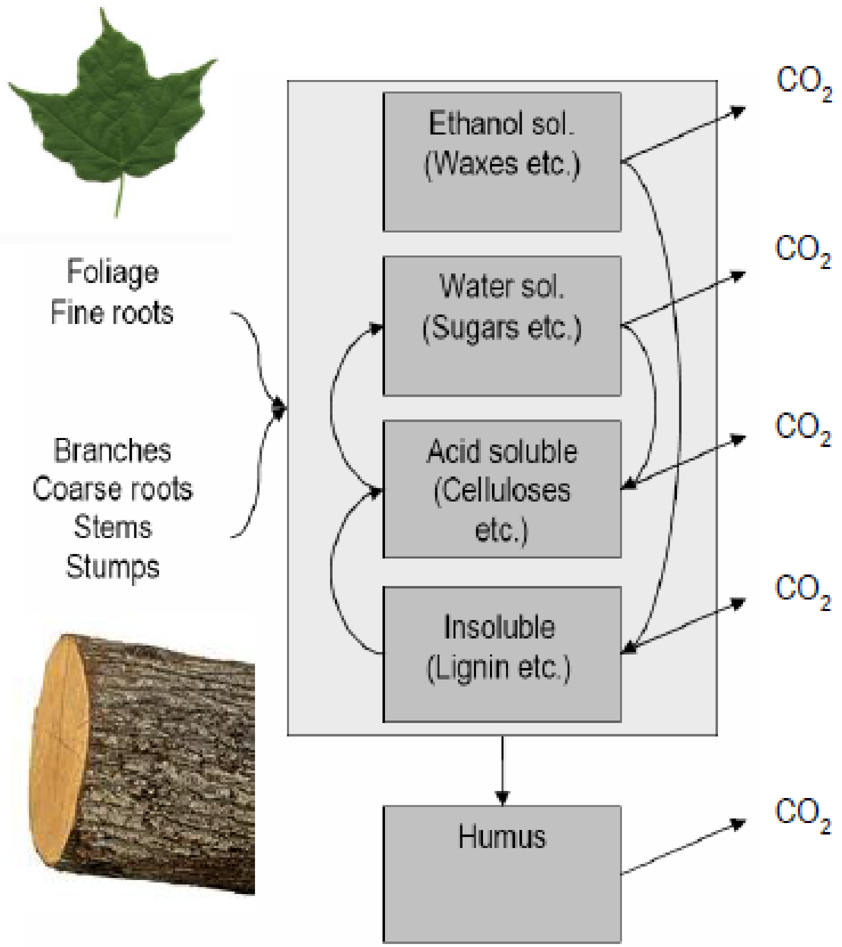


Figure 1. Schematic representation of tree structure as applie[d] in the model.

Yasso 07: a simple dynamic carbon model



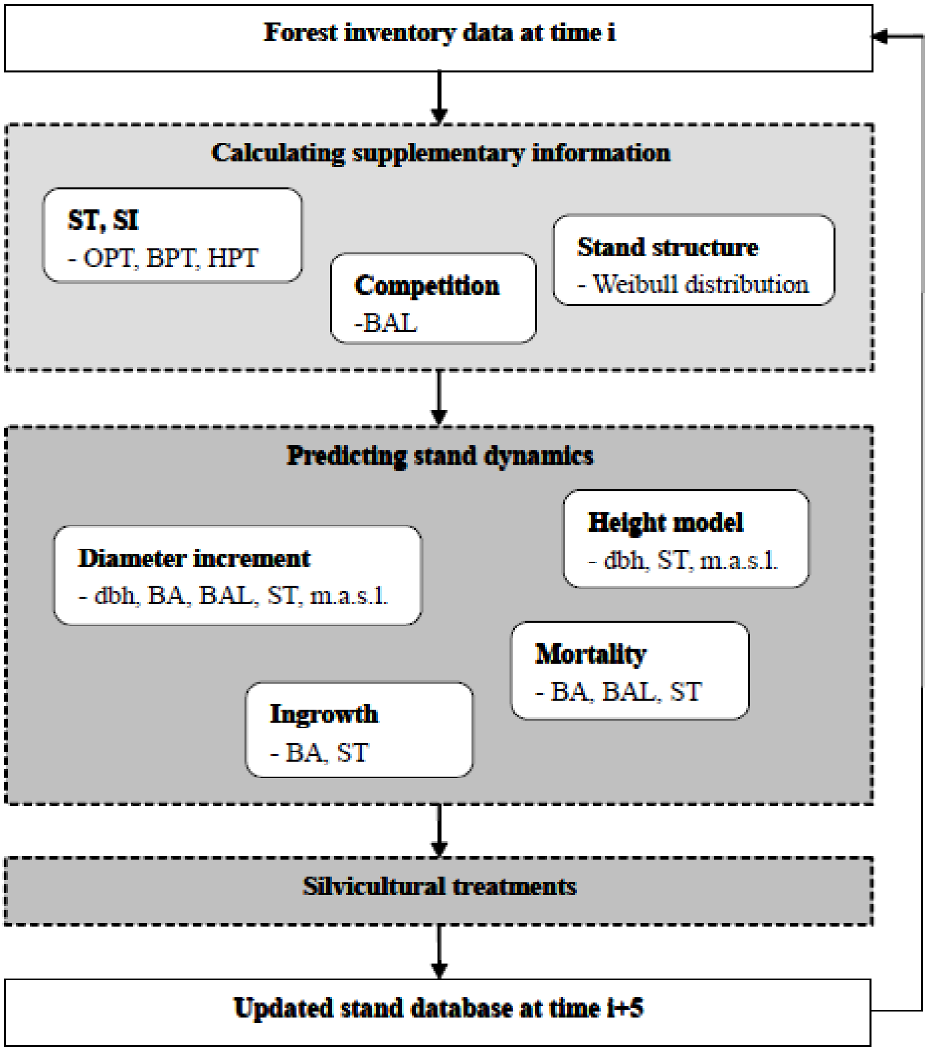
QUASSI (Qinling Uneven-Aged Stand SImulator)

- Collaboration

  LSU

  SCU

  UH



Calibration of PRELES

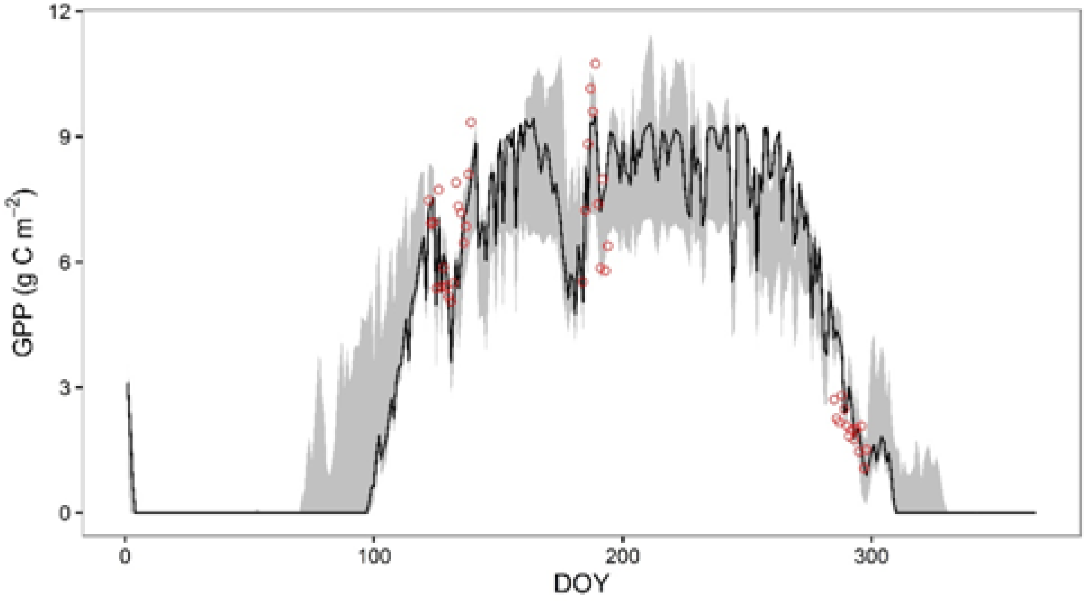
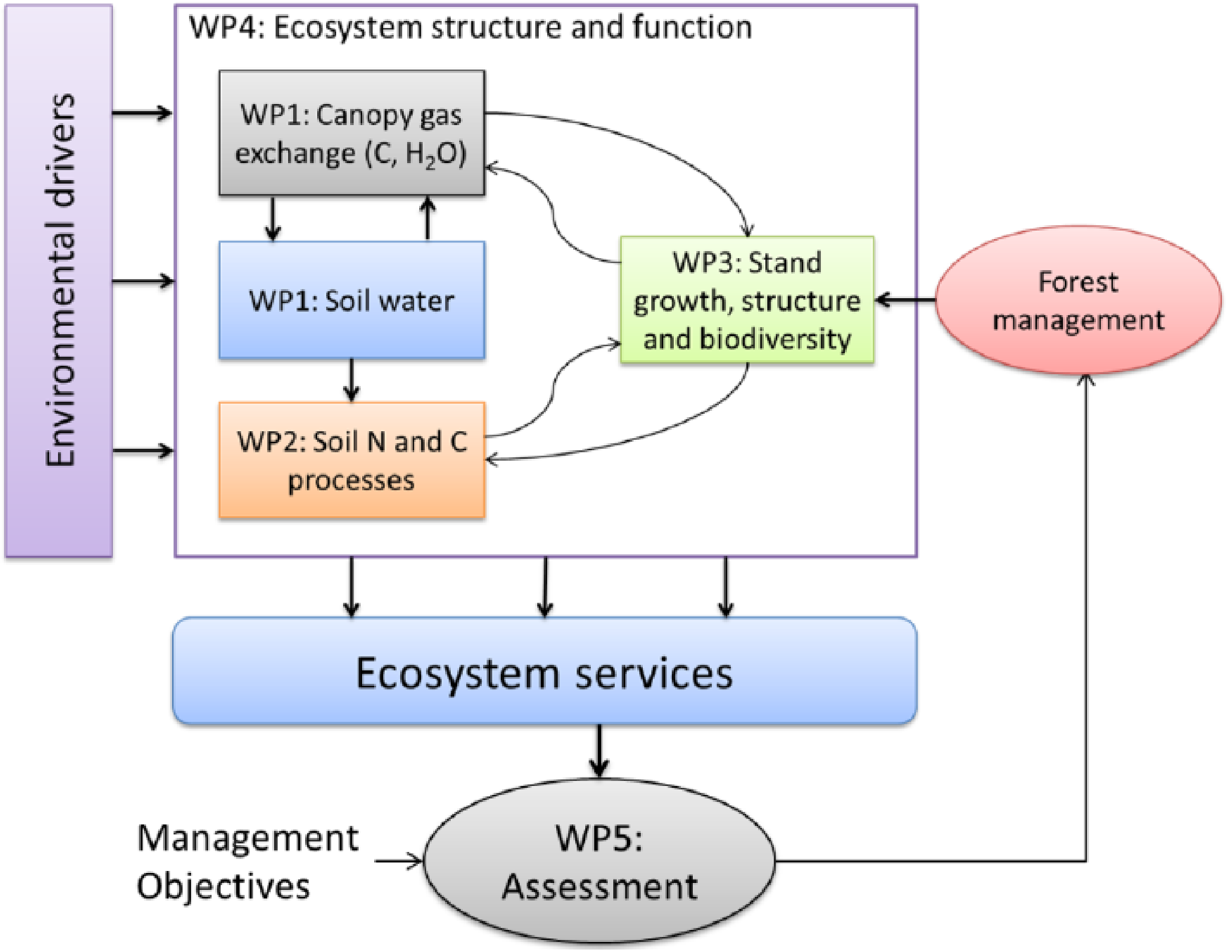


Fig. 6 Calibration of PRELES on Qinglin site. The predictive uncertainty is higher than any other site due to the limited amount of observations (43 days).

MultiFor


Environmental drivers
WP4: Ecosystem structure and function
WP1: Canopy gas exchange (C, H_2O)
WP1: Soil water
WP2: Soil N and C processes
WP3: Stand growth, structure and biodiversity
Forest management
Ecosystem services
Management Objectives
WP5: Assessment

Optimization algorithms in OPTIFOR 2.0

- Differential evolution (Storn and Price 1997)
- Particle swarm (Kennedy and Eberhart 1995)
- Evolution strategy (Bayer and Schwefel 2002)
- The method of Nelder and Mead (Nelder and Mead 1965)

Forest management objectives

Forest management problems

Forest management decisions

Forest management objectives

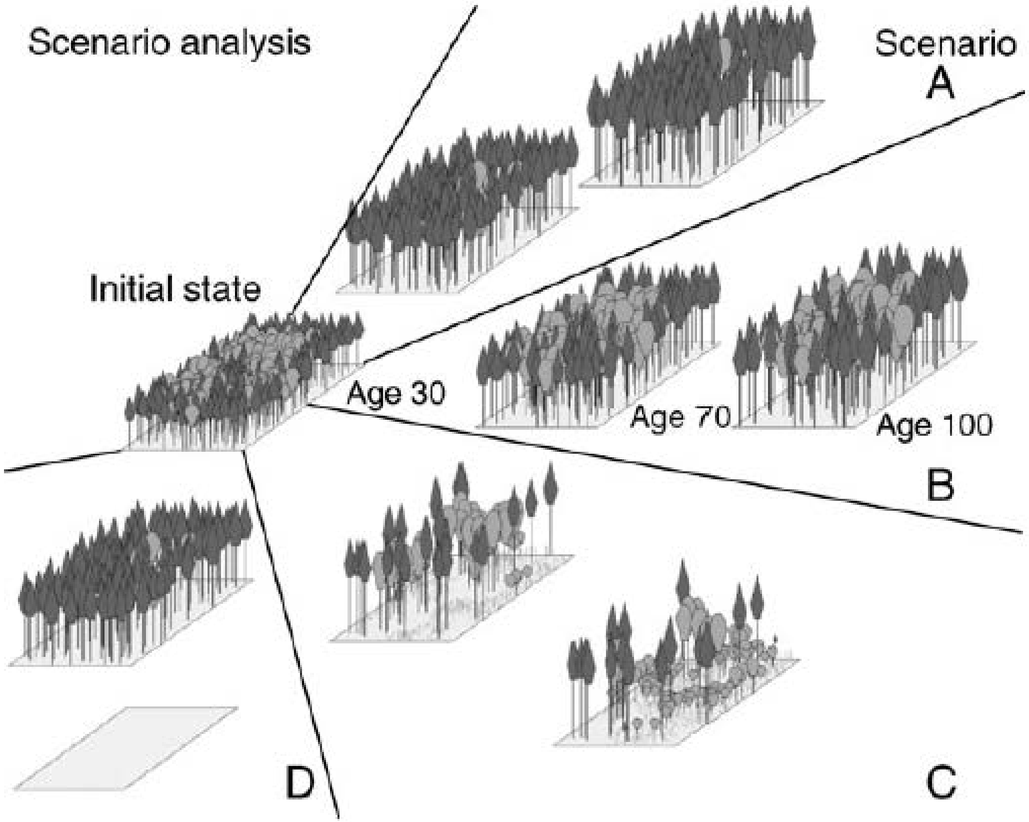


FIG. 2. Scenario analysis with forest stand models. Starting with an *initial state* of an ecosystem, models display the long-term consequences of the different management options A, B, C and D and the consideration of different *objective states*.

Definition

- What does sustainable forest management mean anyway?

- Sustainable timber yield?
- Sustainable forest biodiversity?
- Or sustainable forest ecosystem?

Group work

- Sustainable,
- close-to-nature,
- or adaptive forest management:

- doest it really matter?

Forest ecosystem functions

- IUFRO Division 8.01
- 8.01.00 – Forest ecosystem functions

- 8.01.01 – Old growth forests and forest reserves
- 8.01.02 – Landscape ecology
- 8.01.03 – Forest soils and nutrient cycles
- 8.01.04 – Water supply and quality
- 8.01.05 – Riparian and coastal ecosystems
- 8.01.06 – Boreal forest ecosystems
- 8.01.07 – Hydrologic processes and watershed management

Forest biodiversity

- 8.02.01 – Key factors and ecological functions for forest biodiversity
- 8.02.02 – Forest biodiversity and resilience
- 8.02.03 – Humus and soil biodiversity
- 8.02.04 – Ecology of alien invasives
- 8.02.05 – Wildlife conservation and management
- 8.02.06 – Aquatic biodiversity in forests
- 8.02.07 – Bioenergy productions systems and forest biodiversity
- 8.02.08 – African wildlife conservation and management (AWCM)

Ecosystem services

- Humankind benefits from a multitude of resources and processes that are supplied by ecosystems.
- While scientists and environmentalists have discussed ecosystem services for decades, these services were popularized and their definitions formalized by the United Nations 2005 Millennium Ecosystem Assessment (MA).
- This grouped ecosystem services into four broad categories:
  - provisioning
  - regulating
  - supporting
  - cultural

Forest ecosystem services

- Timber
- Fuel wood
- Hunting
- Preventing erosion
- Amenities
- Biodiversity
- Carbon sequestration
- Mushrooms
- etc.

Multiple roles

Classification of selected forest outcomes

	Method of pricing on public lands		
Location where consumed	Market prices	Administered nominal prices	Not priced
off forest	Timber	Fuelwood	Water
on forest	Hunting and recreation	Forage; developed recreation	Dispersed recreation visual amenities; nongame wildlife; endangered species

Example, A poet and his woods

- The protagonist is a congenial poet-forester who lives in the woods of Northern Wisconsin. Some success in his writing allowed him to buy, about ten years ago, a cabin and <u>90 ha of woods</u> in good productive condition.
- The poet needs to walk the beautiful woods to keep his inspiration alive. But the muses do not always respond and he finds that sales from the woods come very handy to replenish a sometimes empty wallet. In fact, times have been somewhat harder than usual lately. He has firmly decided to get the most he can out of his woods.
- But the arts must go on. <u>The poet does not want to spend more than half of his time in the woods</u>; the rest is for prose and sonnets.

Data

- About 40 ha of the land he owns are covered with red-pine plantations. The other 50 ha contain mixed northern hardwoods.

- Having kept a very good record of his time, he figures that since he bought these woods he has spent approximately 800 days managing the red pine and 1500 days on the hardwoods.

- The total revenue from his forest during the same period was $36,000 from the red pine land and $60,000 from the northern hardwoods.

Management objective

- The poet's objective is to maximize his revenues from the property. But this has a meaning only if the revenues are finite; thus he must mean revenues per unit of time, say per year (meaning an average year, like anyone of the past ten enjoyable years that the poet has spent on his property). Formally, we begin to write the objective as:

- Maximize Z = $ of revenues per year.

- X1 = the number of hectares of red pine to manage
- X2 = the number of hectares of northern hardwoods to manage

Objective function

- The objective function expresses the relationship between Z, the revenues generated by the woods, and the decision variables X1 and X2. Since the poet has earned \$36,000 on 40 ha of red pine and \$60,000 on 50 ha of northern hardwoods during the past 10 years, the average earnings have been \$90 per ha per year (90 \$/ha/y) for red pine, and 120 \$/ha/y for northern hardwoods. We can now write his objective function as:

$$\max \underset{(\$/\mathrm{y})}{Z} = \underset{(\$/\mathrm{ha}/\mathrm{y})}{90} \underset{(\mathrm{ha})}{X_1} + \underset{(\$/\mathrm{ha}/\mathrm{y})}{120} \underset{(\mathrm{ha})}{X_2}$$

Time constraint

- we note that the time he has spent managing red pine during the past 10 years (800 days for 40 ha of land) averages to 2 days per hectare per year (2 d/ha/y). Similarly, he has spent 3 d/ha/y on northern hardwoods (1500 days on 50 ha).
- In terms of the decision variables X1 and X2, the total time spent by the poet-forester to manage his woods is:

$$\underset{(\text{d/ha/y})}{2}\ \underset{(\text{ha})}{X_1} + \underset{(\text{d/ha/y})}{3}\ \underset{(\text{ha})}{X_2} \leq \underset{(\text{d/y})}{180}$$

Land constraints

- Two constraints are very simple. The area managed in each timber type cannot exceed the area available, that is:

- X_1 <= 40 ha of red pine
- X_2 <= 50 ha of northern hardwoods

The optimization problem

- we obtain the complete formulation of the poet-forester problem as: Find the variables X1 and X2, which measure the number of hectares of red-pine and of northern hardwoods to manage, such that:

$$\max Z = 90X_1 + 120X_2$$

$$\text{subject to:}$$

$$X_1 \leq 40$$

$$X_2 \leq 50$$

$$2X_1 + 3X_2 \leq 180$$

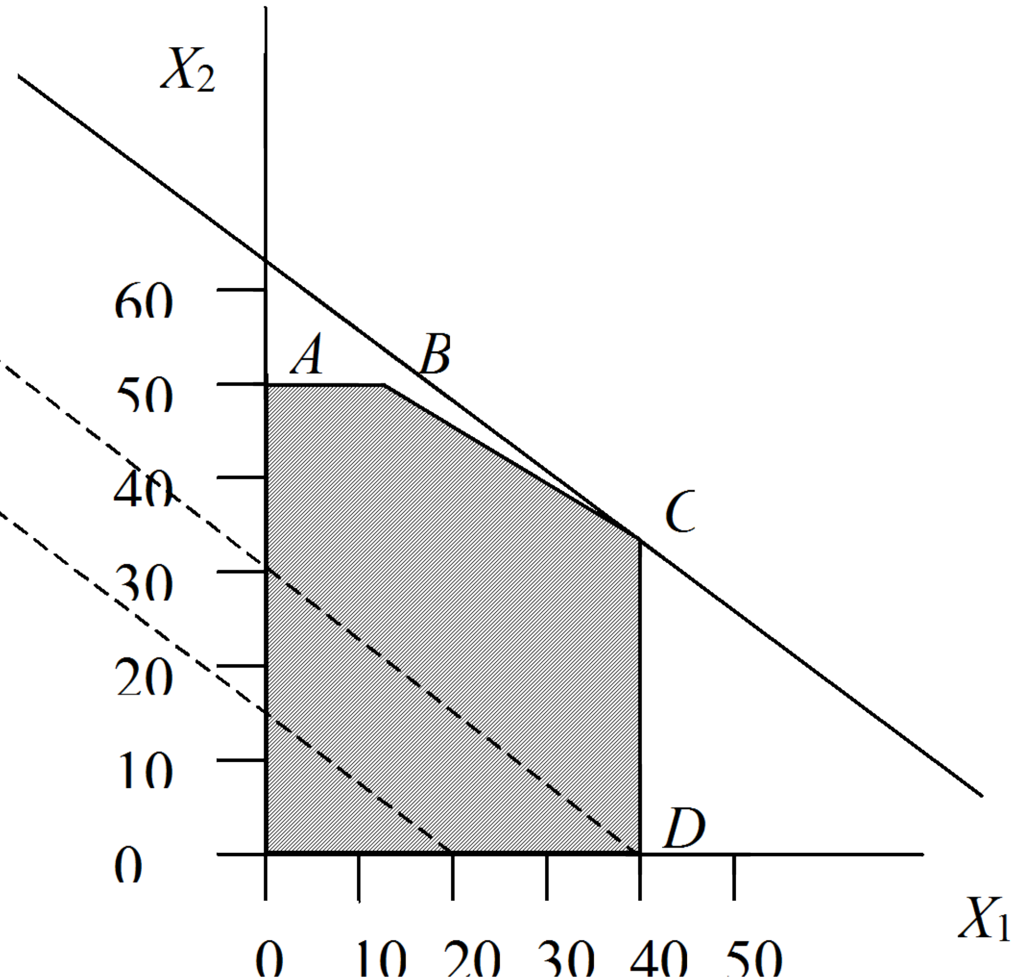
$$X_1, X_2 \geq 0$$

Graphic solution

$$Z = 7600 = 90X_1 + 120X_2$$

$$Z = 3600 = 90X_1 + 120X_2$$

$$Z = 1800 = 90X_1 + 120X_2$$



Conventional objective: timber production

- maximization of bare land value

$$\max_{\{\mathbf{h}_u, t_u, u=1,\ldots,k|\mathbf{Z}_0\}} \mathrm{V} = \frac{\sum_{u=1}^{k}\left[\sum_{i=1}^{n}\sum_{j=1}^{2} p_j \mathrm{g}_{ij}(\mathbf{Z}_{t_u}, \mathbf{h}_u) - \mathrm{c}_u(\mathbf{Z}_{t_u}, \mathbf{h}_u)\right](1+r)^{-t_u} - \mathrm{c}_0(\mathbf{Z}_0)}{1-(1+r)^{-t_k}}$$

Conventional forest management and DP

- In conventional forest management, the dominant model type for growth and yield prediction was whole-stand models.
- This model type is sufficient to achieve the accuracy of volume growth prediction for which timber yield was the main concern of conventional forest management.
- Volume or basal area development is usually simply formulated as a function of time, site type, and stand density in such models.
- In other words, the number of state variables of whole-stand models is limited.

Dynamic programming

- Dynamic programming (DP) was an optimization method widely applied with whole-stand models because of its ability to find a global optimum with a few number of variables.
- Hann and Brodie (1980) reported that DP required more computing hardware capacity or computing time when the number of state variables increased.
- Therefore, earlier studies of stand management optimization concentrated on reducing the dimensionality of optimization problems, either the number of state variables predicted from stand growth models or decision variables optimized by silvicultural treatments (Brodie and Haight 1985).

Intensive forest management and NLP

- Intensive forest management requires detailed information from stand growth predictions with higher resolution, for example, tree diameter distribution, timber assortments and timber grading at tree level.
- It is necessary to apply tree-level growth and yield models, such as tree-list or individual-tree models.
- In the formulation of tree-level growth and yield models, the increment of tree diameter or basal area growth can be modeled as a function of, e.g., diameter at breast height, basal area, site type, site index, basal area in larger trees, crown ratio, crown competition factor, etc.
- Tree-level growth and yield models clearly increase the number of state variables.

Intensive forest management, con't

- Meanwhile, intensive forest management means more intensive silvicultural treatments, such as fertilization, pruning, pre-commercial thinning, and commercial thinning in terms of thinning intensity, thinning type, thinning frequency, and timing of thinning.
- This type of stand management problem is typically non-smooth or non-differentiable because of timber assortments and thinning interventions, and may contribute to more number of state and decision variables formulated in stand management optimization (Hyytiäinen et al. 2004).

Intensive forest management, con’t

- Nonlinear programming (NLP) turns out to be an effective tool handling such complicated optimization problems.
- Roise (1986) and Valsta (1990) both reported that NLP algorithms were more efficient than those of DP.
- Most of conventional and intensive forest management studies assume that stand dynamics can be predicted based on deterministic empirical growth models. This model type heavily depends on empirical observed data.
- As a matter of fact, collecting long-term re-measured data for modeling impacts of thinning or climate effects on regeneration, in-growth, and mortality by various stand density and site conditions often is inefficient and difficult.

Sustainable forest management, gap models

- Applying more detailed succession or process models to explain uncertainties and biological reasons becomes a helpful alternative in contrast to empirical models.
- Gap models perhaps would be one of alternatives to explain succession based on ecological population theory.
- But most of gap models rely on expert opinion for predicting species succession patterns rather than observed data.
- Even calibrated with long-term observed data, individual tree dimensions and stand structure predicted by gap models were still unrealistic (Lindner et al., 1997, Monserud, 2003).

Sustainable forest management, algorithms

- The dimensionality of variables is a choice of a robust and accurate algorithm in stand management optimization in addition to the convexity of the objective function (Cao 2010).
- The HJ algorithm has been well demonstrated earlier with various empirical stand growth models, such as whole-stand models (e.g., Roise 1986), and individual-tree models (e.g., Haight and Monserud 1990).
- As well as heuristic algorithms, such as genetic algorithm (Lu and Eriksson 2000), tabu search (Wikström and Erikson 2000), and simulated annealing (Lockwood and Moore 1993).

Adaptive forest management, process models

- Based on physiological theory, process or mechanistic growth models intend to include key growth processes and underlying causes of forest productivity,
- for example, photosynthesis and respiration, nitrogen cycles, water balance, and carbon balance, and climate effects.
- Although the common purpose of process-based models is to explain scientific reasons rather than growth prediction,
- efforts have also been made to build management-oriented hybrid models by linking processed-based and empirical growth models.

Process-based/mechanistic models, con’t

- it is possible to apply detailed process-based growth models for more complicated forest management situation where is a lack of observed data.
- However, taking stochastic effects into consideration significantly increase the complexity of process-based growth models with a number of parameters,
- for example, 48 parameters for 3-PG (Landsberg and Waring, 1997),
- and 39 parameters for CROBAS (Mäkelä, 1997).

Adaptive forest management, AI algorithms

- With more detailed process-based models (thousands of state variables),
- and more complicated optimization problems (a number of decision variables)

- Differential evolution (Storn and Price 1997),
- Particle swarm optimization (Kennedy and Eberhart 1995),
- Evolution strategy (Bayer and Schwefel 2002)
- Direct and random search (Osyczka 1984)
- Hybrid algorithms

Faustmann equation（1849）

- Bare land value

$$PV_{timber} = \frac{e^{-rt}\left[B(T)-c\right]}{1-e^{-rt}}$$

- Optimalitiy condition

$$\frac{B'(T)}{B(T)-c} = \frac{r}{1-e^{-rt}}$$

Hartman（1976）model

- The discounted perpetual value of the externality

$$PV_{ext} = \frac{Q(T)}{1-e^{-rt}}$$

- The optimality condition

$$\frac{Q'(T)}{Q(T)} = \frac{re^{-rt}}{1-e^{-rt}}$$

- The joint production problem

$$PV_{tot} = PV_{timber} + PV_{ext}$$

Clark (1976) modified model (thinning)

- Common

$$\frac{dV}{dt} = g(t)f(V) \quad V(t_0) = V_0$$

- Modified

$$\frac{dV}{dt} = g(t)f(V) - h(t) \quad t \geq t_0 \quad h(t) \geq 0 \quad V(t_0) = V_0$$

- Objective function

$$PV = \int_{t_0}^{\infty} e^{-rt}\left[p - c(V)\right]h(t)dt$$

- Optimality condition

$$r = \frac{\partial}{\partial V}\left[g(t)f(V)\right]$$

To sustain Eco-econ-soci values

- Multiple parties
- Multiple objectives
- Solving one problem often creates new problems

- Contract theory: Focus on one (or a few) problems
  - The agency
  - The principle
- Transaction cost theory
  - Min. the costs of planning, monitoring, motivating
  - Min. the cost of risk and uncertainty

Social criteria

- Community stability
  - e.g., the national land management policy shift noted above changed harvest levels, which, in turn, affected the local and regional economies as well as the social diensions of impacted communities.
- Community resilience
  - Horned and Haynes (1999) from Shannon and Weaver (1949)
  - D = - sigma_i=1_n(E_i*log(E_i))
  - E_i = the proportion of tot. employment in the are located in the ith industry
- Political considertions

Ecological criteria

- Ecological and environmental goals
  - Maintaining and enhancing forest productivity
  - Conservation of biological diversity
  - Protecting and enhancing environmental conditions
- Biological diversity
  - The landscape level
  - The species level, e.g. Shannon and Weaver (1949)
  - D = sigma_i=1_n(E_i*log(E_i))
  - E_i=the proportion ot total individuals in the area of the ith species
- Environmental protection (short/long term)
  - Vegetation, soil, and watercourses

Economic criteria

- Economic equity
  - e.g., while the nation as a whole may have benefited from improved forestland conditions, sawmill empolyees and loggers, and the rural communiteis in which they lived, bore costs of the logging-ban policy
- Regional economics
  - Defining the region
  - Regional goals and criteria
- Economic efficiency
  - "The greatest good for the greatest number in the long run." -- Gifford Pinchot

Multi-criteria decision methods

- Multi-Objective Programming (MOP)
- Goal programming (GP)
- Compromise Programming (CP)
- Multi-attribute utility theory (MAUT)
- Fuzzy Mult-Criteria Programming (FMCP)
- Analytic hierarchy process (AHP)
- Other Discrete Methods (ODM)
- Data envelopment analysis (DEA)
- Group Decision Making Techniques (GDM)

MCDM papers by foestry topics

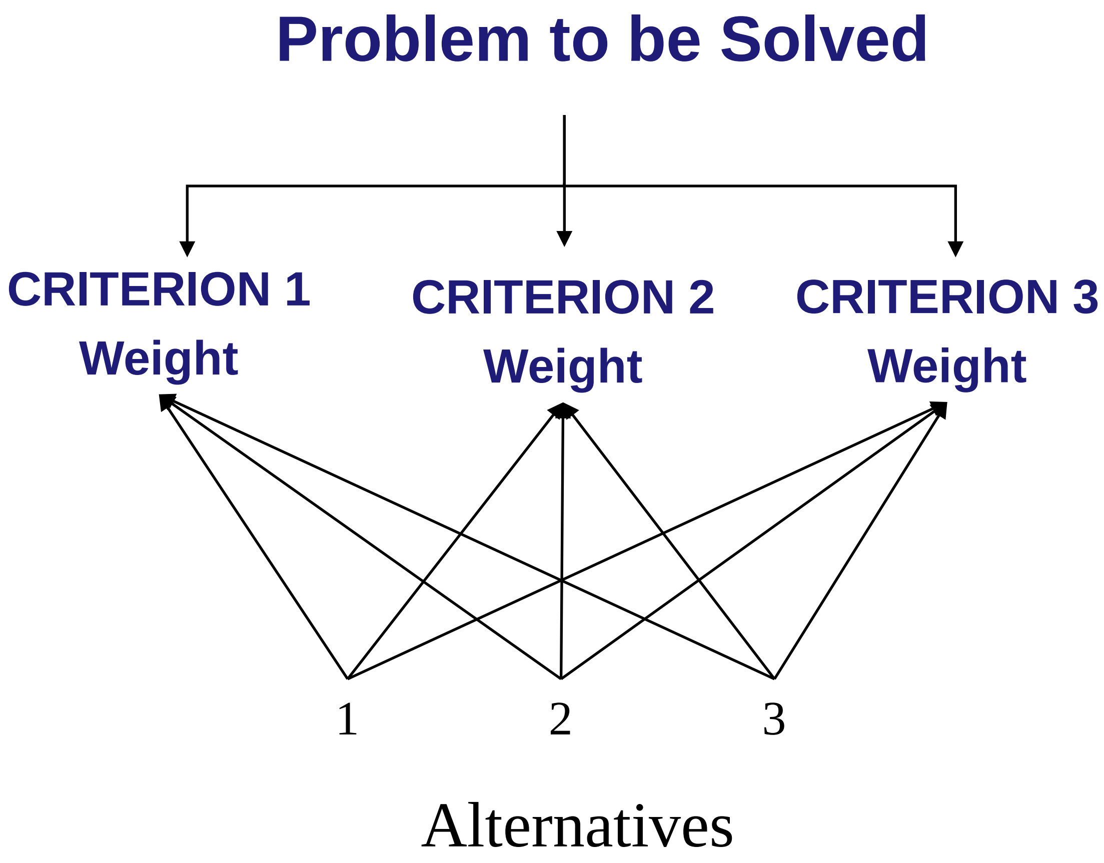
- A (MOP) B (GP) C (CP) D (MAUT) E (FMCP) F (AHP) G (ODM) H (DEA) J (GDM)

	Tot.	A	B	C	D	E	F	G	H	J
Harvest scheduling	17	6	7	1	3	2	2	0	0	0
Ext-harvest scheduling	61	11	14	2	14	5	12	5	0	10
Forest biodiversity	34	6	5	2	9	0	15	2	0	7
Forest sustainability	17	0	2	1	2	3	5	3	0	6
Forestation	11	0	3	1	1	0	2	3	0	1
Regional planning	30	1	7	1	2	0	4	3	13	7
Forestry industry	23	0	1	0	1	0	2	3	18	2
Risk and uncertainty	22	1	0	1	10	1	9	5	0	1
Miscellaneous	40	3	6	1	1	1	5	5	2	12
	255	28	45	10	43	12	56	29	33	46

Analytic hierarchy process (AHP)

- Multifunctional forest management
- Define Criteria
  - NPV, forest health index, PM2.5
- Alternatives
  - timber, carbon balance, biodiversity, recreation

Analytic Hierarchy Process (AHP)



AHP

- Developed by Thomas L. Saaty
- Hierarchy Tree (Decomposition)
- Determining Weights using Pairwise Comparisons
- Synthesizing
- Consistency of Evaluations

Weights

- Scale [1,…,9]:

- 1 Equal Importance
- 3 Weak Importance of one over Another
- 5 Essential or Strong Importance
- 7 Very Strong or Demonstrated Importance
- 9 Absolute Importance

Hierarchy

Multifunctional forest — Overall Goal

Criteria:

- **NPV**
  - timber
  - carbon
  - biodiv.
  - recrea.
- **FH index**
  - timber
  - carbon
  - biodiv.
  - recrea.
- **PM2.5**
  - timber
  - carbon
  - biodiv.
  - recrea.

Alternatives

Pairwise comparison

- FH index is slightly more important (2) than NPV
- NPV is more important (3) than PM2.5
- FH index is clearly more important (4)than PM2.5

	NPV	FH index	PM2.5
NPV	1/1	1/2	3/1
FH index	2/1	1/1	4/1
PM2.5	1/3	1/4	1/1

Eigenvector of Pairwise Comparison Matrix

$$
\begin{array}{ll}
\text{NPV} & \\
\text{FH index} & \\
\text{PM2.5} &
\end{array}
\begin{bmatrix} 0{,}3196 \\ 0{,}5584 \\ 0{,}1220 \end{bmatrix}
\begin{array}{l}
\leftarrow \text{ Next important} \\
\leftarrow \text{ Most important} \\
\leftarrow \text{ Least important}
\end{array}
$$

Hierarchy

Multifunctional forest 1.0 — Overall Goal

Criteria:

- **NPV 0.3196**
  - timber
  - carbon
  - biodiv.
  - recrea.
- **FH index 0.5584**
  - timber
  - carbon
  - biodiv.
  - recrea.
- **PM2.5 0.1220**
  - timber
  - carbon
  - biodiv.
  - recrea.

Alternatives

How about the rank order of Alternatives?

Pairwise comparison

NPV

	Timber	Carbon	Biod.	Recr.
Timber	1/1	1/4	4/1	1/6
Carbon	4/1	1/1	4/1	1/4
Biodiversity	1/4	1/4	1/1	1/5
Recreation	6/1	4/1	5/1	1/1

FH Index

	Timber	Carbon	Biod.	Recr.
Timber	1/1	2/1	5/1	1/1
Carbon	1/2	1/1	3/1	2/1
Biod.	1/5	1/3	1/1	1/4
Recr.	1/1	1/2	4/1	1/1

NPV, FH Index

NPV

		Rank Order
Timber	0,1160	3
Carbon	0,2470	2
Biod.	0,0600	4
Recr.	0,5770	1

FH Index

		Rank Order
Timber	0,3790	1
Carbon	0,2900	2
Biod.	0,0740	4
Recr.	0,2570	3

PM2.5

	PM2.5			Rank Order
Timber	7,0	1/7,0 = 0,1429	0,3010	1
Carbon	8,8	1/8,8 = 0,1136	0,2390	3
Biod.	9,9	1/9,9 = 0,1010	0,2120	4
Recr.	8,5	1/8,5 = 0,1176	0,2480	2
		0,4752	1,0000	

Normalizing

Final hierarchy

Multifunctional forest 1.0

- **NPV 0.3196**
  - Tim. 0.1160
  - Car. 0.2470
  - Bio. 0.0600
  - Rec. 0.5770
- **FH index 0.5584**
  - Tim. 0.3790
  - Car. 0.2900
  - Bio. 0.0740
  - Rec. 0.2570
- **PM2.5 0.1220**
  - Tim. 0.3010
  - Car. 0.2390
  - Bio. 0.2120
  - Rec. 0.2480

What to choose?

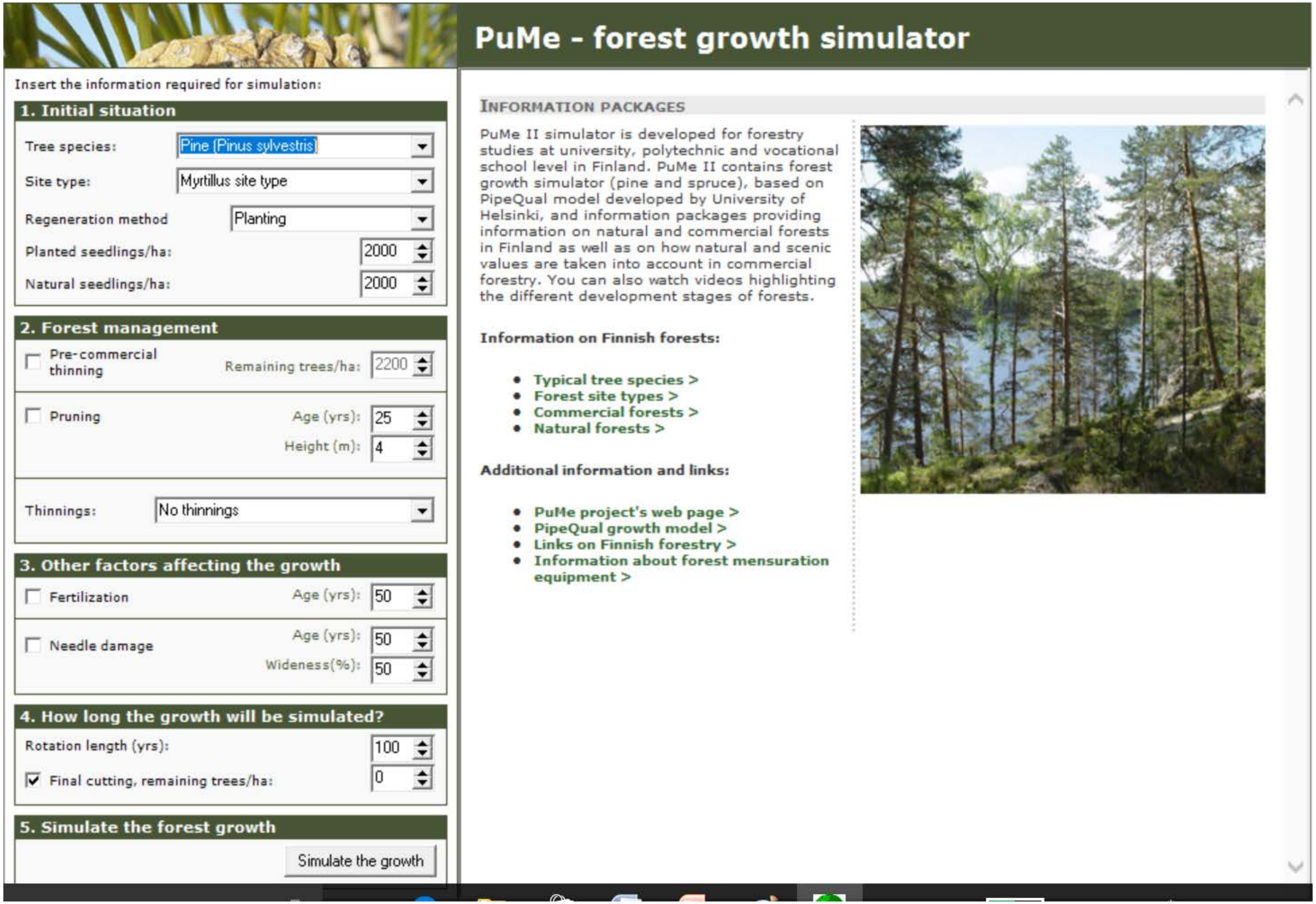
Final score

1. Recreation	0,3582
2. Timber	0,2854
3. Carbon	0,2700
4. Biodiversity	0,0864

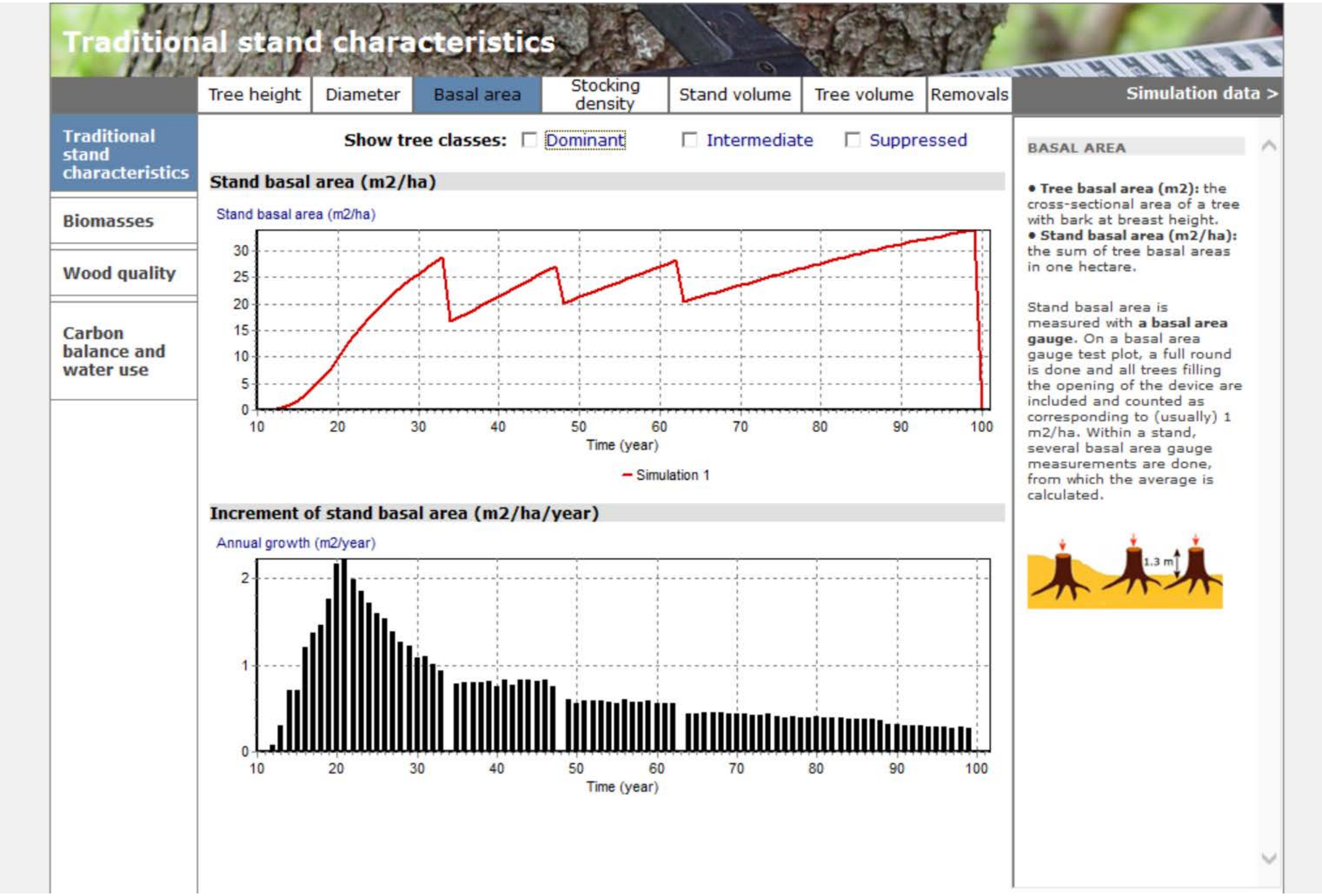
PuMe II demo

- Student version
- Process-based
- Wood quality
- Soil carbon
- Water use

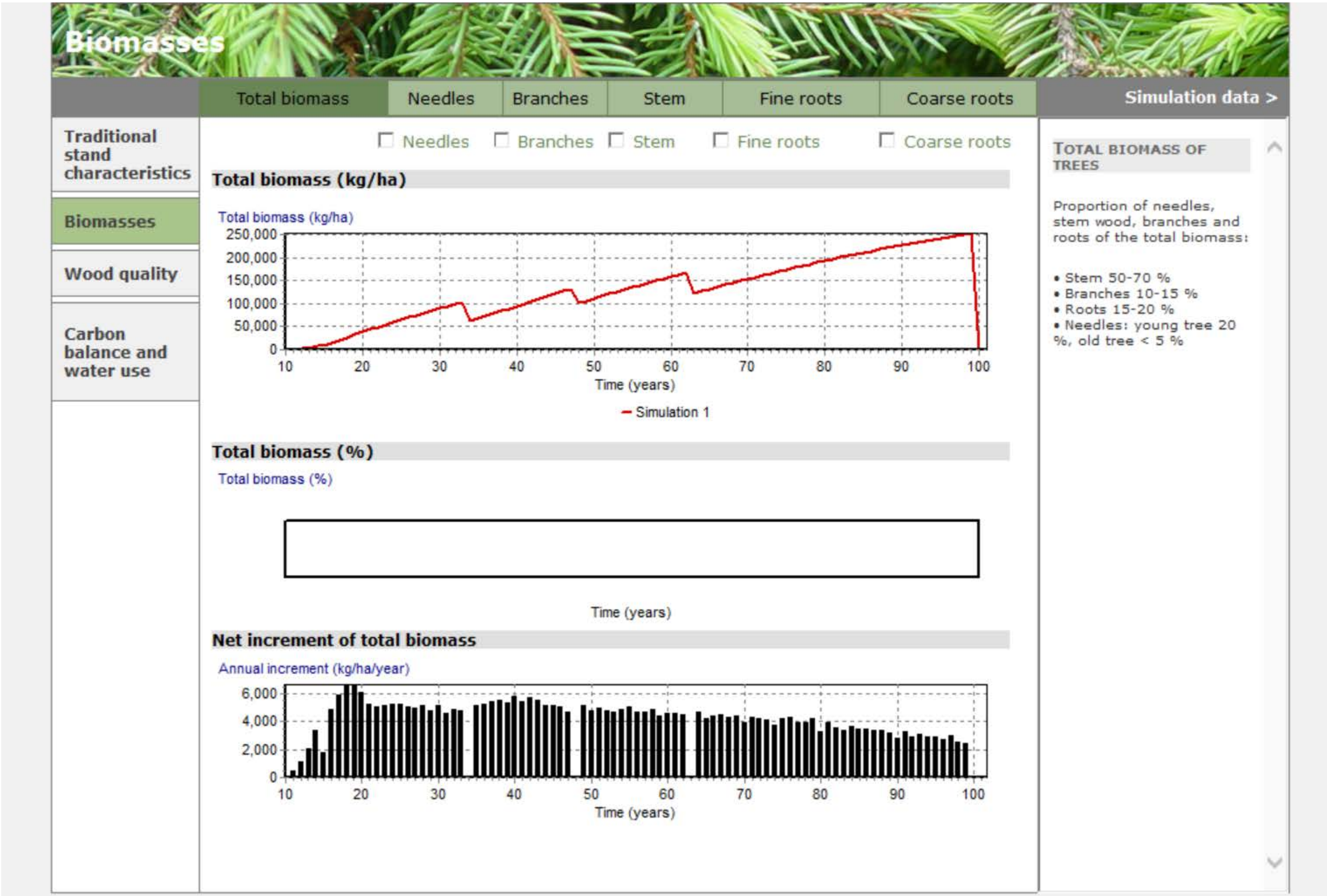
PuMe II


PuMe - forest growth simulator
Insert the information required for simulation:
1. Initial situation
Tree species:
Pine (Pinus sylvestris)
Site type:
Myrtillus site type
Regeneration method
Planting
Planted seedlings/ha:
2000
Natural seedlings/ha:
2000
2. Forest management
Pre-commercial thinning
Remaining trees/ha:
2200
Pruning
Age (yrs):
25
Height (m):
4
Thinnings:
No thinnings
3. Other factors affecting the growth
Fertilization
Age (yrs):
50
Needle damage
Age (yrs):
50
Wideness(%):
50
4. How long the growth will be simulated?
Rotation length (yrs):
100
Final cutting, remaining trees/ha:
0
5. Simulate the forest growth
Simulate the growth
INFORMATION PACKAGES
PuMe II simulator is developed for forestry studies at university, polytechnic and vocational school level in Finland. PuMe II contains forest growth simulator (pine and spruce), based on PipeQual model developed by University of Helsinki, and information packages providing information on natural and commercial forests in Finland as well as on how natural and scenic values are taken into account in commercial forestry. You can also watch videos highlighting the different development stages of forests.
Information on Finnish forests:
Typical tree species >
Forest site types >
Commercial forests >
Natural forests >
Additional information and links:
PuMe project's web page >
PipeQual growth model >
Links on Finnish forestry >
Information about forest mensuration equipment >

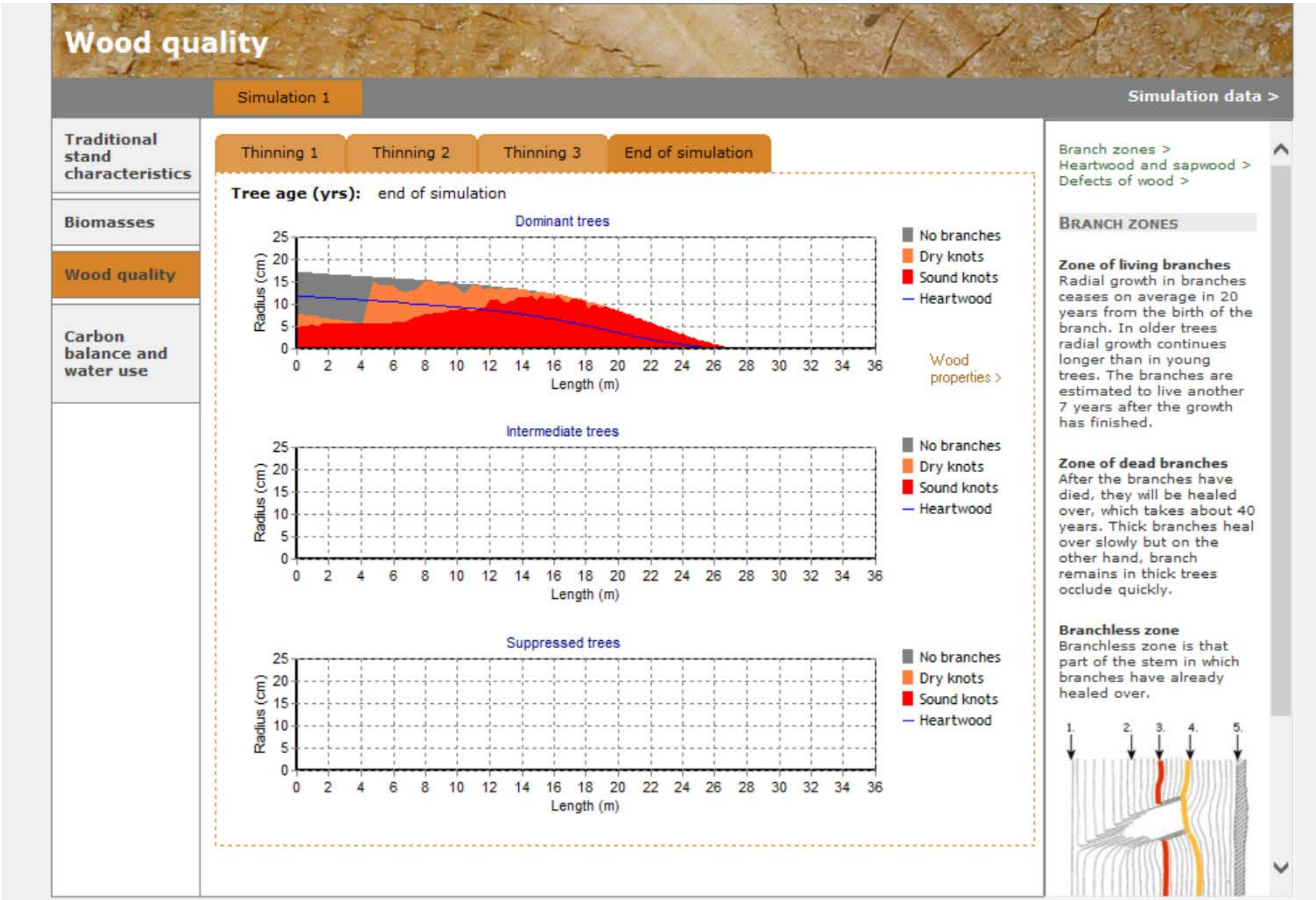
Silvicultural recommendations



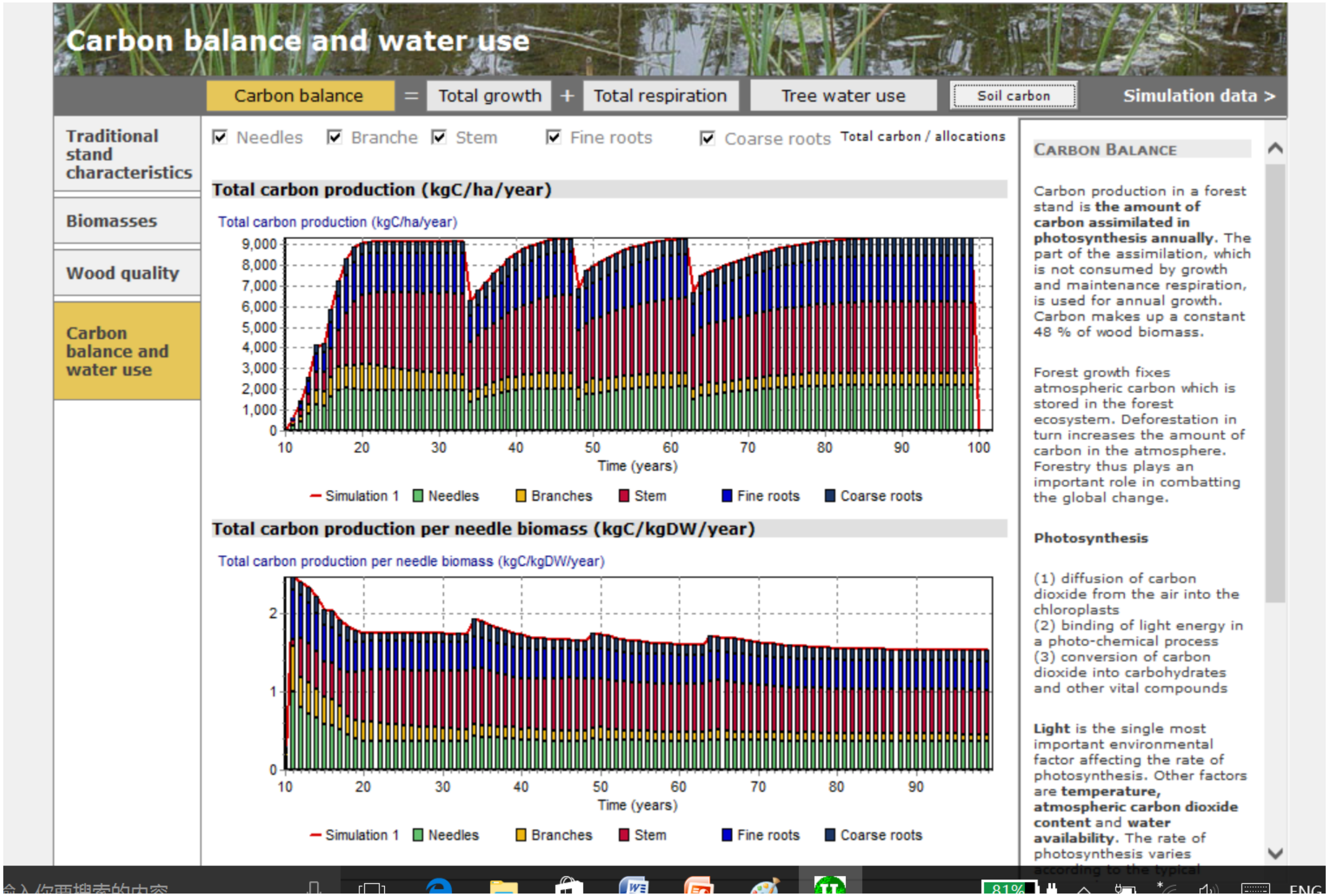
Biomass production



Wood quality



Carbon balance and water use



Optimization modeling techniques

- Linear vs. Nonlinear
- Convex vs. Non-convex
- Strictly convex vs. Pseudo-convex vs. Quasi-convex
- One-dimensional vs. Multidimensional
- Deterministic vs. Stochastic
- Constrained vs. Unconstrained
- Single-objective vs. Multi-objective

- Source: Bazaraa et al. (1993), Taha (1997), Bertsekas (1999), Boyd and Vandenberghe (2004)

NLP: theory and algorithms

- Overview
- Mathematical background
- Convex sets
- Convex functions
- The FJ and KKT optimility conditions
- Lagrangian duality and saddle point
- Algorithms
- Unconstraint optimization
- Penality and barrier functions
- Interior point method

Overview

Optimization problem has the form

$$\begin{array}{ll} \text{minimize} & f_0(x) \\ \text{subject to} & f_i(x) \le b_i, i = 1, \ldots, m. \end{array}$$

where $x \in \mathbb{R}^n$ is the optimization variable, $f_0(x) : \mathbb{R}^n \mapsto \mathbb{R}$ is the objective function,$f_i : \mathbb{R}^n \mapsto \mathbb{R}$ are (inequality) constraint functions and b_i are the limits, or bounds, for the constraints.

A vector x^* is called optimal, or a solution if it has the smallest objective value among all vectors z that satisfy the constraints: for any z with $f_1(z) \le b_1, f_2(z) \le b_2, \ldots, f_m(z) \le b_m$ we have $f_0(z) \ge f_0(x^*)$.

Optimization classifications

Optimization problems can be divided into classes based on the properties of the objective and constraint functions.

A linear problem satisfies

$$f_i(\alpha x + \beta y) = \alpha f_i(x) + \beta f_i(y),$$

for all $x, y \in \mathbb{R}^n$ and for al $\alpha, \beta \in \mathbb{R}$. Problems that do not satisfy the above are called non-linear.

A convex problem satisfies

$$f_i(\alpha x + \beta y) \leq \alpha f_i(x) + \beta f_i(y),$$

for all $x, y \in \mathbb{R}^n$ and for all $\alpha, \beta \geq 0$ and $\alpha + \beta = 1$. Problems that do not satisfy the above are called non-convex.

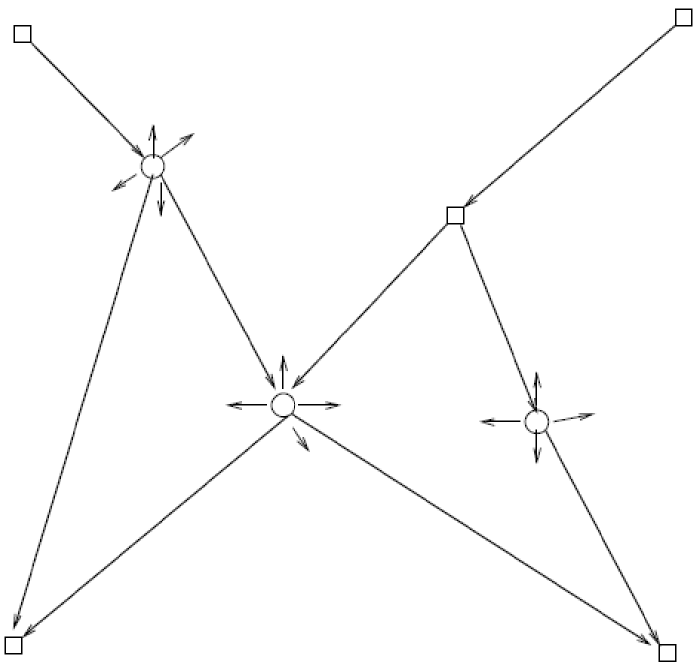
Note: convexity is more general than linearity: instead of an equality, we only require an inequality to be satisfied, and only for a certain values of α and β.

Historical notes

- The traditional division in optimization literature was linear vs. non-linear problems as the former were thought to be "easiter" to solve than the latter.
- The more recent division is between convex and non-convex problems, as it has been found that non-linear problems that are convex are often "almost as easy" to solve as linear problems, while non-convex non-linear problems often pose problems.
- One of the aims of this course is to cover a significant portion of these efficient NLP techniques.

Application -- facility location

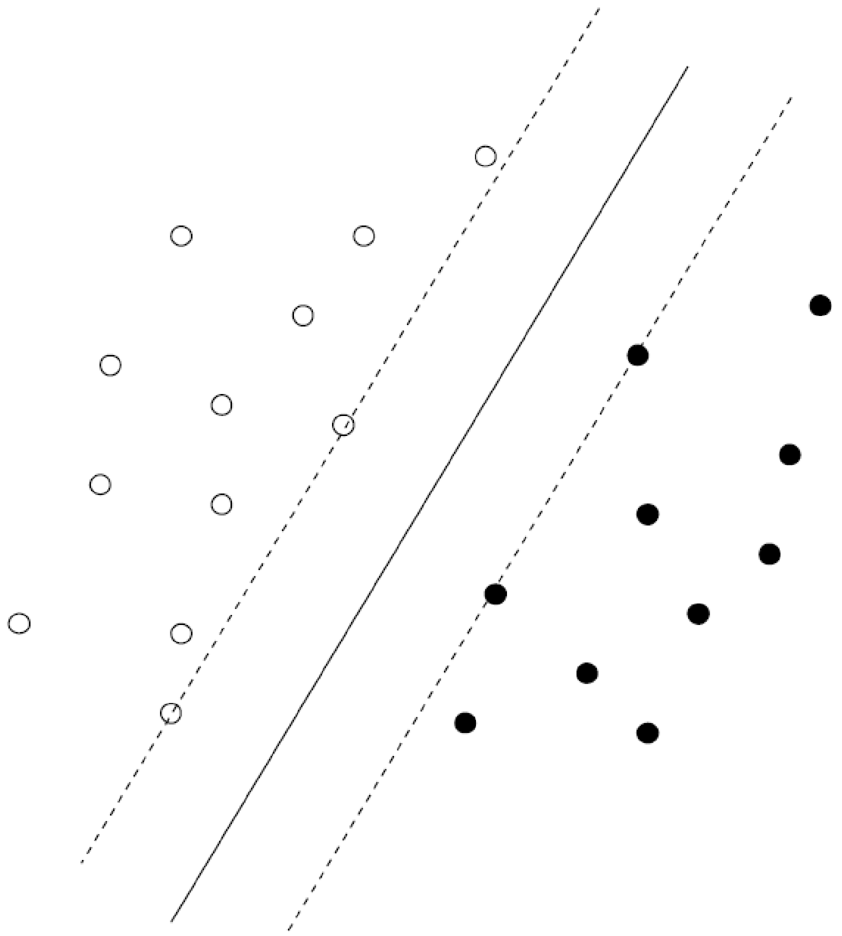
- In facility location problems one considers a set of facilities that need to b e placed optimally with respect to some a priori fixed locations.
- In minimax delay placement, one has a directed graph of nodes on a plane with some of the node locations fixed and some being free.
- The airm is to minimize the longest path between a source and sink node.



Application -- support vector classification

Given a set of labeled points $(x, y), x \in \mathbb{R}^n, y \in \{-1, +1\}$, a support vector machine aims at finding the hyperplane separating the differently labeled points in a way that the minimum distance (or margin) from a point to the hyperplane is maximized.

This maximization problem is a convex optimization problem, a convex quadratic programme to be exact.



Application -- portforlio optimization

- We have a set of assets or stocks held over a period of time.
- Assume prices change according to a probability distribution with known mean and variance. Then the return of the portfolio is a random variable as well.
- The classical Markovitz porfolio optimization problem of finding the distribution of investments among the set of assets that minimizes the variance of the return, subject to the return being above given threshold, is a convex (quadratic) optimization problem.

Concepts

- Vectors and matrices
- Norms
- Open and closed sets, supremum and infimum
- Functions
- Derivatives

Set notation, supremum, infimum

- Set membership $x \in X$, set union $X_1 \cup X_2$, set intersection $X_1 \cap X_2$, quantifiers $\exists$ (there exists), $\forall$ (for all), empty set $\emptyset$.
- Set of real numbers $\mathbb{R}$, extended real numbers $\mathbb{R} \cup \{\infty\} \cup \{-\infty\}$
- Intervals: Open $(a, b) = \{x|a < x < b\}$, closed $[a, b] = \{x|a \leq x \leq b\}$, and half-open $[a, b) = \{x|a \leq x < b\}, (a, b] = \{x|a < x \leq b\}$.
- Supremum of a non-empty set of real numbers X, denoted $\sup X$ is the least number y that satisfies $x \leq y$ for all $x \in X$. Infimum of a non-empty set, denoted $\inf X$ is the largest number y such that $x \geq y$ for all $x \in X$. If X is unbounded from above, $\sup X = \infty$. If X is unbounded from below $\inf X = -\infty$.
- For empty sets we use the conventions $\inf \emptyset = \infty$ and $\sup \emptyset = -\infty$.
- If the supremum (resp. infimum) belongs to the set, supremum equals the maximum element (resp. minimum element) of the set.

Vectors

- We denote by $\mathbb{R}^n$ the set of n-dimensional real vectors. For any $x \in \mathbb{R}^n$ we use x_i to indicate the i'th coordinate (also called component), we also write $x = (x_1, \ldots, x_n)$.
- Vectors will be viewed as column vectors. For any vector $x \in \mathbb{R}^n$, the transpose x^T denotes the n-dimensional row vector.
- The inner product of two vectors $x, y \in \mathbb{R}^n$is defined by

$$x^T y = \sum_{i=1} x_i y_i,$$

.

- Vectors x, y and called orthogonal if $x^T y = 0$.
- For vector x the notation $x > 0$ and $x \geq 0$ denote that all its components are positive and nonnegative, respectively. For any two vectors x, y the notation $x > y$ denotes that the difference vector $d = x - y$ satisfies $d > 0$. The notation $x \geq y$, $x < y$ and $x \leq y$ is defined analogously

Vector sum, Cartesian product

If X is a set and λ is a scalar, we denote by λX the set $\{\lambda x | x \in X\}$.

If X_1 and X_2 are two subsets of $\mathbb{R}^n$, we denote by $X_1 + X_2$ the set

$$\{x_1 + x_2 | x_1 \in X_1, x_2 \in X_2\},$$

which is referred to as the vector sum of X_1 and X_2.

When one of the sets consists of a single vector $\bar{x}$ we use the simplified notation $\bar{x} + X$ to denote $\{\bar{x} + x | x \in X\}$. We also denote by $X_1 - X_2$ the set $\{x_1 - x_2 | x_1 \in X_1, x_2 \in X_2\}$

Given sets $X_i \subset \mathbb{R}^{n_i}, i = 1, \ldots, m$, the Cartesian product of the sets, denoted by $X_1 \times \cdots \times X_m$ is the set

$$\{(x_1, \ldots, x_m) | x_i \in X_i, i = 1, \ldots, m\},$$

which is a subset of $\mathbb{R}^{n_1 + \cdots + n_m}$.

Subspaces and linear independence

- A non-empty subset S of $\mathbb{R}^n$ is a subspace if $ax + by \in S$ for every $x, y \in S$ and every $a, b \in \mathbb{R}$.
- An affine set or linear manifold in $\mathbb{R}^n$ is a translated subspace, i.e., a set X of the form $X = \bar{x} + S = \{\bar{x} + x | x \in S\}$ where $\bar{x} \in \mathbb{R}^n$ and $S \subset \mathbb{R}^n$. S is called the subspace parallel to X.
- The span of a finite collection $\{x_1, \ldots, x_m\}$ of elements or $\mathbb{R}^n$ is the subspace consisting of all vectors y of the form $y = \sum_{k=1}^{m} \alpha_k x_k$, where $\alpha_k \in \mathbb{R}$.
- The vectors $x_1, \ldots, x_k$ are called linearly independent if there exists no set of scalars α_1, α_k, at least one of which is non-zero, such that $\sum_{k=1}^{m} \alpha_k x_k = 0$.

Basis, dimension and orthogonal complement

A set of linearly independent vectors in subspace S whose span is equal to S is called a basis for S.

Every basis of a given subspace has the same number of vectors, this number is called the dimension of S.

The dimension of an affine set $\bar{x} + S$ is the dimension of the corresponding subspace S.

Every subspace of non-zero dimension has a basis that is orthogonal, i.e. $x^T z = 0$ for any two vectors x, z of the basis.

Given any set X, the set of vectors that are orthogonal to all elements of X,

$$X^\perp = \{y | y^T x = 0, \forall x \in X\},$$

is called the orthogonal complement of X. Any vector can be uniquely decomposed as a sum of a vector from a subspace and its orthogonal complement.

Matrix range, null space and rank

We denote by $\mathbb{R}^{m\times n}$ the set of real matrices with m rows and n columns. For any $A \in \mathbb{R}^{m\times n}$ we use a_{ij} to indicate the component on the i'th row and j'th column. We may also write $[A]_{ij}$ to denote the same element.

The range space of A, denoted $\mathcal{R}(A)$, is the set of all vectors $y \in \mathbb{R}^m$ such that $Ax = y, \forall x \in \mathbb{R}^n$.

The null space of A, denoted $\mathcal{N}(A)$, is the set of all $x \in \mathbb{R}^n$ such that $Ax = 0$.

The rank of A is the dimension of $\mathcal{R}(A)$.

The range space and null space of a matrix are tied together via:

$$\mathcal{R}(A) = \mathcal{N}(A^T)^\perp$$

Euclidean norm and angle

The Euclidean norm, or ℓ_2-norm, of a vector $x \in \mathbb{R}^n$ is defined as

$$||x||_2 = (x^T x)^{1/2} = (x_1^2 + x_2^2 + \cdots + x_n^2)^{1/2},$$

i.e. the square root of the inner product of the vector with itself.

The *cosine angle* between x and y is given by

$$\cos \angle(x, y) = \frac{x^T y}{||x||_2 \, ||y||_2} = \underline{x}^T \underline{y},$$

where $\underline{x} = x / \, ||x||_2$ and $\underline{y} = y / \, ||y||_2$.

The distance of two vectors is defined as the norm of their difference vector: $\mathbf{dist}(x, y) = ||x - y||$

Norm equivalence

The Euclidean norm belongsd to the family of p-norms

$$||x||_p = (|x_1|^p + \cdots + |x_n|^p)^{1/p}$$

Other important p-norms for vectors are the ℓ_1-norm

$$||x||_1 = \sum_{i=1}^{n} |x_i|,$$

and the ℓ_∞ norm

$$||x||_\infty = \max_i |x_i|$$

The norms are equivalent in the following way. Suppose that $||\cdot||_a$ and $||\cdot||_b$ are norms on $\mathbb{R}^n$. Then it can be proven that there exist positive constants α and β such that for all $x \in \mathbb{R}^n$,

$$\alpha\, ||x||_a \leq ||x\,|_b \leq \beta\, ||x||_a$$

Matrix norms

As well as vectors, norms can be defined for matrices.

Perhaps the most important is the Frobenius norm:

$$||X||_F = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} x_i j^2 \right)^{1/2},$$

that can be seen as the counterpart of Euclidean norm for matrices.

Other matrix norms include the sum-absolute-value norm (cf. ℓ_1-norm for vectors)

$$||X||_{sav} = \sum_{i=1}^{m} \sum_{j=1}^{n} |x_{ij}|,$$

and the maximum-absolute-value norm (cf. ℓ_{infty}-norm)

$$||X||_{mav} = \max x_{ij} | i = 1, \ldots, m, j = 1, \ldots, n$$

Symmetric eigenvalue decomposition and defineteness

Let $\mathbb{S}^n$ denote the set of symmetric matrices in $\mathbb{R}^{n\times n}$. Every $A \in S^n$ can be decomposed as

$$A = Q\Lambda Q^T,$$

where Q is an orthogonal matrix (columns are orthogonal and have Euclidean norm of 1)and Λ is a diagonal matrix, with the eigenvalues (λ_1, λ_n) of A as the diagonal elements.

A matrix $A \in \mathbb{S}^n$ is called positive definite (pos. semi-definite), if all its eigenvalues are positive (non-negative). Similarly a matrix $A \in \mathbb{S}^n$ is called negative definite (neg. semi-definite) if all its eigenvalues are negative (non-positive).

Alternatively, the matrix A is seen to be positive definite, if the quadratic form $x^T Ax > 0$ for all $x > 0$. The other defineteness classes can be defined analogously.

Interior of a set

An element $x \in C \subseteq \mathbb{R}^n$ is called an *interior* point of C if there exists an $\epsilon > 0$ for which

$$\{y | \; ||y - x||_2 \leq \epsilon\} \subseteq C,$$

in other words, there exists a ball centered at x that lies entirely in C. The set of all points *interior* to C is called the *interior* of C and denoted **int** C.

All norms generate the same set of interior points (this is a consequence of the equivalence of the norms)

Open and closed sets

A set C is *open* if $\textbf{int}\ C = C$, that is, every point in C is an interior point. A set $C \subseteq \mathbb{R}^n$ is *closed* if its complement $\mathbb{R}^n \setminus C$ is open.

The *closure* of a set C is defined as

$$\textbf{cl}\ C = \mathbb{R}^n \setminus \textbf{int}(\mathbb{R} \setminus\ C),$$

in other words, as the complement of the interior of the complement of C.

A point x is in the closure of C if for every $\epsilon > 0$, there is a $y \in C$ with $||x - y||_2 \leq \epsilon$

The *boundary* of the set C is defined as

$$\textbf{bd}\ C = \textbf{cl}\ C \setminus \textbf{int}\ C$$

A point $x \in \textbf{bd}\ C$ is called a *boundary point.*

Characterizing open and closed sets

Characterization using convergent sequences and limit points: A set C is closed if and only if contains the limit point of every convergent sequence in it. The boundary of the set is the set of all limit points of convergent sequences in C.

Characterization using the boundary concept: C is closed if $\mathbf{bd} \ \in C$. It is open if it contains no boundary points $C \cap \mathbf{bd}C = \emptyset$

Functions and continuity

- $f : X \mapsto Y$ denotes a function that has a domain X, denoted $\mathbf{dom} f$ and range Y.
- If U and V are subsets of X and Y, respectively, the set $\{f(x)|x \in U\}$ is the image of U (under f) and the set $\{x|f(x) \in V\}$ is the inverse image (or preimage) of set V.
- A function $f : \mathbb{R}^n \mapsto \mathbb{R}^m$ is continuous at $x \in \mathbf{dom} f$ if for all $\epsilon > 0$ there exists a δ such that

$$y \in \mathbf{dom} f, ||y - x|| \leq \delta \Rightarrow ||f(y) - f(x)|| \leq \epsilon$$

.

- Continuity can be expressed in terms of limits: whenever the sequence $x_1, x_2, \cdots \in \mathbf{dom} f$ converges to to a point in $x \in \mathbf{dom} f$, the sequence $f(x_1), f(x_2), \ldots$ converges to $f(x)$
- A function is continuous if it is continuous in every point of its domain

Closed functions

A function $f : \mathbb{R}^n \mapsto \mathbb{R}$ is said to be *closed* if, for each $\alpha \in \mathbb{R}$, the sublevel set

$$\{x \in \mathbf{dom}\ f \,|\, f(x) \leq a\}$$

is closed.

If $f : \mathbb{R}^n \mapsto \mathbb{R}$ is continuous, and $\mathbf{dom}\ f$ is closed, then f is closed.

If $f : \mathbb{R}^n \mapsto \mathbb{R}$ is continuous, and $\mathbf{dom}\ f$ is open, then f is closed if and only if f converges to ∞ along every sequence converging to a boundary point of $\mathbf{dom} f$.

Examples

- The function $f : \mathbb{R} \mapsto \mathbb{R}$ with $f(x) = x \log x$, $\mathbf{dom}\ f = \mathbb{R}_{++}$, is not closed.
- The function $f : \mathbb{R} \mapsto \mathbb{R}$ with

$$f(x) = \begin{cases} x \log x, & x > 0 \\ 0, & x = 0 \end{cases}, \mathbf{dom}\ f = \mathbb{R}_{+},$$

  is closed
- The function $f(x) = -\log x$, $\mathbf{dom}\ f = \mathbb{R}_{++}$, is closed.

Derivative and gradient

Suppose $f : \mathbb{R}^n \mapsto \mathbb{R}^n$ and $x \in \textbf{int dom } f$. The derivative, or *Jacobian*, of f at x is the matrix $Df(x) \in \mathbb{R}^{m \times n}$, given by

$$Df(x)_{ij} = \frac{\partial f_i(x)}{\partial x_j}, i = 1 \ldots m, j = 1 \ldots, n.$$

provided the partial derivatives exist,in which case, f is said to be differentiable at x.

The function f is *differentiable* is **dom** f is open, and it is differentiable at every point in its domain.

The affine function of z given by

$$f(x) + Df(x)(z - x)$$

is called the *first order approximation* of f at (or near) x.

Gradient

When f is real-valued $f : \mathbb{R}^n \mapsto \mathbb{R}$, the Jacobian is a $1 \times n$ matrix, that is a row vector. Its transpose is called the *gradient* of f:

$$\nabla f(x) = Df(x)^T.$$

Example: Consider the quadratic function $f : \mathbb{R}^n \mapsto \mathbb{R}$,

$$f(x) = 1/2x^T Hx + q^T x + r,$$

where $H \in S^n$, $q \in \mathbb{R}^n$, and $r \in \mathbb{R}^n$.

The gradient of f is

$$\nabla f(x) = Hx + q$$

Second derivative

The second derivative, or *Hessian* matrix of $f : \mathbb{R}^n \mapsto \mathbb{R}$ at $x \in \mathbf{int\,dom}\, f$, is given by

$$[\nabla^2 f(x)]_{ij} = \frac{\partial^2 f(x)}{\partial x_i \partial x_j}, i = 1, \ldots, n, j = 1, \ldots, n,$$

provided that f is twice differentiable at x

The *second order approximation of* f, at or near x, is the quadratic function of z, given by

$$\hat{f}(z) = f(x) + \nabla f(x)^T (z - x) + 1/2 (z - x)^T \nabla^2 f(x)(z - x).$$

Example: the second derivative of

$$f(x) = 1/2 x^T H x + q^T x + r,$$

is the matrix H.

The H&J Algorithm

Initialization Step

- Choose a scalar $\varepsilon > 0$ to be used in terminating the algorithm. Choose a starting point x1, let y1 = x1, let k=j=1, and got to the main step.

Main step

- Let λ_j be an optimal solution to the problem to minimize $f(y_j + \lambda d_j)$ s.t. λ in E^1, and let $y_{j+1} = y_j + \lambda_j * d_j$. If $j < n$, replace j by j+1, and repeat step 1. Otherwise, if $j = n$, let $x_{k+1} = y_{n+1}$. If $||x_{k+1} – x_k|| < \varepsilon$, stop; otherwise, go to step 2.
- Let $d = x_{k+1} – x_k$, and let $\hat{\lambda}$ be an optimal solution to the problem to minimize $f(x_{k+1} + \lambda d)$ s.t. λ in E^1. Let $y_1 = x_{k+1} + \hat{\lambda} * d$, let j=1, replace k by k+1, and repeat step 1.

Method of Hooke and Jeeves

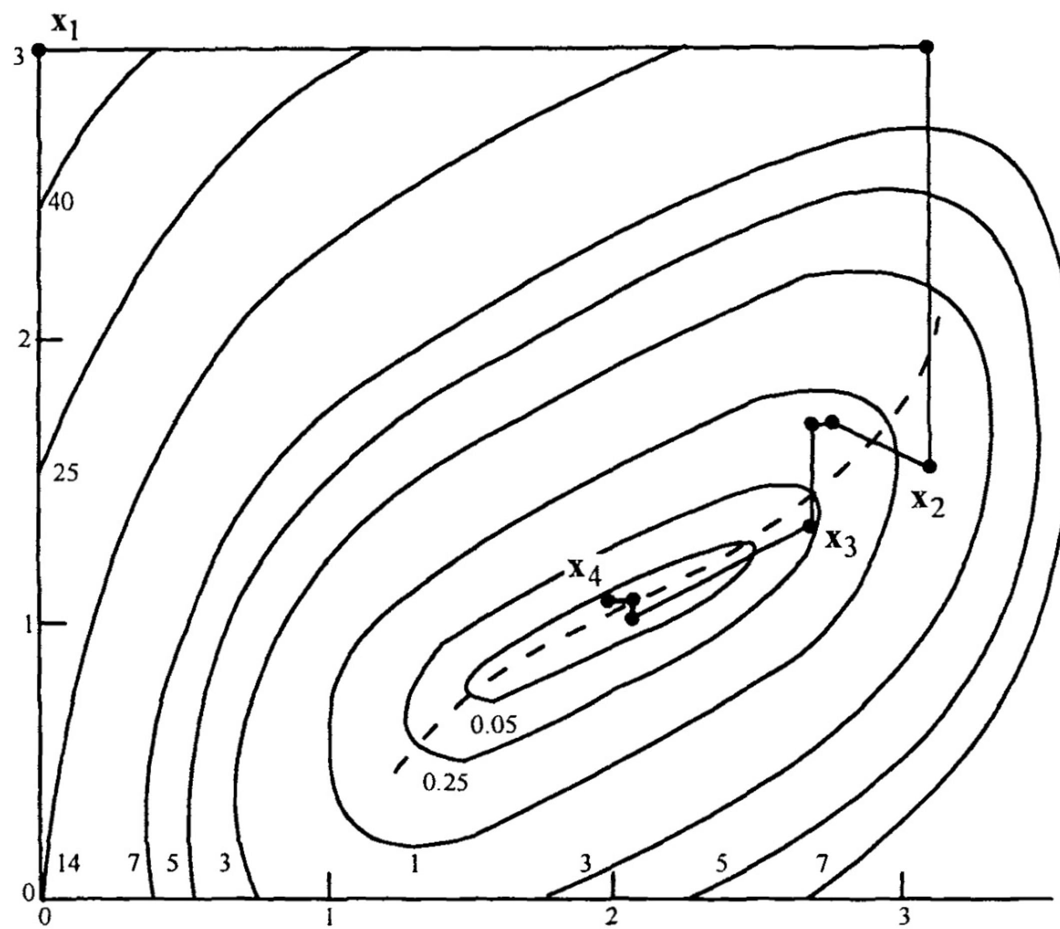


Figure 8.10 Method of Hooke and Jeeves using line searches. Method of Hooke and Jeeves with Discrete Steps

Direct and random search

- Osyczka (1984) modified the version of discrete steps of the Hooke and Jeeves Method (1961) to overcome local optimum problems.
- The Osyczka's (1984) direct and random search algorithm (DRS) is a hybrid algorithm based on neighborhood search, shotgun search and Hooke and Jeeves' direct search.
- By integrating neighborhood search and random search into direct search phases, DRS has proved to be a successful method for solving forest management problems.
- The details of DRS were explained in Valsta (1992).

Particle Swarm Optimization

- Particle Swarm Optimization (PSO) is a stochastic global optimization algorithm inspired by swarm behavior in birds, insects, fish, even human behavior (Kennedy and Eberhart, 1995).
- In PSO, each particle (individual) adjusts its position and velocity, moves to some global objective through information exchange between its neighbor particles and the whole swarm (population).
- PSO carries out a five steps search:

PSO, con't

- 1) Randomly generate initial swarm (population) which consists of m particles (individuals), each particle $\boldsymbol{x}_i=(x_{i1},x_{i2},\ldots x_{in})$ has velocity $\boldsymbol{v}_i=(v_{i1},v_{i2},\ldots v_{in})$.
- 2) Evaluate each particle, store the previous best position for each particle $\boldsymbol{pbest}_i=(p_{i1},p_{i2},\ldots,p_{in})$, and find the global best for the entire population $\boldsymbol{gbest}=(g_1,g_2,\ldots,g_n)$.
- 3) Update the $i+1$th generation $\boldsymbol{x}_{i+1}=\boldsymbol{x}_i+\boldsymbol{v}_{i+1}$, where $\boldsymbol{v}_{i+1}$ is updated as

$$\boldsymbol{v}_{i+1}= w\boldsymbol{v}_i+c_1r_1(\boldsymbol{pbest}_i-\boldsymbol{x}_i)+c_2r_2(\boldsymbol{gbest}-\boldsymbol{x}_i),$$

where $w=(0.4+(0.9-0.4)(80-i)/80)$ is the inertia factor decreased linearly from 0.9 to 0.4, c_1 and c_2 are constants, r_1, r_2 are random values between [0,1].

PSO, con't

4) Evaluate every new particle, $\boldsymbol{x}_i = \boldsymbol{x}_{i+1}$ if $f(\boldsymbol{x}_{i+1}) < f(\boldsymbol{x}_i)$, otherwise $\boldsymbol{x}_{i+1} = \boldsymbol{x}_i$. Compare the best value $f(\boldsymbol{x}_{i+1})$ with $f(\boldsymbol{pbest}_i)$ and $f(\boldsymbol{gbest})$, if $f(\boldsymbol{x}_{i+1}) < f(\boldsymbol{pbest}_i)$, $\boldsymbol{pbest}_{i+1} = \boldsymbol{x}_{i+1}$, if $f(\boldsymbol{x}_{i+1}) < f(\boldsymbol{gbest})$, $\boldsymbol{gbest} = \boldsymbol{x}_{i+1}$.

5) Check whether the number of iterations reaches up its maximum limit. If not, go to step 3.

Differential Evolution

- Storn and Price (1995) proposed Differential Evolution (DE), a stochastic evolutionary algorithm to solve global optimization problems.
- In DE an offspring individual (candidate solution) is generated through mutation and crossover with the weighted difference of parent solutions.
- The offspring may replace its parent through competitive selection.
- The most applied mutation strategies are rand/1, best/1, current to best/1, best/2, and rand/2 schemes (for details, see Liu et al., 2010).
- In this version we used the mutation strategy of current to best/1 scheme rather than the rand/1 scheme used in Pukkala (2009). The method of Differential Evolution (DE) performs a six steps search:

DE, con't

- 1) Randomly generate initial population (initial parent individuals) $\boldsymbol{x}_i=(x_{i1},x_{i2},\ldots x_{in})$.
- 2) Evaluate the initial population, calculate every individual function value $f(\boldsymbol{x}_i)$, and record the optimized value and the previous best individual $\boldsymbol{pbest}_i$.
- 3) Randomly select two remainder individuals $\boldsymbol{x}_{r1}$ and $\boldsymbol{x}_{r2}$, and calculate for the mutant individual $\boldsymbol{y}_i=(y_{i1},y_{i2},\ldots y_{in})$ using the current to best/1 scheme,

$$\boldsymbol{y}_i=\boldsymbol{x}_i+\alpha(\boldsymbol{pbest}_i-\boldsymbol{x}_i)+\beta(\boldsymbol{x}_{r1}-\boldsymbol{x}_{r2}),$$

where α is a random number between [0,1], β=0.8.

DE, con't

- 4) Generate the offspring individual $\boldsymbol{x}_i'$ by a crossover operation on $\boldsymbol{x}_i$ and $\boldsymbol{y}_i$ with a crossover probability parameter (in this study CR=0.5) determining the genes of $\boldsymbol{x}_i'$ are inherited from $\boldsymbol{x}_i$ or $\boldsymbol{y}_i$. Let $x_{ij}'=y_{ij}$, if a random real number from [0,1] is less than CR, otherwise, $x_{ij}'=x_{ij}$.
- 5) Select the best individual for the next generation $\boldsymbol{x}_{i+1}$ by the competition between the offspring individual $\boldsymbol{x}_i'$ and the parent individual $\boldsymbol{x}_i$. If $f(\boldsymbol{x}_i')\leq f(\boldsymbol{x}_i)$, $\boldsymbol{x}_{i+1}=\boldsymbol{x}_i'$, otherwise, $\boldsymbol{x}_{i+1}=\boldsymbol{x}_i$.
- 6) Check whether the number of iterations reaches the maximum limit. If not, go to step 3.

Evolution Strategy

- The Evolution Strategy (ES) uses strategy parameters to determine how a recombinant is mutated.
- ES generates an offspring as a mutated recombination from two parents. One of the parents is the previous best individual, and another one is randomly drawn from the remaining individuals.
- The offspring then compete with the parents. If the offspring is better, the mutated solution replaces the worst solution of the parent population. The best solution at the last generation is the optimal solution.
- ES conducts a five steps search:

ES, con't

- 1) Randomly generate initial population $\boldsymbol{x}_i$. The initial strategy parameters $\boldsymbol{\sigma}_i$ was calculated from $\boldsymbol{\sigma}_i=\alpha\boldsymbol{x}_i$, where α=0.2.
- 2) Obtain the previous best individual $\boldsymbol{pbest}_i$ and $\boldsymbol{\sigma best}_i$ strategy parameter values. Recombine the selected parents, the best individual $\boldsymbol{pbest}_i$ and random individual $\boldsymbol{x}_r$, obtain the recombined individual $\boldsymbol{x}_m=0.5(\boldsymbol{pbest}_i+\boldsymbol{x}_r)$, and strategy parameters

$$\boldsymbol{\sigma}_m=0.5(\boldsymbol{\sigma best}_i+\boldsymbol{\sigma}_r)\times\exp(\tau_g\times N(0,1)+\tau_l\times N(0,1)),$$

where the global study parameter τ_g is $1/\sqrt{(2\times n_d)}$, the local study parameter τ_l is $1/\sqrt{(2\sqrt{n_d})}$, $N(0,1)$ is a normally distributed random number.

ES, con't

- 3) Mutate an offspring individual $\boldsymbol{x}' = \boldsymbol{x}_m + \boldsymbol{\sigma}_m \times N(0,1)$.
- 4) Evaluate the new individual $\boldsymbol{x}'$. Replace the worst solution of the parent generation if $f(\boldsymbol{x}')$ is less than the worst function value.
- 5) Check whether the number of iterations reaches its maximum limit. If not, go to step 2.

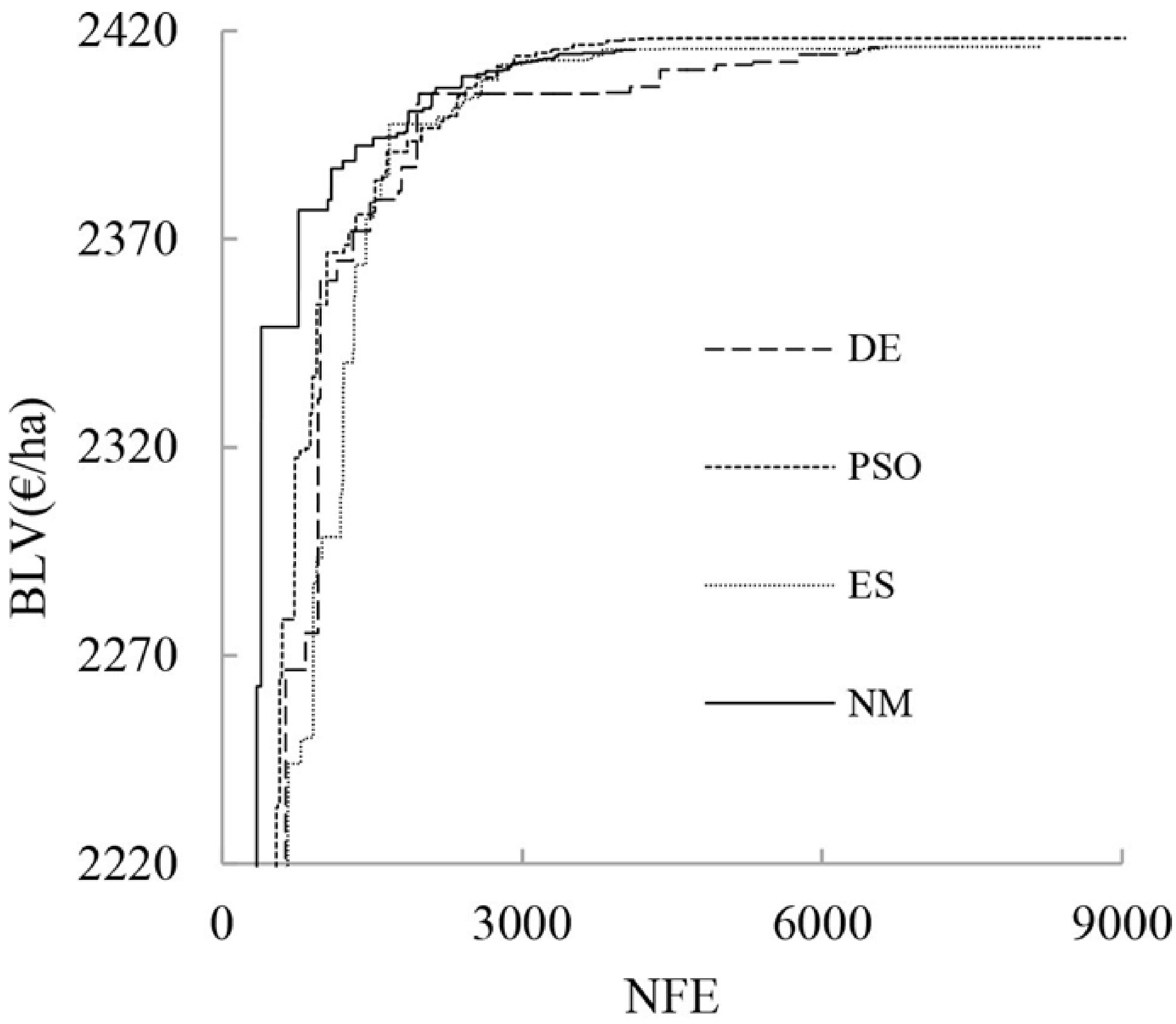
Nelder-Mead

- Similar to ES, Nelder-Mead (NM) also uses a new candidate solution to replace the worst solution of all solutions at every iteration.
- In NM the new candidate solution is calculated based on the centroid solution and the best solution through reflection, expansion, and contraction operations.
- In case none of better new candidate solutions can be found in the reflection, expansion, and contraction operations, NM carries out an additional shrinking operation for a new iteration by updating all candidate solutions except the best solution.
- In NM all operations are calculated without stochasticity.
- NM implements a six steps search (Lagarias et al. 1998):

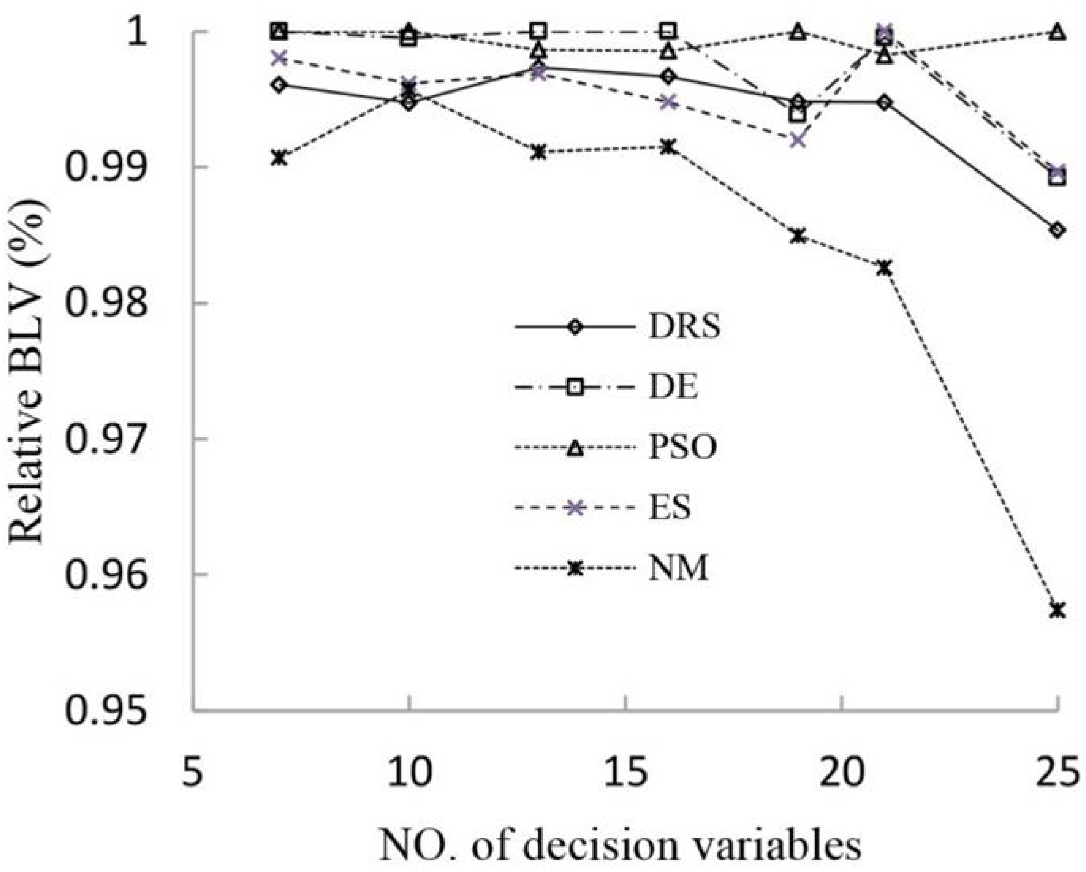
NM, con't

- 1) Randomly generate initial population. Select the best, the worst, the second worst solutions $\boldsymbol{x}_b$, $\boldsymbol{x}_w$, $\boldsymbol{x}_{sw}$ from all candidate solutions by their function values $f(\boldsymbol{x}_b)$, $f(\boldsymbol{x}_w)$, $f(\boldsymbol{x}_{sw})$.
- 2) Calculate the reflection point $\boldsymbol{x}_{rf}=(1+\rho)\boldsymbol{x}_m-\rho\boldsymbol{x}_w$, where the reflection parameter ρ=1.4, and the centroid point (average except the worst point $\boldsymbol{x}_w$) $\boldsymbol{x}_m=\Sigma_{i\neq w}(\boldsymbol{x}_i/(n_d-1))$. If $f(\boldsymbol{x}_b)<f(\boldsymbol{x}_{rf})<f(\boldsymbol{x}_{sw})$, replace $\boldsymbol{x}_w$, with $\boldsymbol{x}_{rf}$ and terminate the iteration.
- 3) If $f(\boldsymbol{x}_{rf})<f(\boldsymbol{x}_b)$, compute expansion point $\boldsymbol{x}_e=\chi\boldsymbol{x}_{rf}+(1-\chi)\boldsymbol{x}_m$, where the expansion parameter χ=2.5. If $f(\boldsymbol{x}_e)\leq f(\boldsymbol{x}_b)$, replace $\boldsymbol{x}_w$ with $\boldsymbol{x}_e$ and terminate the iteration; else replace $\boldsymbol{x}_w$, with $\boldsymbol{x}_{rf}$ and terminate the iteration.

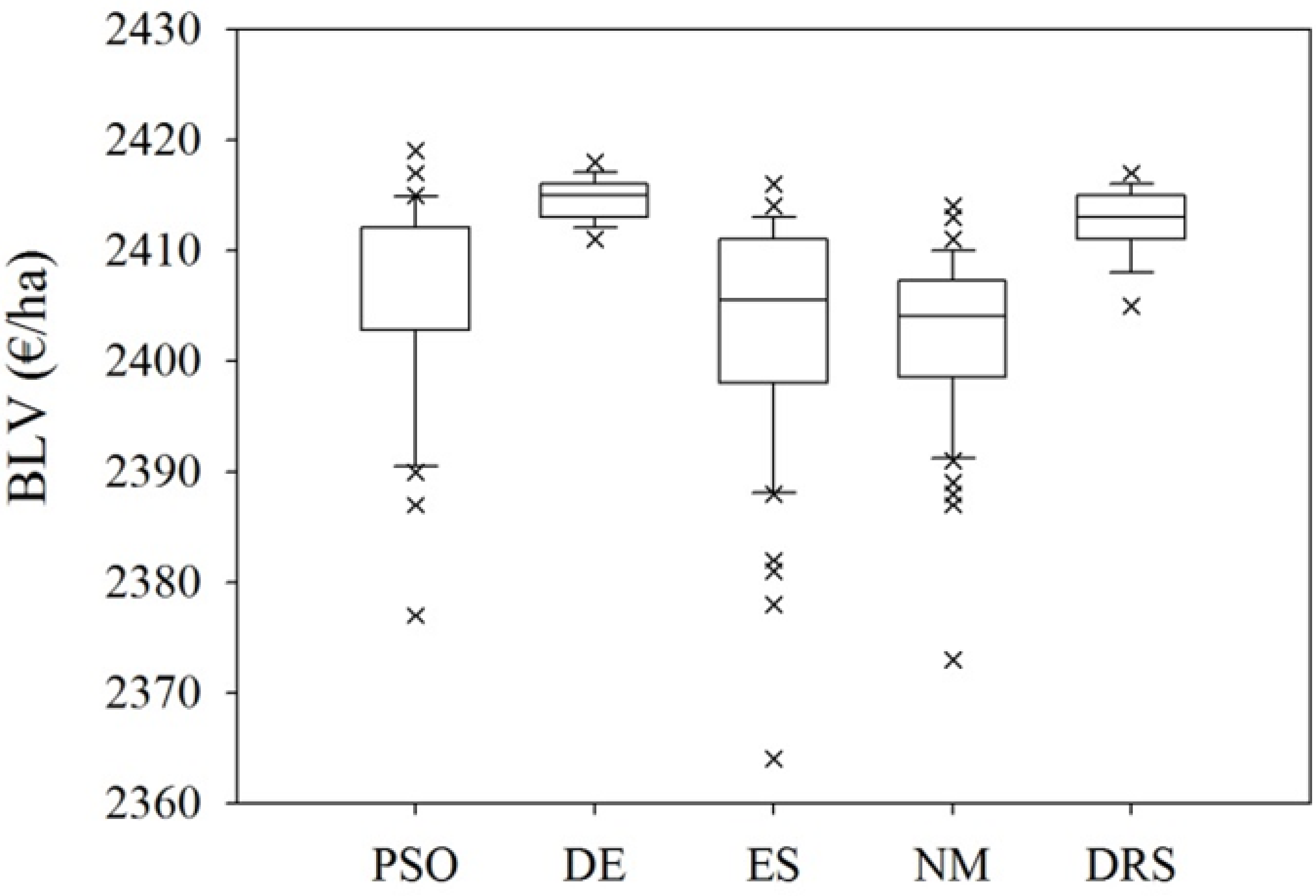
Number of function evaluations



Effects of decision variables



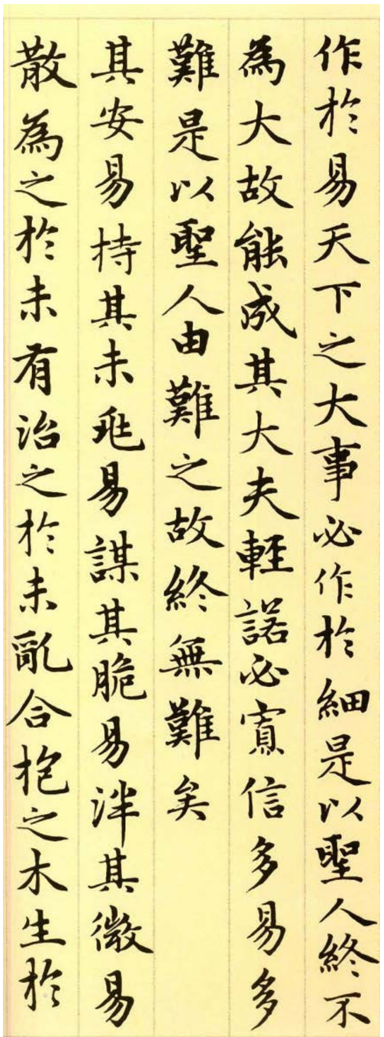
Robustness of algorithms



NM, con't

- 4) If $f(\boldsymbol{x}_{rf})>f(\boldsymbol{x}_{sw})$, compute inside contraction point $\boldsymbol{x}_c=\gamma\boldsymbol{x}_w+(1-\gamma)\boldsymbol{x}_m$, where the contract parameter $\gamma=0.5$. If $f(\boldsymbol{x}_c)\leq f(\boldsymbol{x}_w)$, replace $\boldsymbol{x}_w$ with $\boldsymbol{x}_c$ and terminate the iteration, else go to step 5. If $f(\boldsymbol{x}_{sw})\leq f(\boldsymbol{x}_{rf})<f(\boldsymbol{x}_w)$, compute outside contraction point $\boldsymbol{x}_c=\gamma\boldsymbol{x}_{rf}+(1-\gamma)\boldsymbol{x}_m$. If $f(\boldsymbol{x}_c)\leq f(\boldsymbol{x}_{rf})$, replace $\boldsymbol{x}_w$ with $\boldsymbol{x}_c$ and terminate the iteration, else go to step 5.
- 5) Compute $\boldsymbol{x}_i$ ($i\neq b$) with $\boldsymbol{x}_b$ and shrinkage parameter $\delta=0.8$ for the new generation $\boldsymbol{x}_i'=\boldsymbol{x}_b+\delta(\boldsymbol{x}_i-\boldsymbol{x}_b)$, and begin a new iteration.
- 6) Check whether the number of iterations reaches its maximum limit. If not, go to step 2.

Forest management planning


作於易天下之大事必作於細是以聖人終不
為大故能成其大夫輕諾必寡信多易多
難是以聖人由難之故終無難矣
其安易持其未兆易謀其脆易泮其微易
散為之於未有治之於未亂合抱之木生於

Therefore deal with things before they happen; Create order before there is confusion.

Observation data

- Which type of information is more important in forestry?

- Forest stand data from
  - Earlier inventory
  - Later inventory

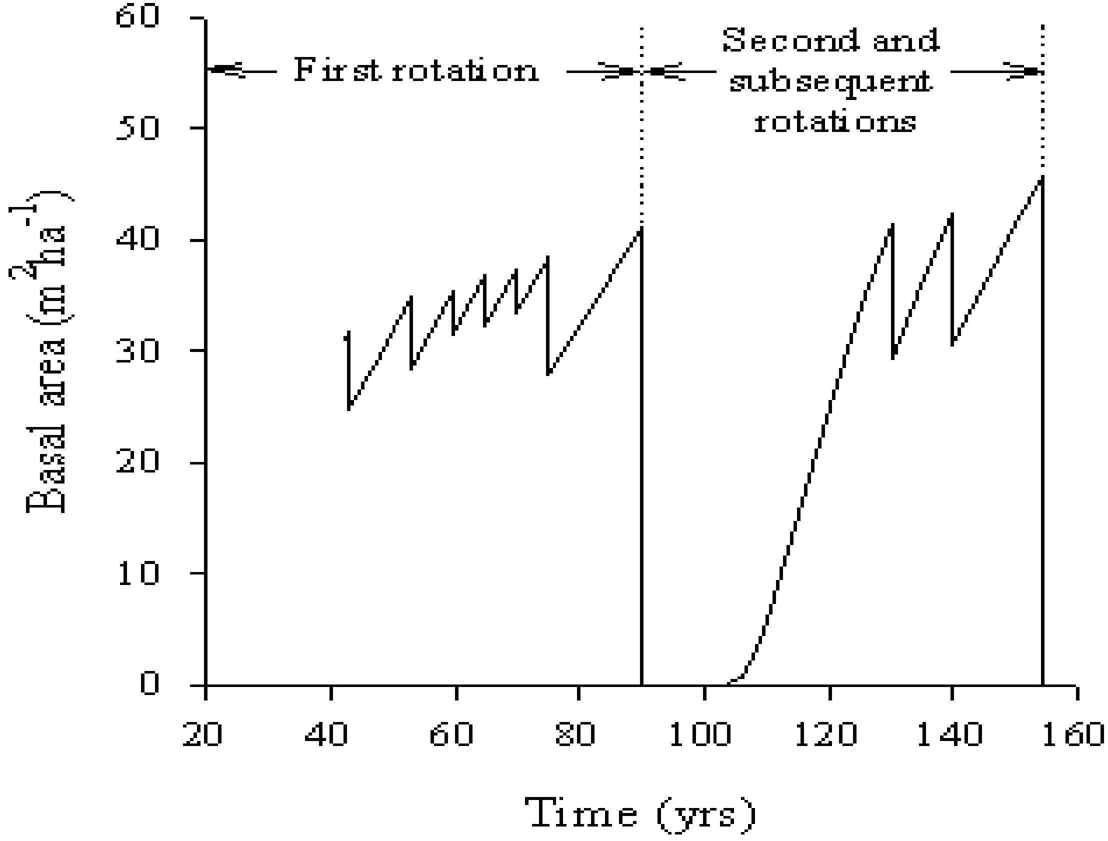
4.04.00 – Forest management planning

- 4.04.02 – Planning and economics of fast-growing plantation forests
- 4.04.03 – SilvaPlan: Forest management planning terminology
- 4.04.04 – Sustainable forest management scheduling
- 4.04.06 – Nature conservation planning
- 4.04.07 – Risk analysis
- 4.04.08 – Adaptation to climate change

Even-aged management

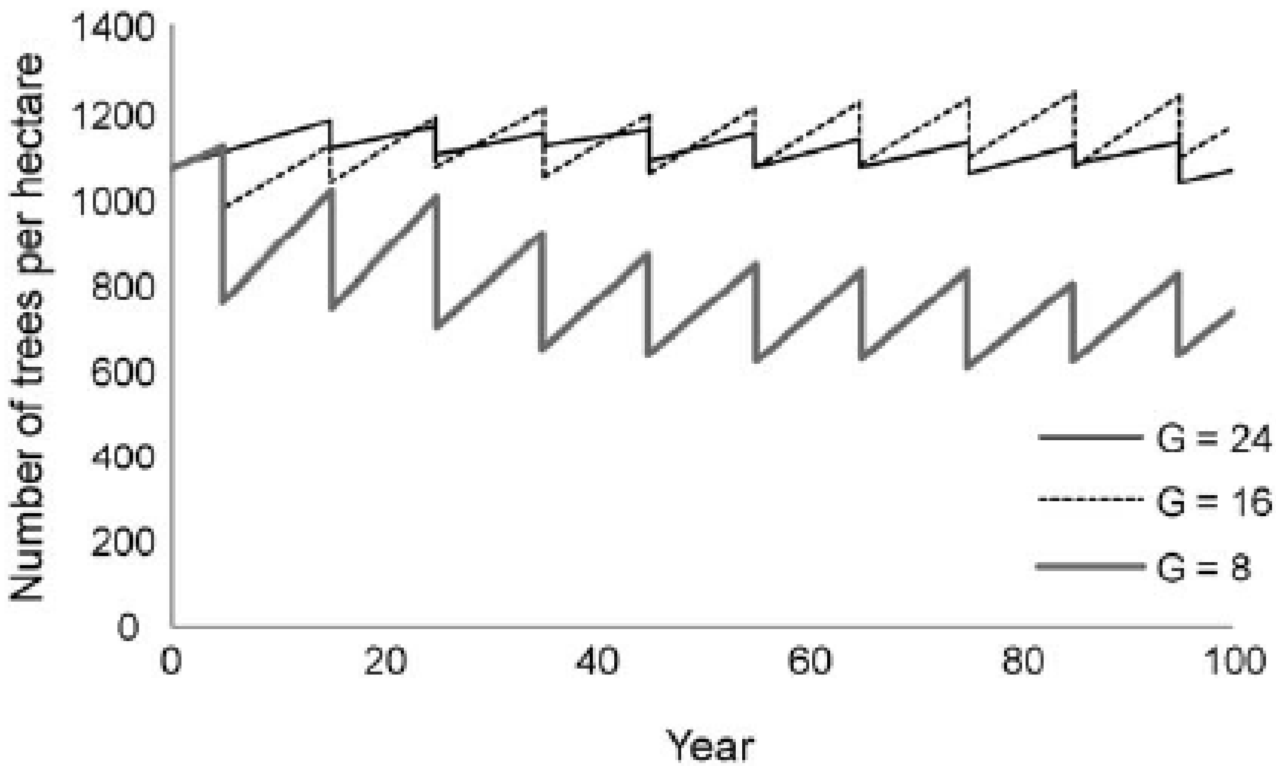
- Even-aged management deals with forests composed of even-aged stands.
- In such stands, individual trees originate at about the same time, either naturally or artificially.
- In addition, stands have a specific termination date at which time all remaining trees are cut.
- This complete harvest is called a *clear-cut*.

同龄林经营 Even-aged management



Source: Cao et al. (2006), Fig. 1

异龄林经营 Uneven-aged management



Source: Pukkala et al. (2009), Fig. 8

Regeneration methods

- Regeneration of even-aged stands may be done by planting or seeding. The latter may be natural. For example, in a *shelterwood* system a few old trees are left during the period of regeneration to provide seed and protect the young seedlings.
- Natural regeneration may continue for a few years after initial planting or seeding.
- Nevertheless, the basic management remains the same, it leads to a total harvest and a main crop when the stand has reached rotation age.
- Light cuts called “thinnings” are sometimes done in even-aged stands before the final harvest.

EXAMPLE: Converting forests

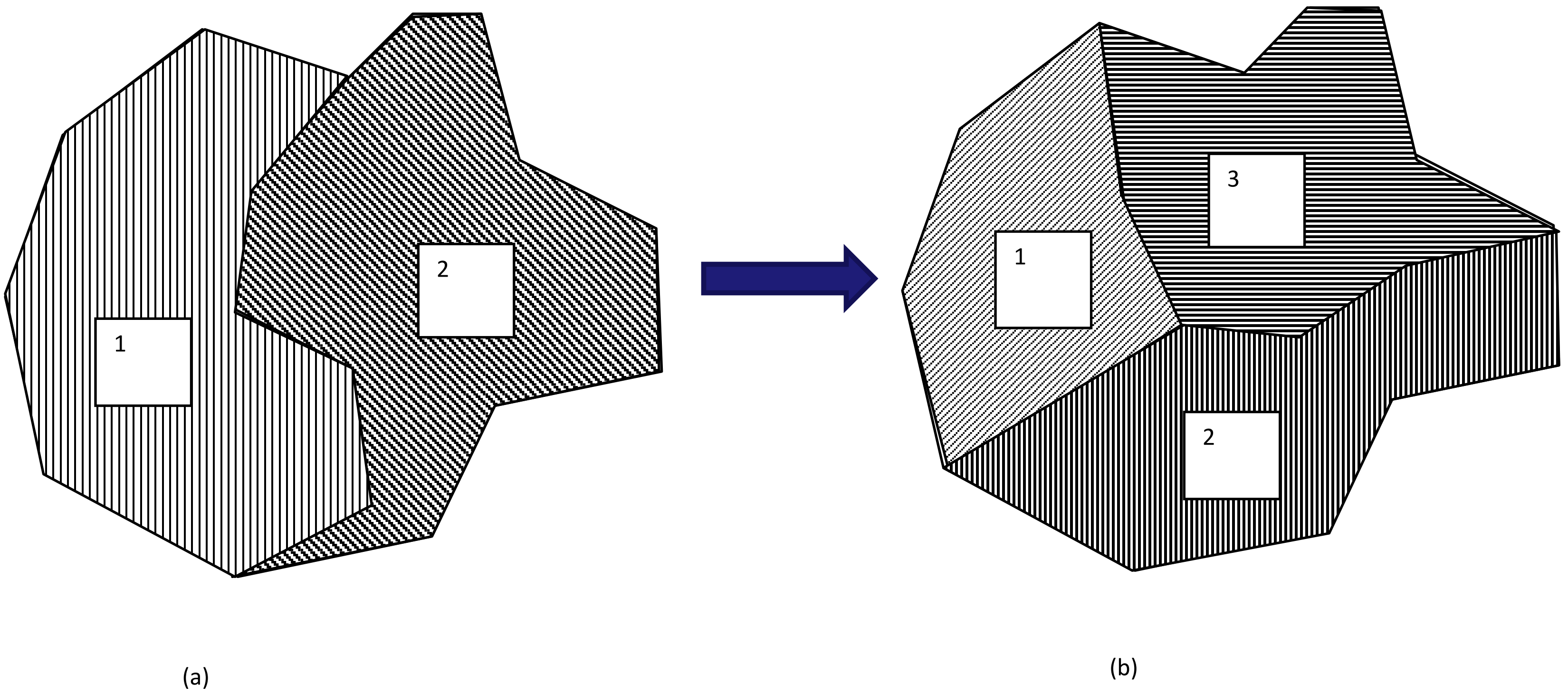


Figure 4.1 Hardwood forest with (a) two initial age classes, converted to a regulated pine plantation with (b) three age classes.

The even-aged management problem

- There are only two compartments on this forest, labeled 1 and 2. Compartment 1 has an area of 120 ha, compartment 2 has 180 ha. Southern hardwoods of low quality currently cover the two compartments. However, they are on distinct soils and timber grows better on compartment 1 than on compartment 2.
- One objective of the owner of this property is to convert the entire area to a pine plantation during a period of 15 years. The forest created at the end of this period should be *regulated*, with a rotation age of 15 years.
- That is to say, one third of the forest should be covered with trees 0 to 5 years old, a third with trees 6 to 10 years old and another third with trees 11 to 15 years old. This would lead to a pattern of age-classes like that shown in Fig. (*b*).

Even-aged management, con’t

- Finally, the owner desires to maximize the amount of wood that will be produced from his forest during the period of conversion to pine.
- However, the owner will not cut any of the pine stands before they are 15 years old.
- We shall learn how to represent this problem as a linear program with decision variables, constraints, and an objective function, and how to solve it to find the best solution.

Model formulation

- let *Xij* be the area to be cut from compartment *i*, in period *j*, where *i* and *j* are integer subscripts. Here, *i* may take the value 1 or 2 since there are only two compartments in the initial forest.
- Let us use a time unit of 5 years in this example. Therefore, *j* can take the values 1, 2 or 3 depending on whether a cut occurs during he first 5 years of the plan, the second 5 years, or the third. As soon as an area is cut over, it is immediately replanted with pine trees, so X_{ij} is also the area replanted in compartment *i* during period *j*.
- Thus, all the possible harvests and reforestations in compartment 1 are defined by the three decision variables: X_{11}, X_{12}, and X_{13}, while those possible on compartment 2 are: X_{21}, X_{22}, and X_{23}.

The optimization problem

- Objective function:

$Z = 16\ X_{11} + 23\ X_{12} + 33\ X_{13} + 24\ X_{21} + 32\ X_{22} + 45\ X_{23}$ tons

where $16\ X_{11} + 23\ X_{12} + 33\ X_{13}$, and $24\ X_{21} + 32\ X_{22} + 45\ X_{23}$ the tonnage of hardwood cut from compartments 1, and 2.

- Constraints:

$X_{11} + X_{12} + X_{13} = 120$ ha,

$X_{21} + X_{22} + X_{23} = 180$ ha

$X_{11} + X_{21} = 100$ ha

$X_{12} + X_{22} = 100$ ha

$X_{13} + X_{23} = 100$ ha

OPTIMIZED HARVEST PLAN: AREA DATA

	Ha per 5-year period			
Compartment	1	2	3	Total ha
1	100	20	0	120
2	0	80	100	180
	——	——	——	
Total	100	100	100	300

OPTIMIZED HARVEST PLAN: TONNAGE DATA

	Tons per 5-year period			
Compartment	1	2	3	Total tons
1	1600	460	0	2060
2	0	2560	4500	7060
	——	——	——	
Total	1600	3020	4500	9120

Reforestation

- Silvicultural treatments
- Logging
- Administration
- Financial costs
- Taxes

Regeneration

- Clearing and soil preparation (scarification, harrowing, mounding, ploughing, 0 year after clearcut, by tree species, and site types);
- seeding or planting (1st year after clearcut, by tree species, and site types);
- tending (7-14 years after clearcut, by tree species, and site types);
- pruning, fertilization, and forest road construction.

Natural vs. artificial regeneration

Table 3. Unit costs, timing, and probability of occurrence of stand establishment and silvicultural activities.

					Probability of occurrence in H100 class					H50 class	
		2003	2003		Norway spruce		Scots pine			Birch & others	
Type of work	Timing, year	Total area, 1000ha	Total cost, €1000	Unit cost, €/ha	21–27	30–33	15–21	24	27–30	22-24	26-30
Natural regeneration		37			(%)						
Clearing	0			**113**	60	60	75	75	75	60	60
Soil preparation	0			**189**	40	20	75	75	75	40	20
Artificial regeneration		119			(%)						
Clearing	0			**113**	60	60	75	75	75	60	60
Soil preparation	0			**189**	40	20	75	75	75	40	20
Seeding	1	32	5858	**183**	0	0	100	50	0	0	0
Planting	1	87	51740	**595**	100	100	0	50	100	100	100
Weeding (1, 2, 3)	2, 2, 3	6	841	**140**	60	60	30	30	90	60	60
Tending (1, 2, 3)	7,11,14	138	41786	**303**	90	90	80	80	80	90	90
Fixed annual cost (€/ha/yr)				**4**	100	100	100	100	100	100	100

Note: Fixed annual cost and probability of occurrence are extended from Hyytiäinen and Tahvonen (2001), Salminen (1993), and Hyytiäinen et al. (2005).
Source: Finnish Statistical Yearbook of Forestry 2004

Logging

- Logging method
- Terrain class
- Average travel distance
- Thinnings/partial cut (mean tree size, by tree species, and site types);
- Final harvesting (mean tree size, by tree species, and site types).

A simple logging model

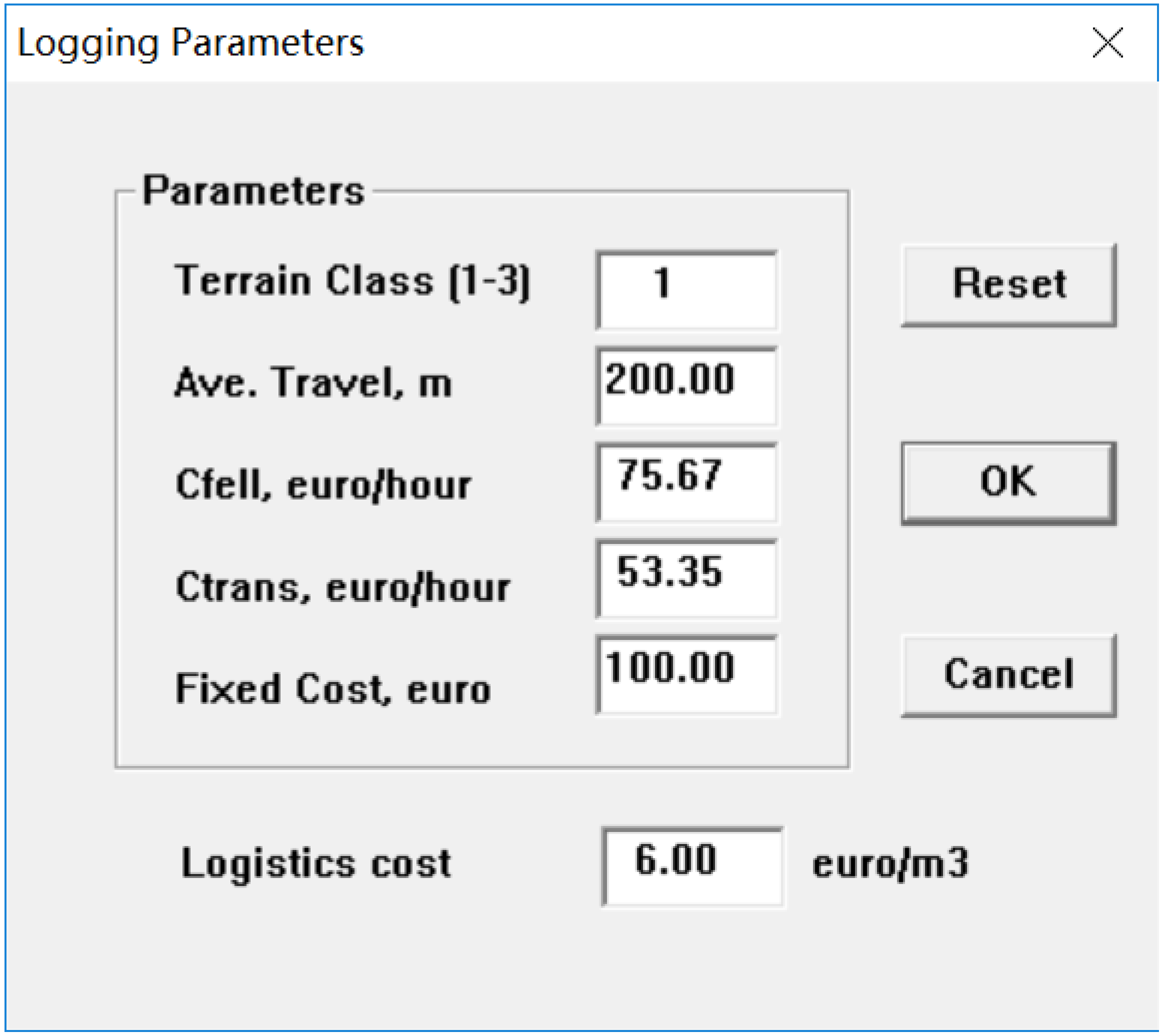
- The absolute effect of logging conditions on harvest revenues, b_{ih} ($/m^3), is a function of the average size of the removed trunks in liters, v_i, and the removed total volume, x_i, expressed in m^3.
- the logging cost for thinning and clearcutting. The equations are, for thinning (equation 1), (h = 1):

$$b_{i1} = (1.413 + 4.005\, \ln(0.008908\, x_i) + 4.634\, \ln(0.0422\, v_i) - 2622/v_i)\, x_i\,,$$

and for clearcutting (equation 2), (h = 2):

$$b_{i2} = (-0.3191 + 1.602\, \ln(0.004259\, x_i) + 2.408\, \ln(9.579\, v_i) - 2642/v_i)\, x_i\,.$$

The logging model



Effects of initial states on optimal thinning

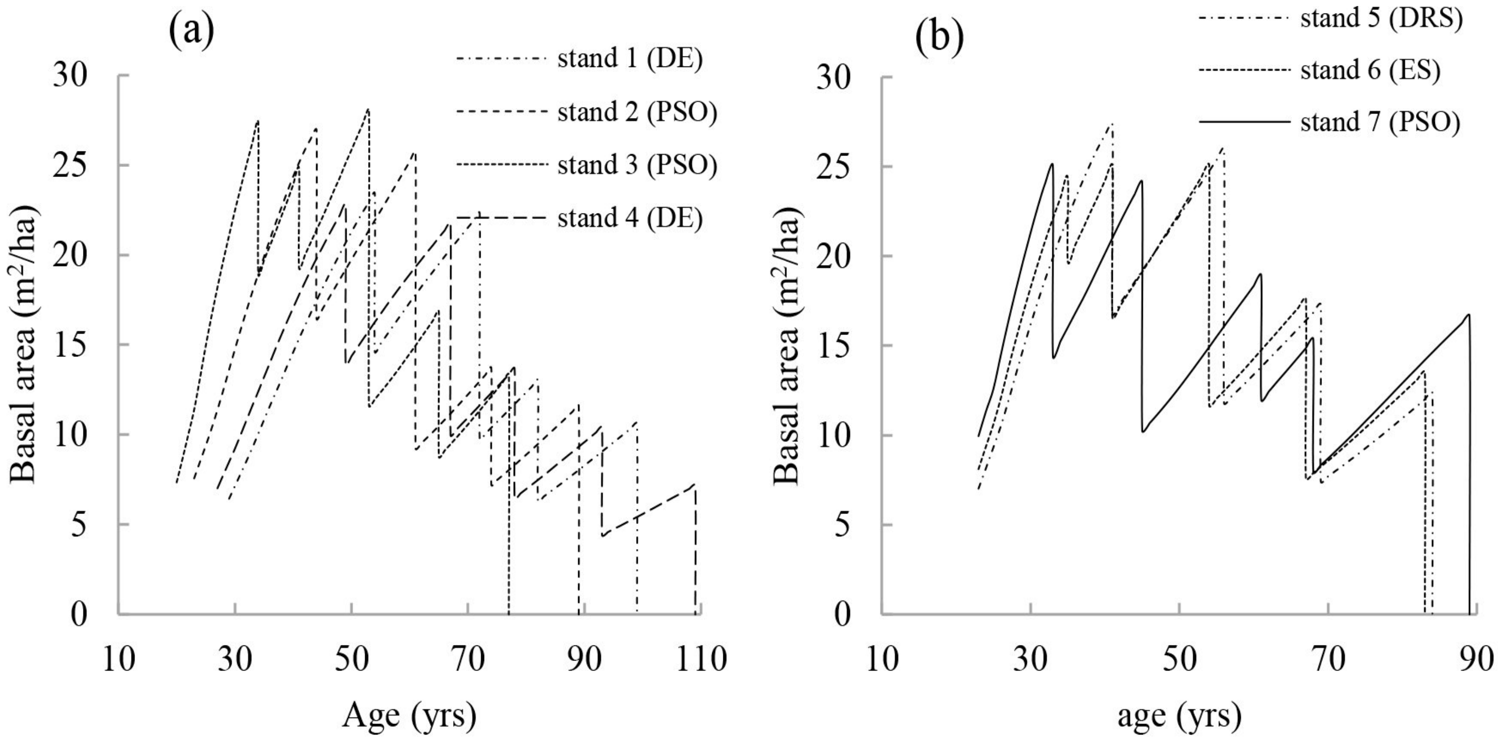
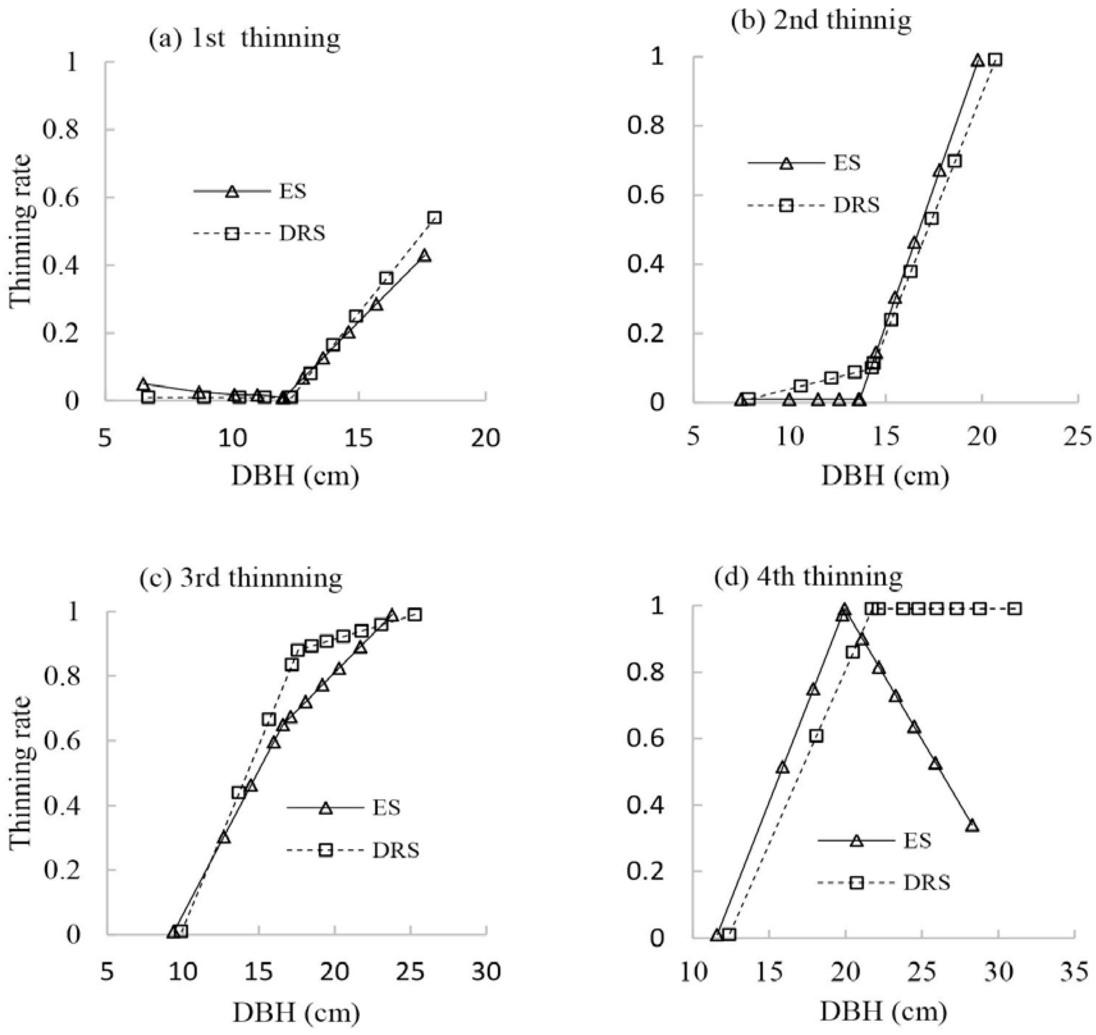
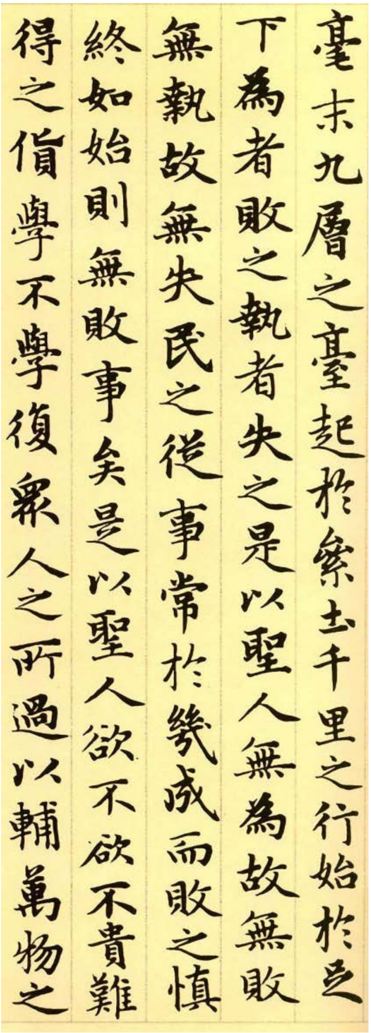


Fig.1. Basal area development by optimal solutions for stands 1-4 (a) and stands 5-7 (b).

Type of thinning



Simulation optimization systems



毫末九層之臺起於累土千里之行始於足
下為者敗之執者失之是以聖人無為故無敗
無執故無失民之從事常於幾成而敗之慎
終如始則無敗事矣是以聖人欲不欲不貴難
得之貨學不學復衆人之所過以輔萬物之

Yet a tree broader than a man can embrace is born of a tiny shoot; A dam greater than a river can overflow starts with a clod of earth; A journey of a thousand miles begins at the spot under one's feet.

Decision support systems

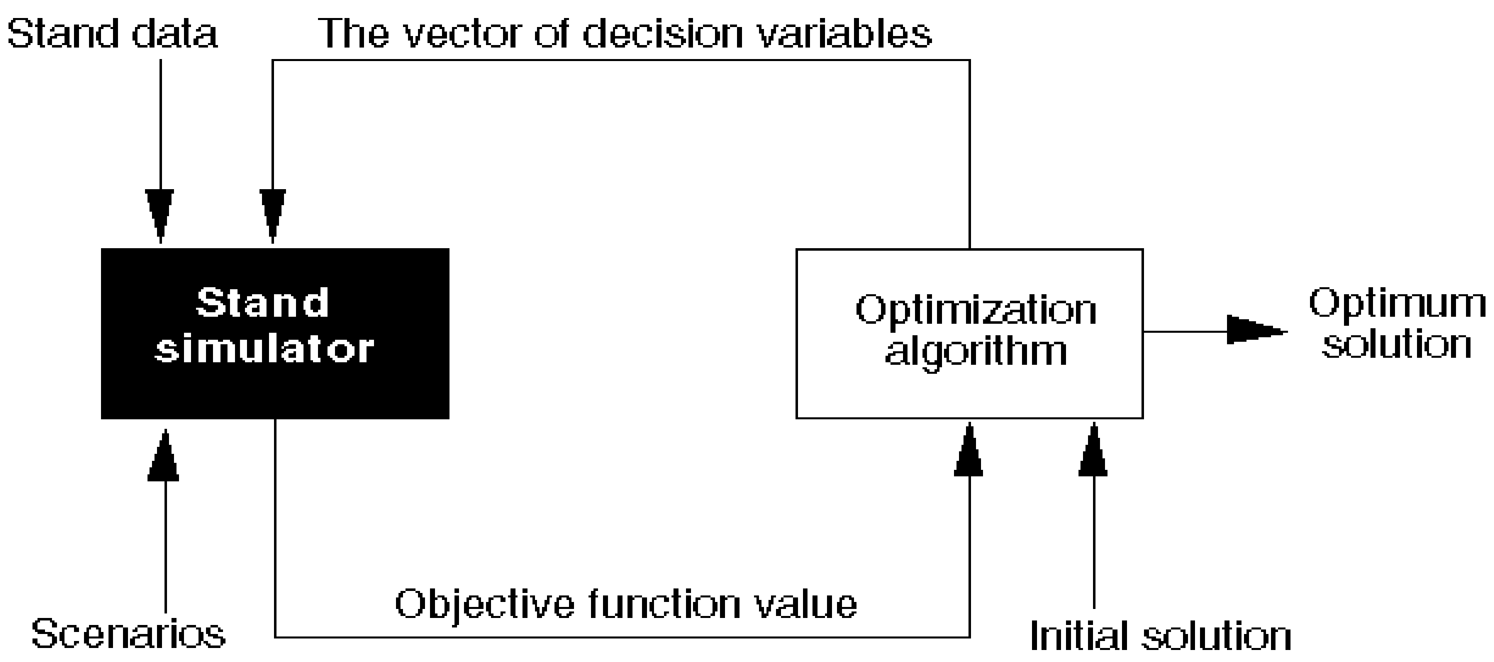
- Inventory DSS
- Simulation DSS
- Two-level DSS
- Theoretical optimization DSS
- Simulation optimization DSS

A simulation optimization approach

- USA (e.g., Amidon and Akin, 1968; Brodie et al., 1978; Kao and Brodie, 1979,1980; Roise, 1986; Haight, 1987,1991,1993; Haight and Monserud ,1990; Arthaud and Pelkki ,1996)
- Sweden (e.g., Eriksson 1994,1997; Gong, 1998; Wikström 2000,2001; Lu and Gong, 2003)
- Norway (e.g., Risvand ,1969; Solberg and Haight, 1991; Hoen and Solberg, 1994)
- Denmark (e.g., Thorsen and Helles, 1998; Strange et al., 1999)
- The Netherlands (e.g., Brazee and Bulte, 2000)
- Finland (e.g., Kilkki and Väisänen, 1969; Valsta, 1986,1990,1992; Salminen, 1993; Miina, 1996; Pukkala and Miina, 1997,1998; Hyytiäinen et al., 2003, 2004; Cao et al., 2006; Pukkala, 2009; Cao et al., 2010)

Structure of simulation-optimization system

- Valsta (1992)



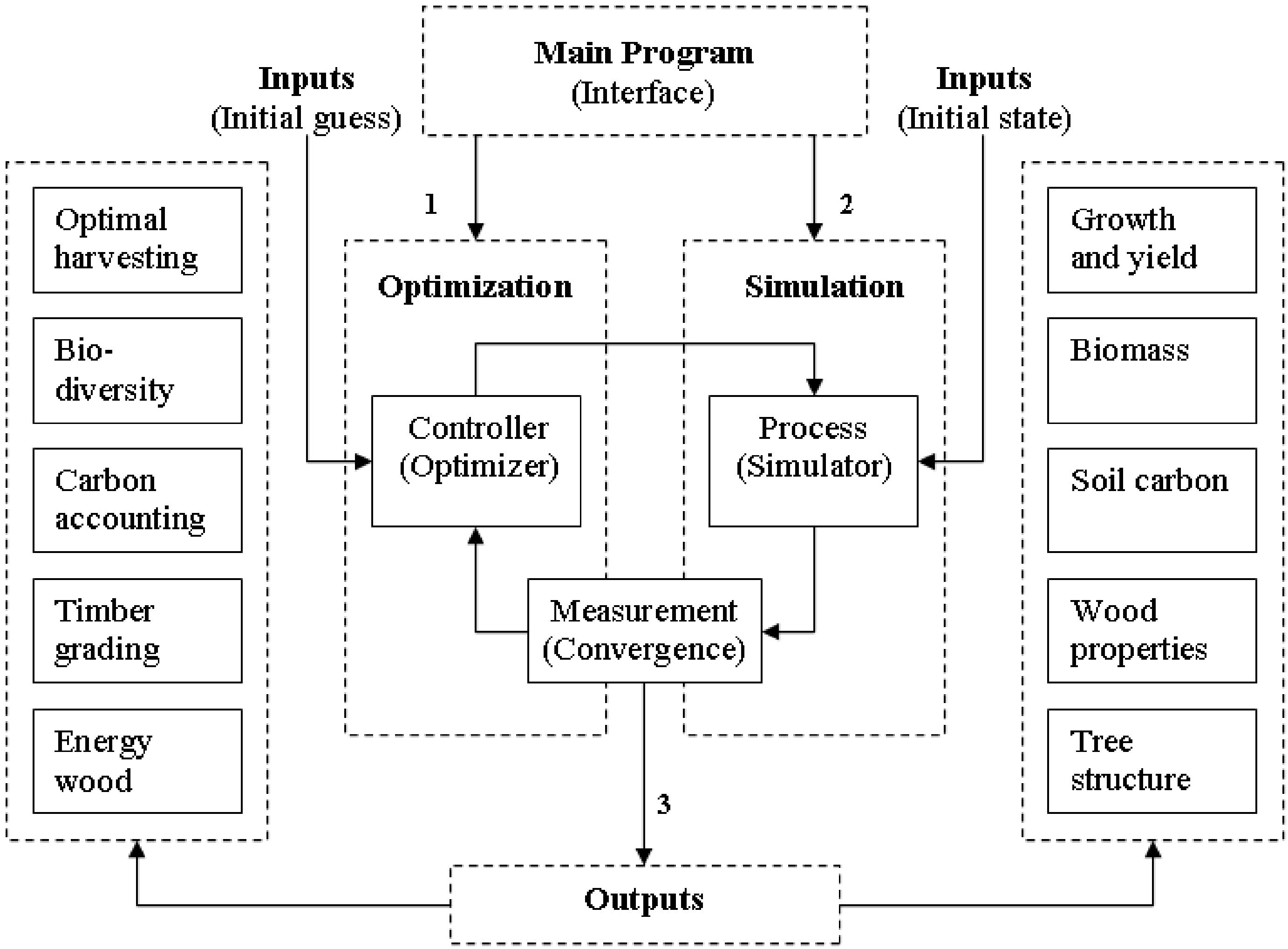
Forest decision-support systems

- http://fp0804.emu.ee/wiki/index.php/Category:DSS
- EFISCEN
http://www.efi.int/portal/completed_projects/efiscen/
- FVS http://www.fs.fed.us/fmsc/fvs/
- MOTTI, SIMO, MONSU , SMA
http://www.metla.fi/metinfo/motti/index-en.htm
www.simo-project.org
http://fp0804.emu.ee/wiki/index.php/Monsu
http://www.helsinki.fi/forestsciences/research/projects/sma/programme/index.htm
- OptiFor
http://www.optifor.cn

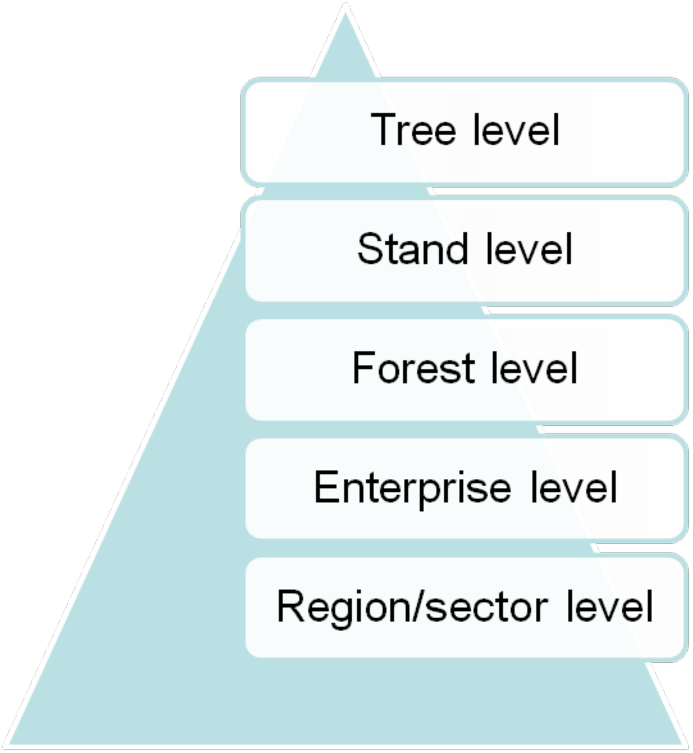
The optimization problem

- Forest management objectives
  - Problems to be solved
- Silvicultural operations
  - Decision variables
- Stand simulator
  - State variables
- Biological and economic data
  - Initial states

OptiFor simulation-optimization system



Decision levels



Source: Valsta (1993)

Decision makers

- Decision makers categories
  - risk averse,
  - risk neutral
  - risk preferring.
- Maximizing expected value
  - risk neutraler
- Maximizing the maximum value (Maxmax)
  - extreme risk lover
- Maximizing the minimum value (Maxmin)
  - extreme risk averter

Decision-making under risks

- Production risk
  - Silvicultural operations
  - Biotic and Abiotic risks
- Price risk
  - Timber market
  - Financial market
  - Long-term investment
- Institutional risk
  - Common forestry policy
- Risk and insurance
  - Forest fire insurance

Risk analysis

- The degree of risk in a revenue is the amount of variation in its possible outcomes. A risk-free return has no variation.
- The expected value of any risky variable is the sum of the possible values multiplied by their probabilities of occurrence.
- Most people are risk-averse: they prefer less variation in revenues.
- The certainty-equivalent of a risky revenue is a sure dollar amount giving the investor the same satisfaction as the risky revenue.

Risk analysis, con't

- For a risk-averse person, the certainty-equivalent of a risky revenue will be less than its expected value.
- Risk-free revenues and certainty-equivalents should be discounted with a risk-free interest rate to arrive at the correct present value.
- The correct present value of a risky revenue is its expected value discounted with a risk-adjusted discount rate (RADR). For a risk-averse investor, this RADR exceeds the risk-free discount rate.
- Make sure you're discounting expected values, not optimistic values.

Risk-adjusted discount rate

- The further in the future a risky revenue is, the lower the correct RADR is, given the same degree of risk and risk aversion.
- Thus, forestry's long production periods may often require lower RADRs than average short-term industrial RADRs.
- There's no such thing as "the" correct RADR for forestry's expected values. In reality, a different RADR should be used for each cash flow, depending on its probability distribution, on its time from the present, and on the decision maker.
- We can hope to give only rough guidelines for different situations.

RADA, con't

- If data are available, try, using computer simulations, to construct probability distributions of net present value or internal rate of return for projects, and let decision makers compare them.
- While it's correct for a risk-averse investor to increase the discount rate for risky future revenues, the discount rate should be lowered for risky future costs that are independent of revenues.

Predicting biotic forest damages

- mycelium originating from old-growth stumps may be viable for up to 60-120 yrs, and spread after felling of butt-rotted trees (Stenlid & Redfern 1998).

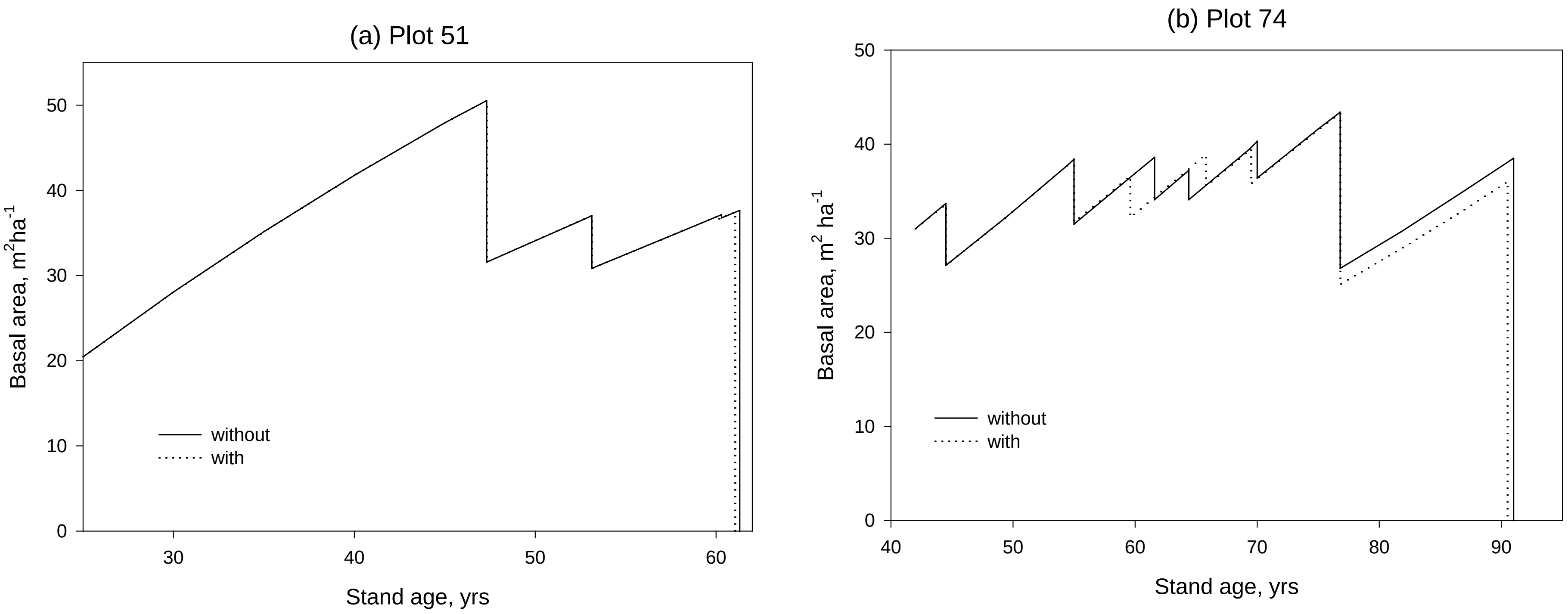
- Frequent summer thinnings without stump treatment are the most important operation increasing the proportion of trees with butt rot at the end of rotation (Swedjemark & Stenlid 1993, Venn & Solheim 1994, Vollbrecht & Agestam 1995).

Optimal harvesting with butt rot effects

- Cao, T., 2003. Optimal harvesting for even-aged Norway spruce using an individual-tree model. Finnish Forest Research Institute, Research Papers 897. 44 pp. ISBN 951-40-1886-9, ISSN 0358-4283.

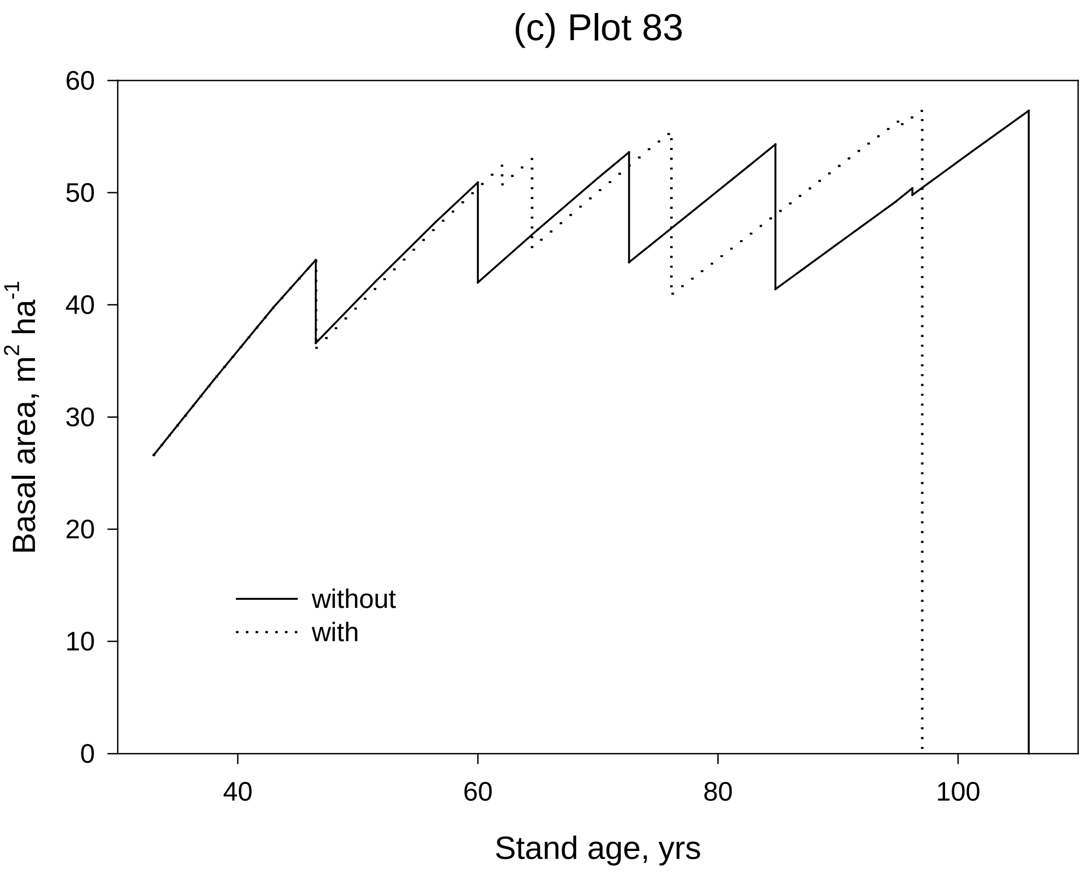
- A comparison of optimal harvesting with and without butt rot effects

Butt rot effects (Cao 2003)



Basal area development in optimal solutions, with or without butt rot effects, 3% rate of interest (Cao 2003).

Butt rot effects, con't



Basal area development in optimal solutions, with or without butt rot effects, 3% rate of interest (Cao 2003).

Modelling the spread of butt rot

- Möykkynen, T., Miina, J. & Pukkala, T., Von Weissennerg, K., 1998. Modelling the spread of butt rot in a *Picea abies* stand in Finland to evaluate the profitability of stump protection against *Heterobasidion annosum*. For. Ecol. Manage. 106, 247–257.

- Möykkynen, T., Miina, J. & Pukkala, T. 2000. Opimizing the Management of a *Picea abies* Stand Under Risk of Butt Rot. Forest Pathology 30: 65-76.

Stand Management Under Risk of Butt Rot

- A simulation model was developed to predict the growth of a Norway spruce stand under risk of butt rot caused by *Heterobasidion annosum* stump infection and logging injuries.

- The simulation model was distance-dependent.
- The spread of butt rot through root contacts depended on tree location. Infection of stumps and injured trees,
- The spread of butt rot in the stand were stochastic processes whereas tree growth and mortality were treated as deterministic processes.

Simulation of spread of butt rot

- The progress of decay in the root system was 30 cm per year (ISOMÄKI and KALLIO 1974; STENLID 1987; SWEDJEMARK and STENLID 1993).

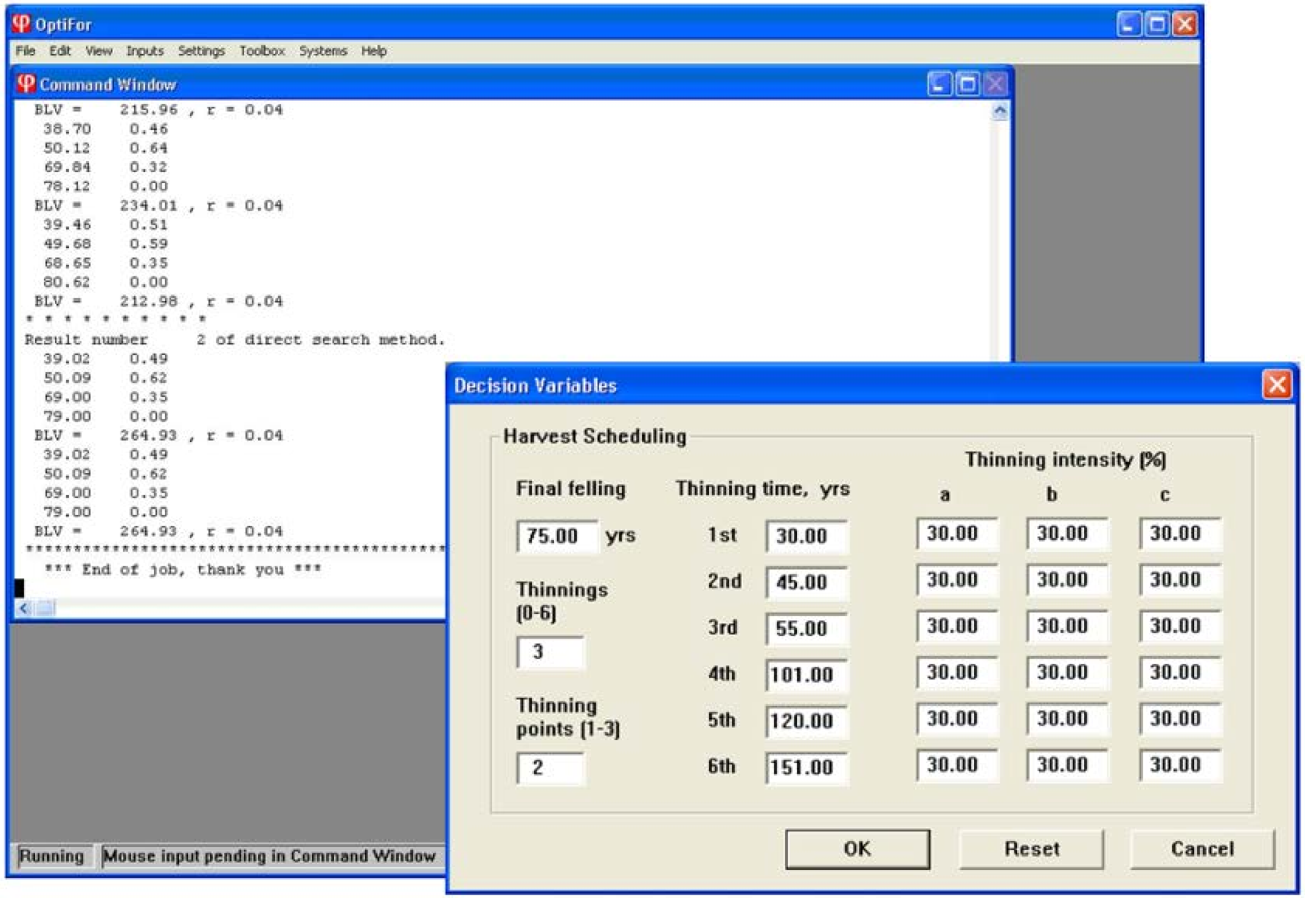
- h_rot = 4.78*t_d^0.5 +1.59 * t_d^0.5 *d_d
- d_rot = 0.05*h_rot

- For example, a tree infected 10 years ago with d.b.h. of 2 dm has a decay cone which is 25.2dm high with a diameter of 1.26dm at the stump level.

OptiFor demo

- A simulation-optimization tool for forest resources management
- Website: www.optifor.cn
- Documentation
- Student version
- Scientific version

OptiFor


OptiFor
File Edit View Inputs Settings Toolbox Systems Help
Command Window
BLV = 215.96 , r = 0.04
38.70 0.46
50.12 0.64
69.84 0.32
78.12 0.00
BLV = 234.01 , r = 0.04
39.46 0.51
49.68 0.59
68.65 0.35
80.62 0.00
BLV = 212.98 , r = 0.04
Result number 2 of direct search method.
39.02 0.49
50.09 0.62
69.00 0.35
79.00 0.00
BLV = 264.93 , r = 0.04
39.02 0.49
50.09 0.62
69.00 0.35
79.00 0.00
BLV = 264.93 , r = 0.04
*** End of job, thank you ***
Running
Mouse input pending in Command Window
Decision Variables
Harvest Scheduling
Final felling
75.00 yrs
Thinnings (0-6)
3
Thinning points (1-3)
2
Thinning time, yrs
1st 30.00
2nd 45.00
3rd 55.00
4th 101.00
5th 120.00
6th 151.00
Thinning intensity (%)
a b c
30.00 30.00 30.00
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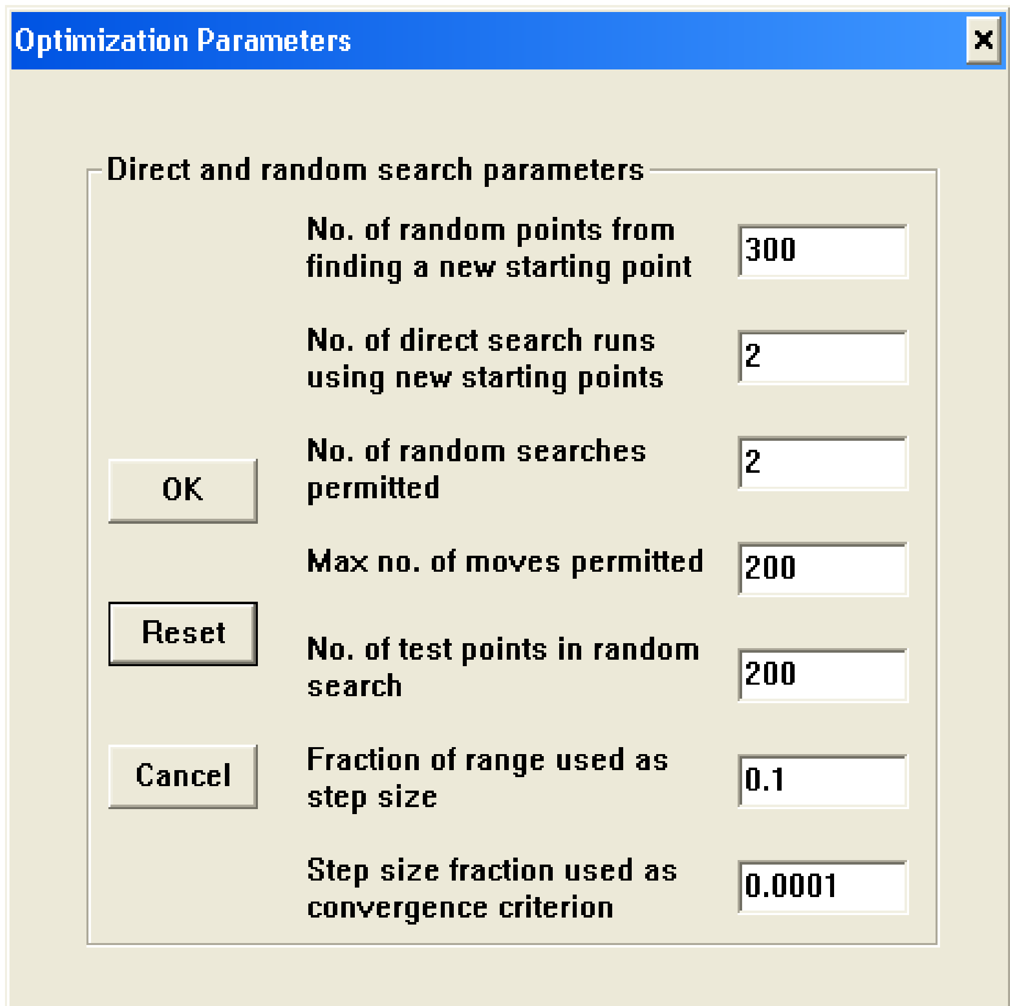
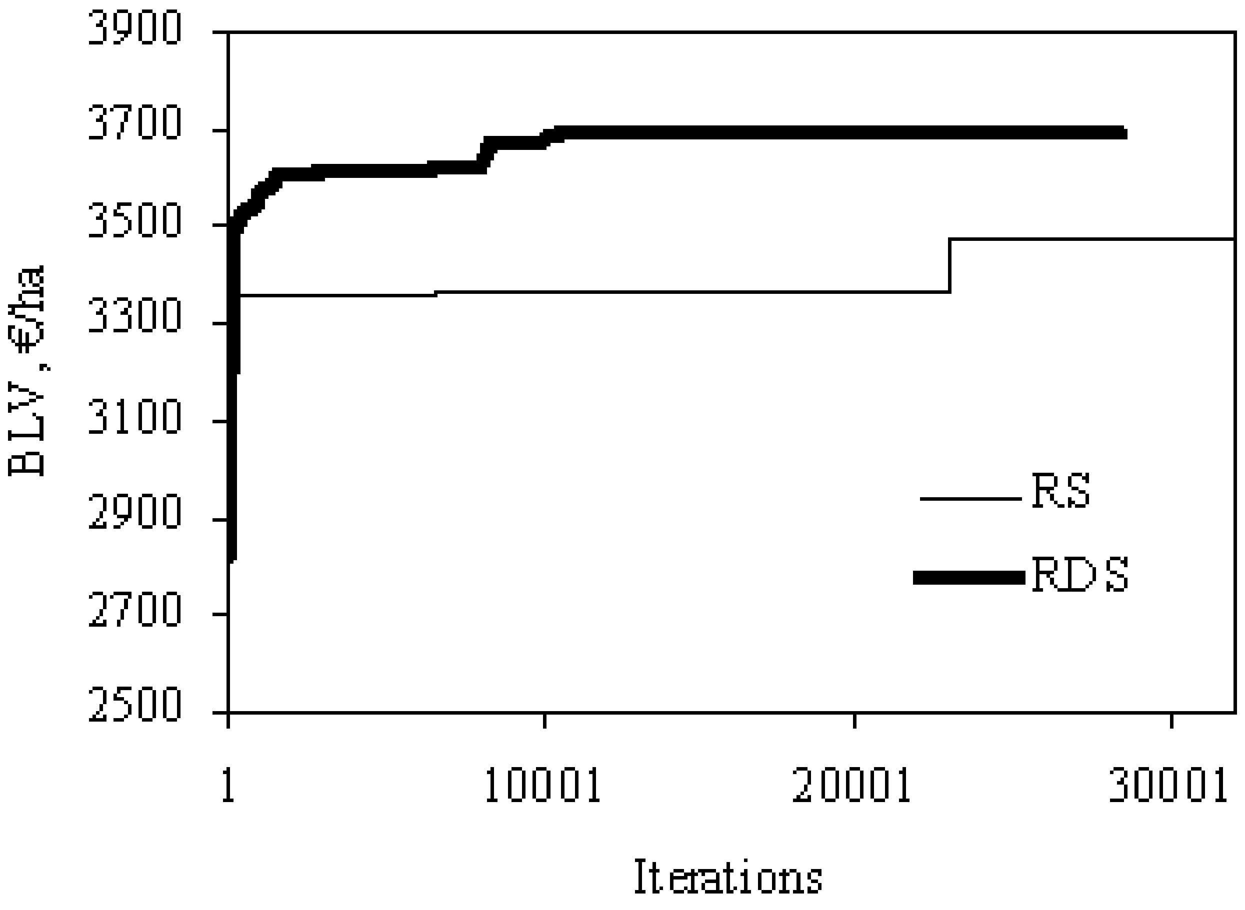
OK Reset Cancel

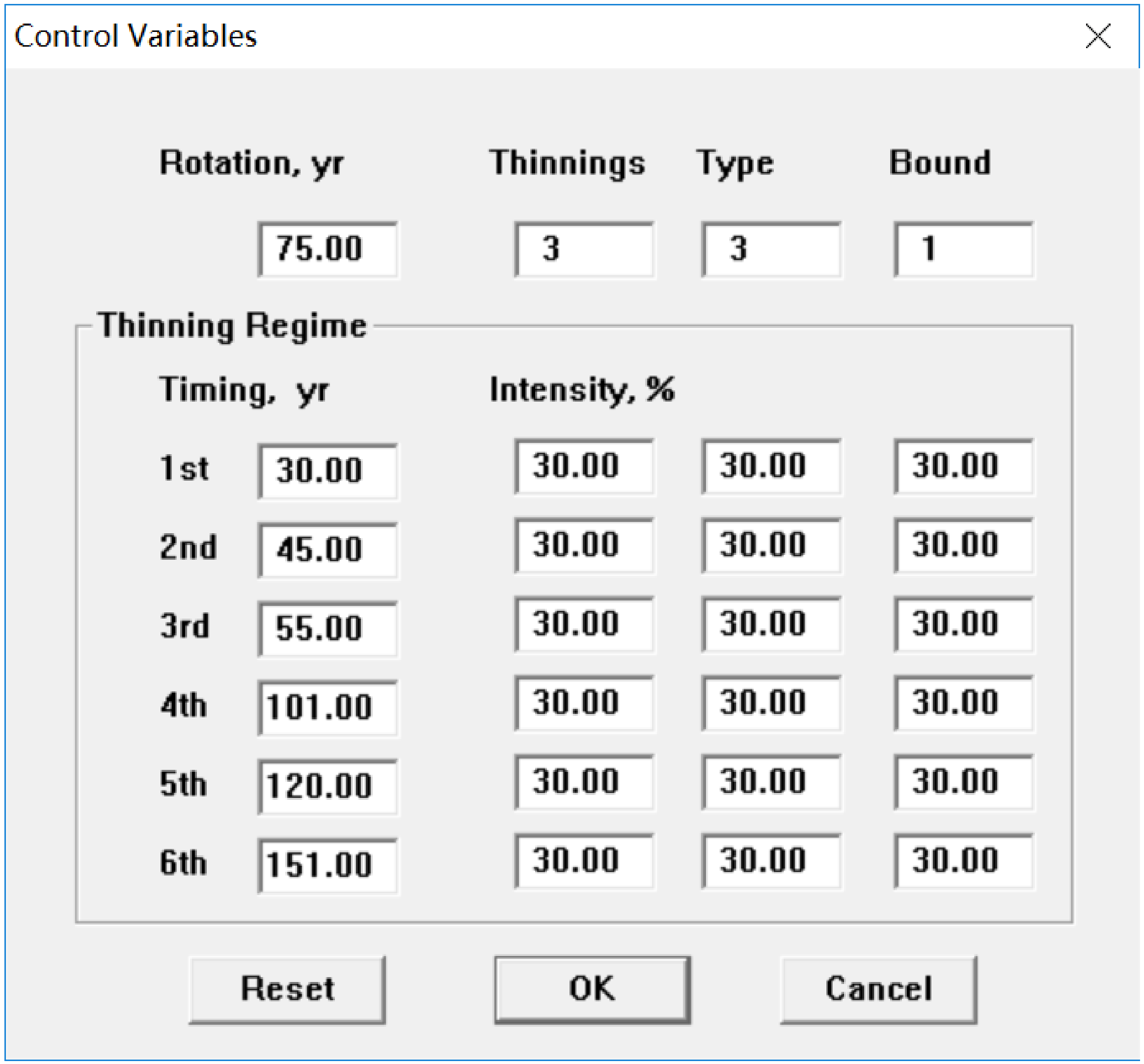
HJ algorithm parameters



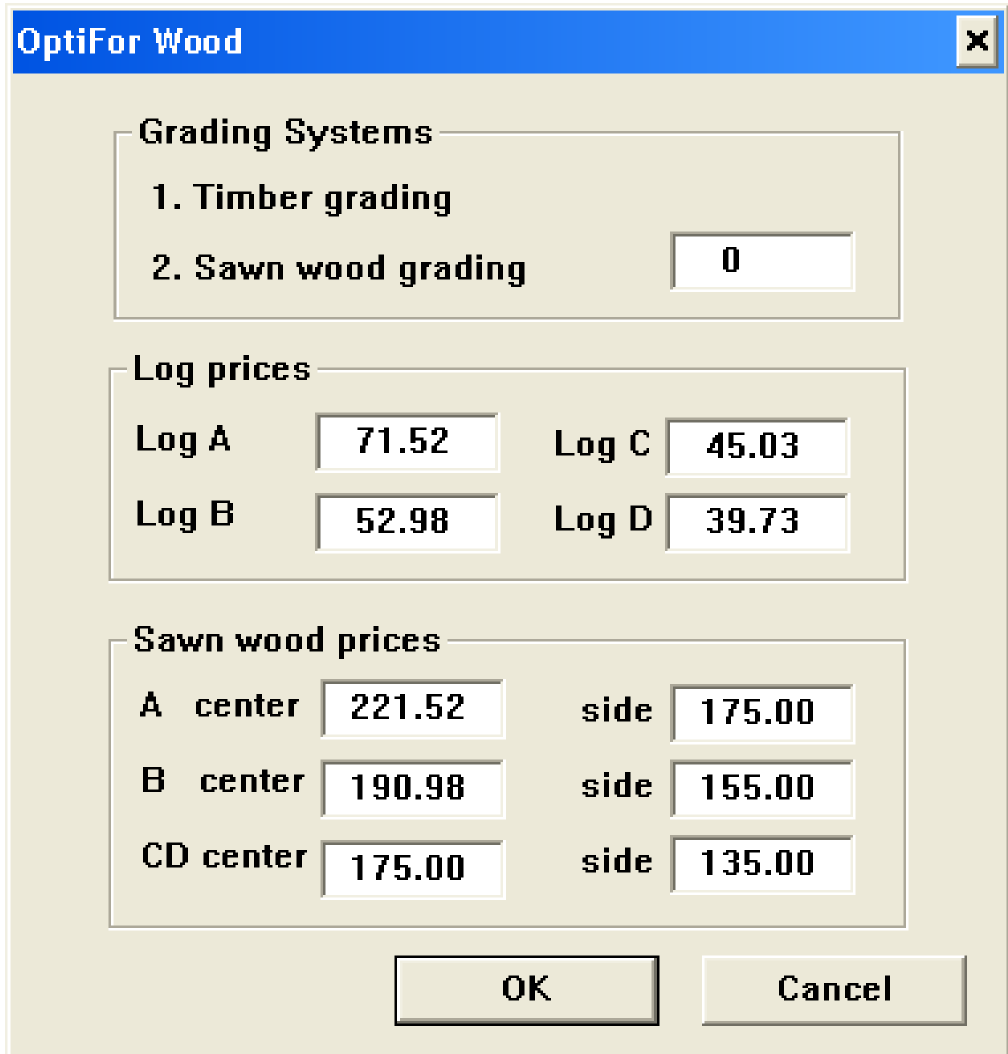
Comparison of optimization algorithms



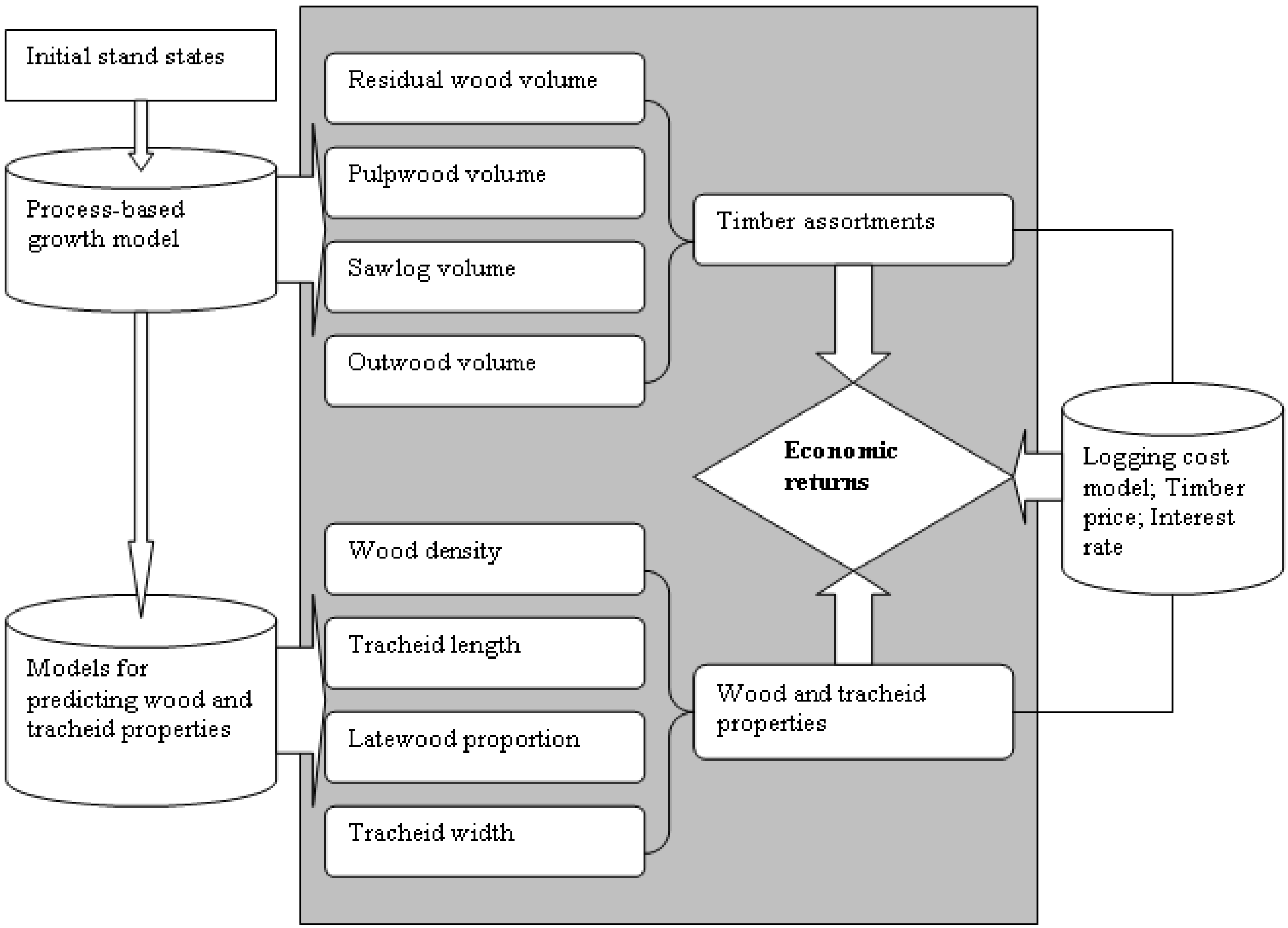
Harvest scheduling



OptiFor Wood



Cao et al. (2008)



Hurttala et al. (2017)

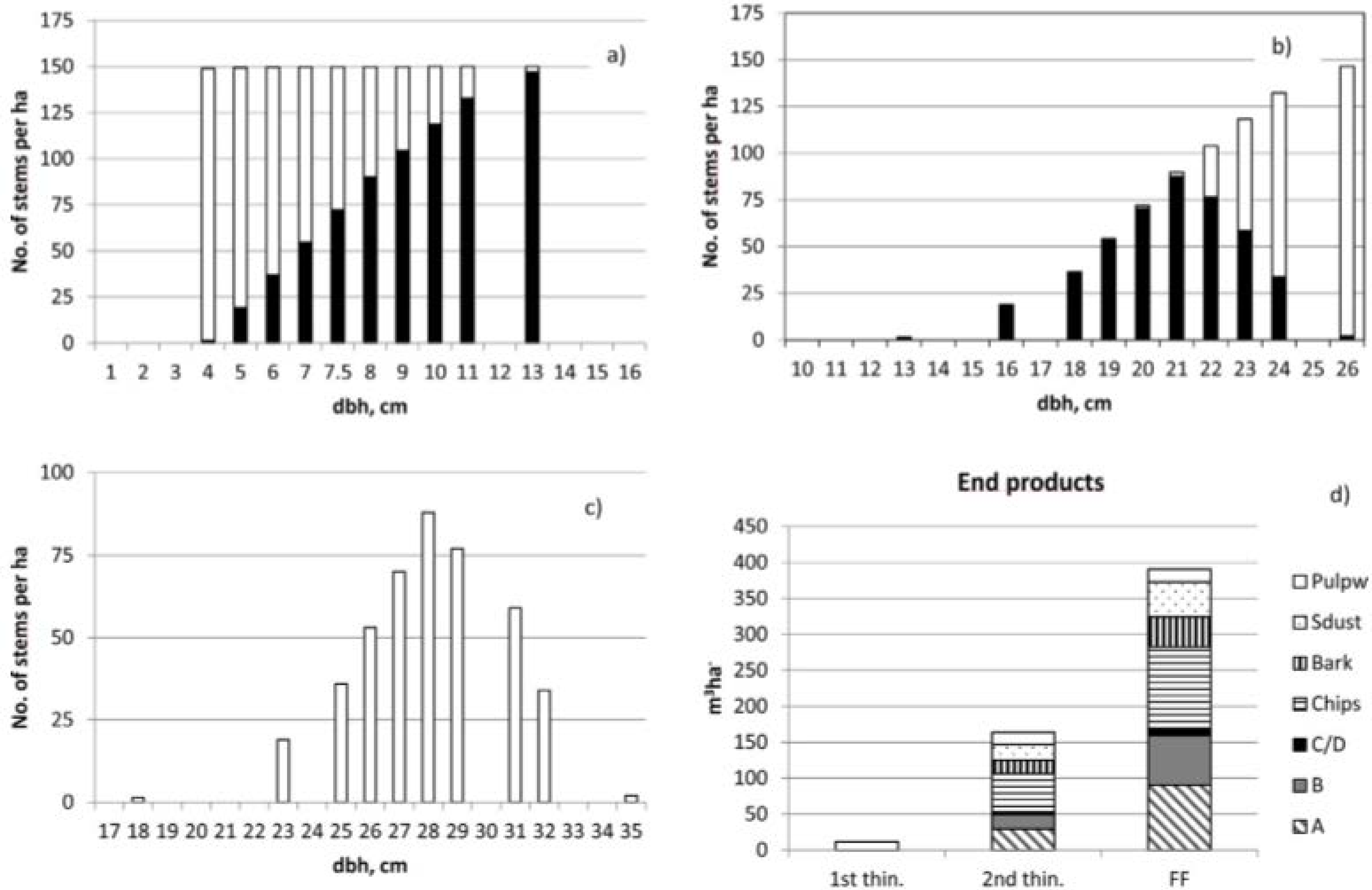
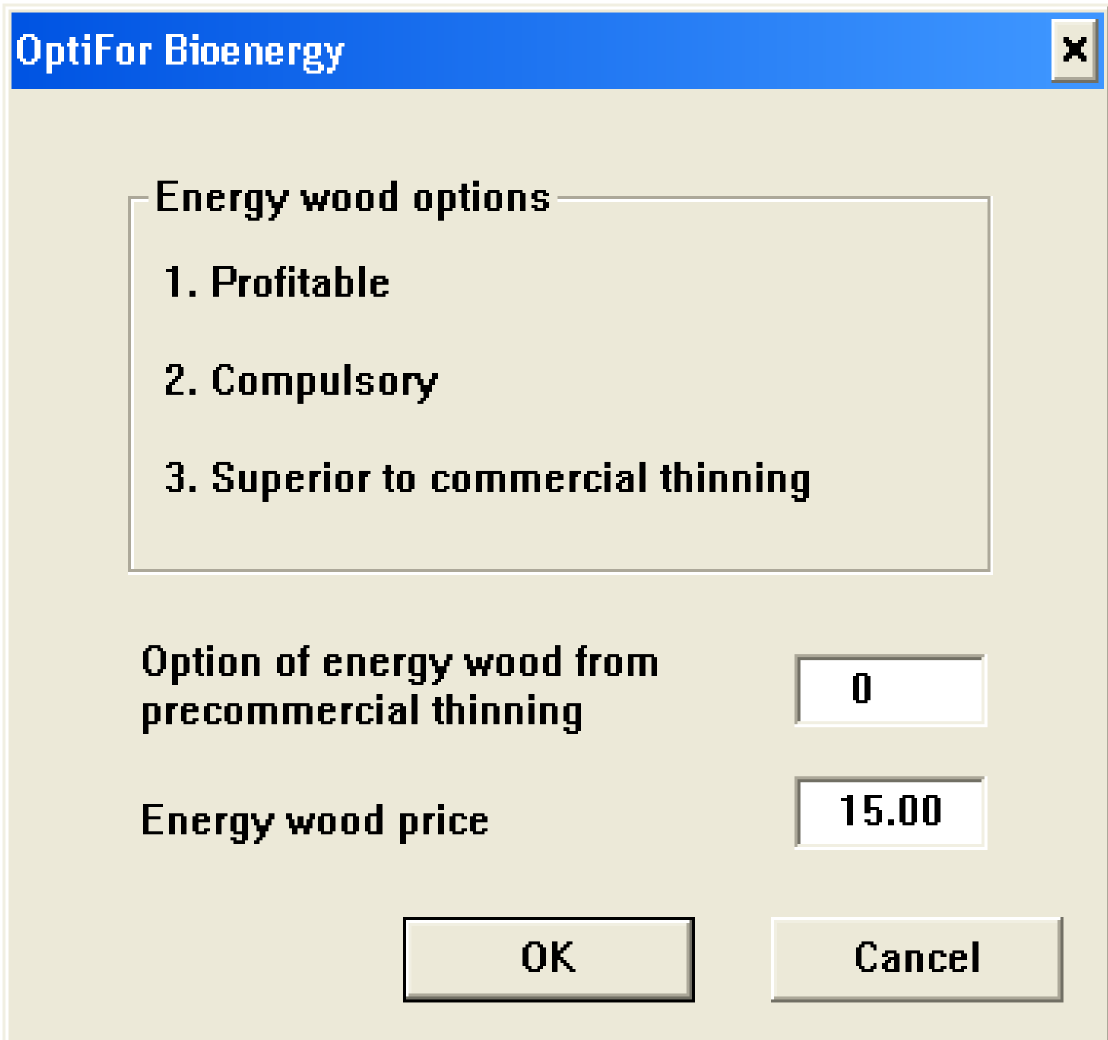


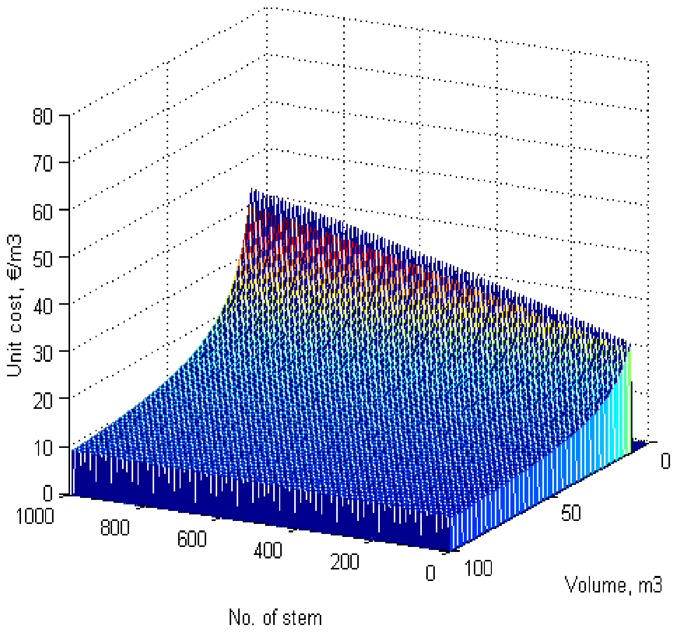
Fig. 4. Removals (open bars) in size classes (a–c) and yields of end products in thinnings and final felling in the QPC on MTS$_{1500}$ (d).

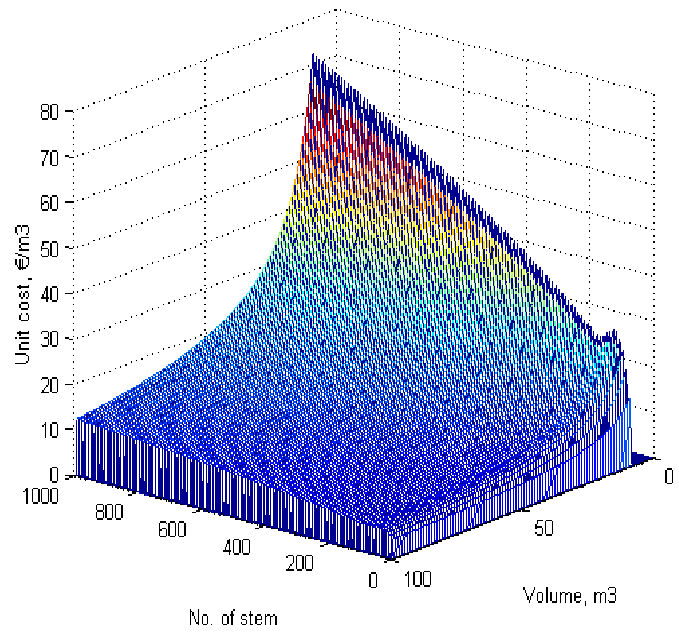
OptiFor Bioenergy



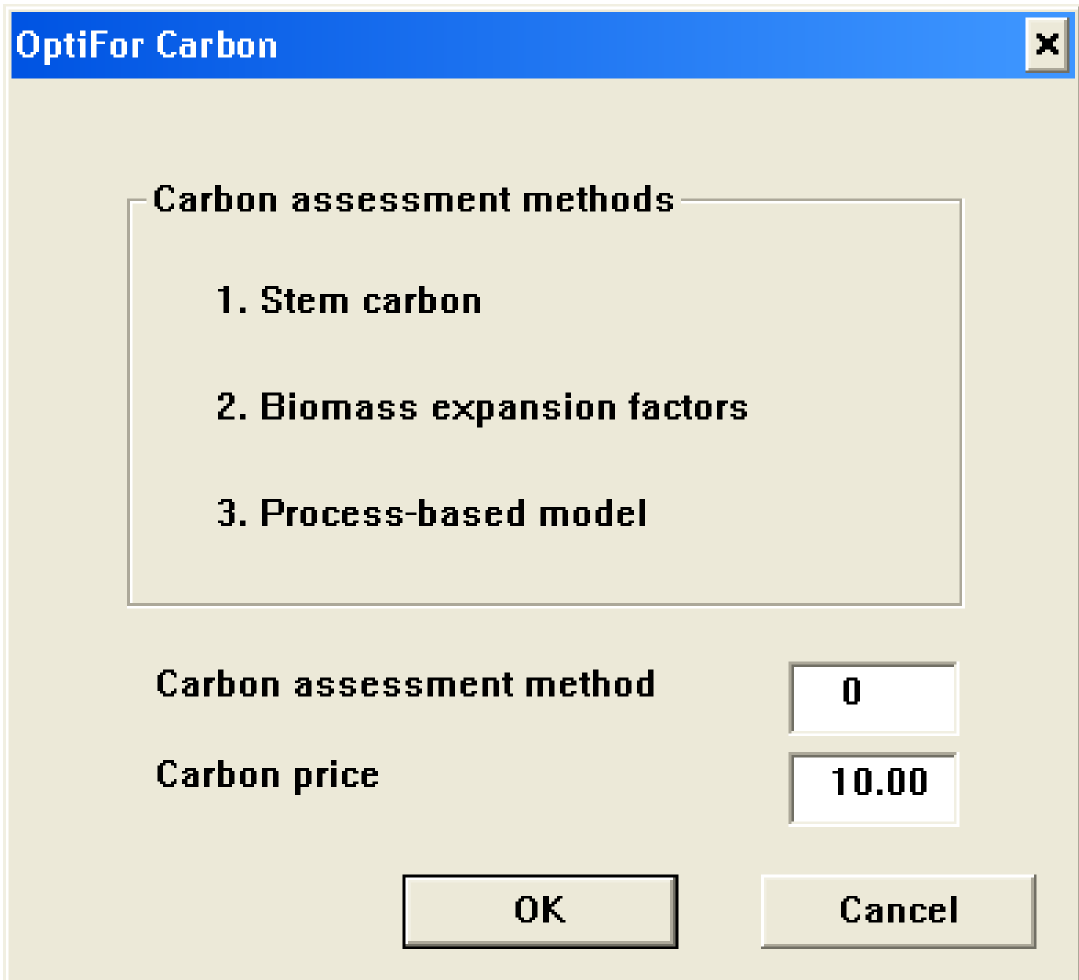
Cao et al. (2015)

- (a) Energy wood harvesting (b) Conventional thinning

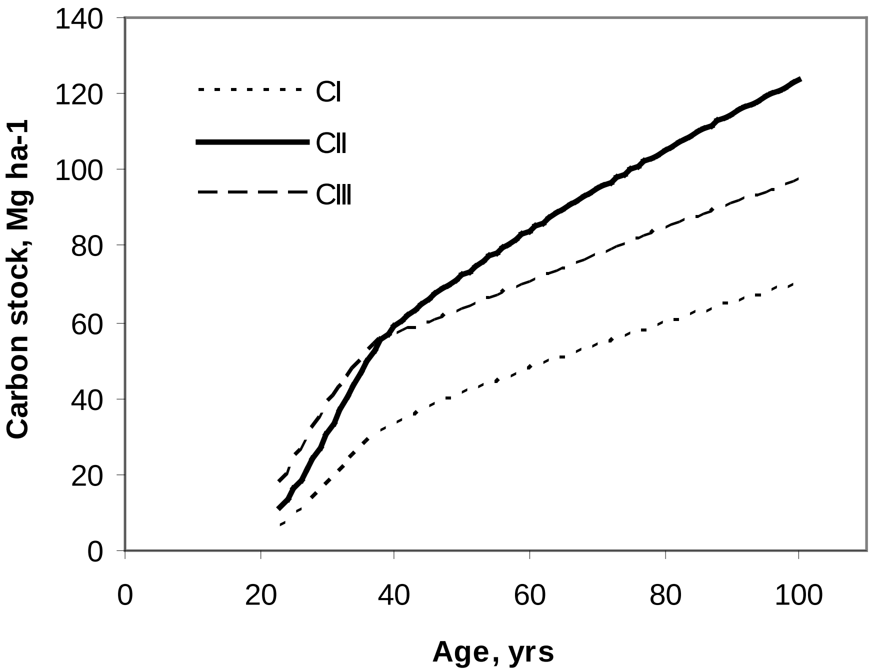


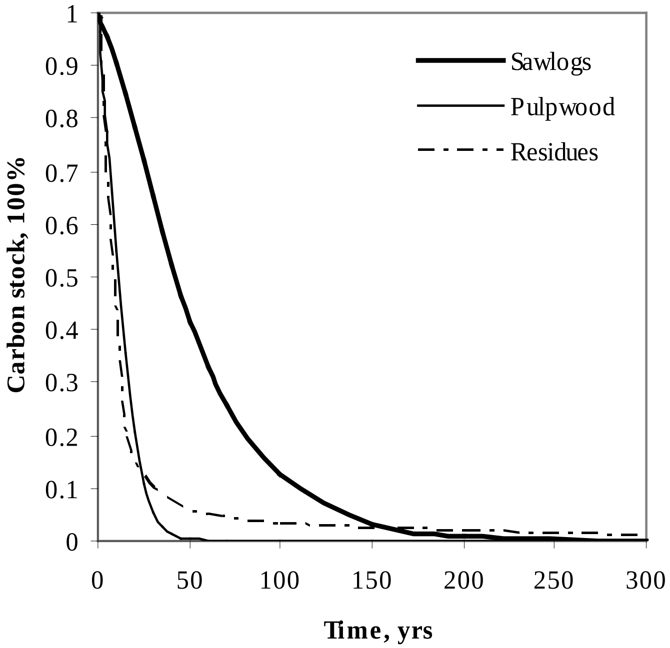


OptiFor Carbon

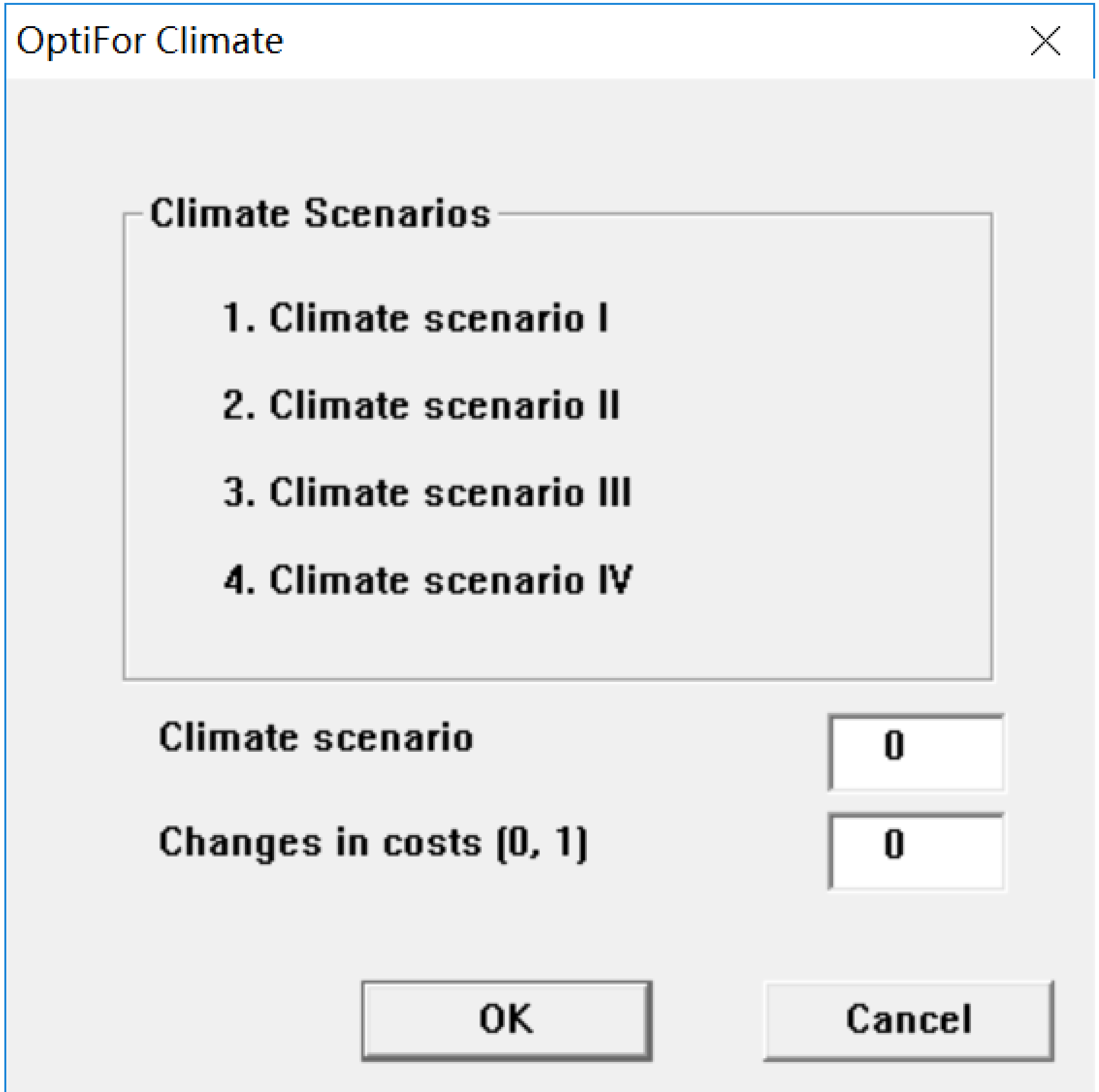


Cao et al. (2010)





OptiFor Climate



Climate-sensitive model

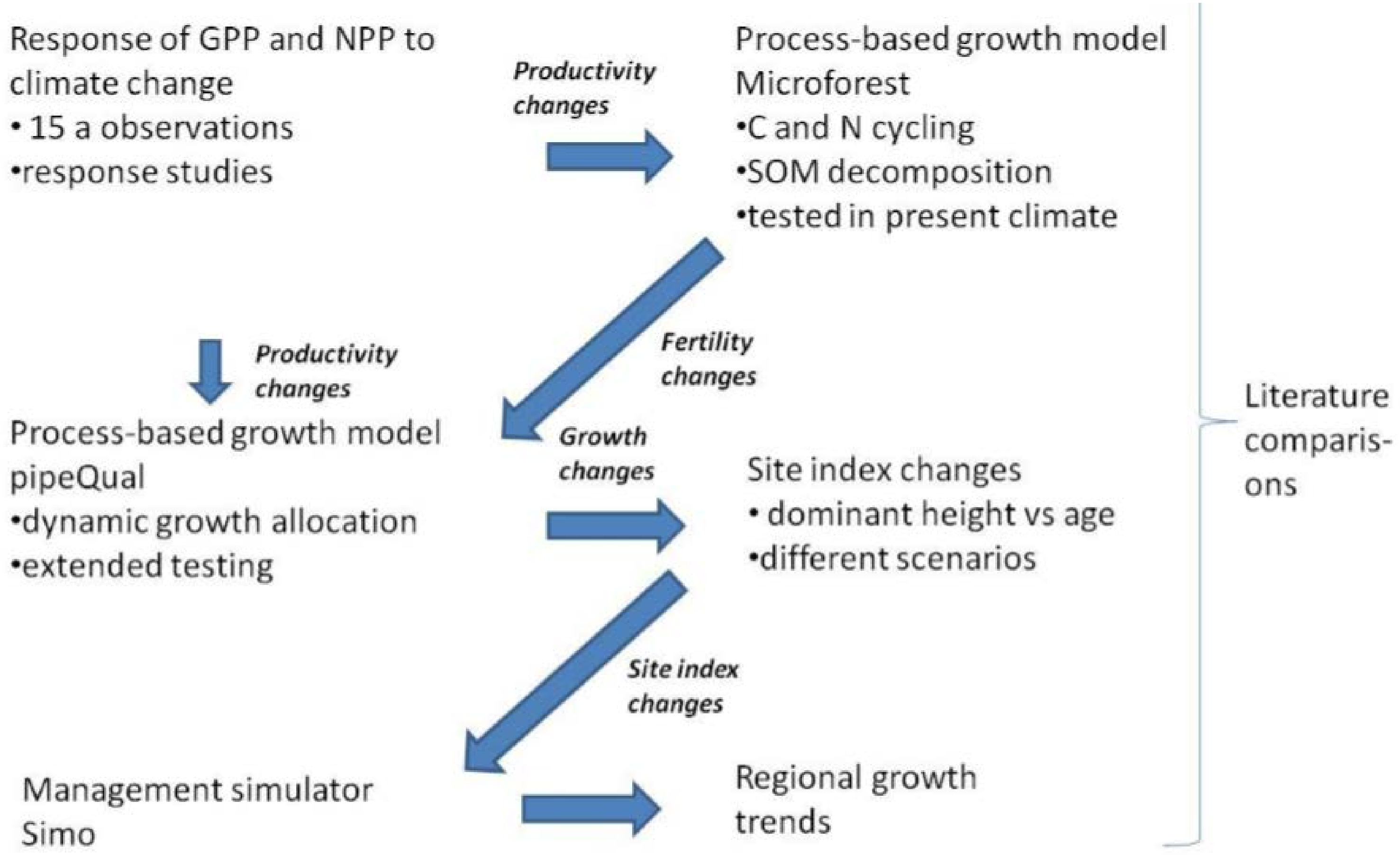
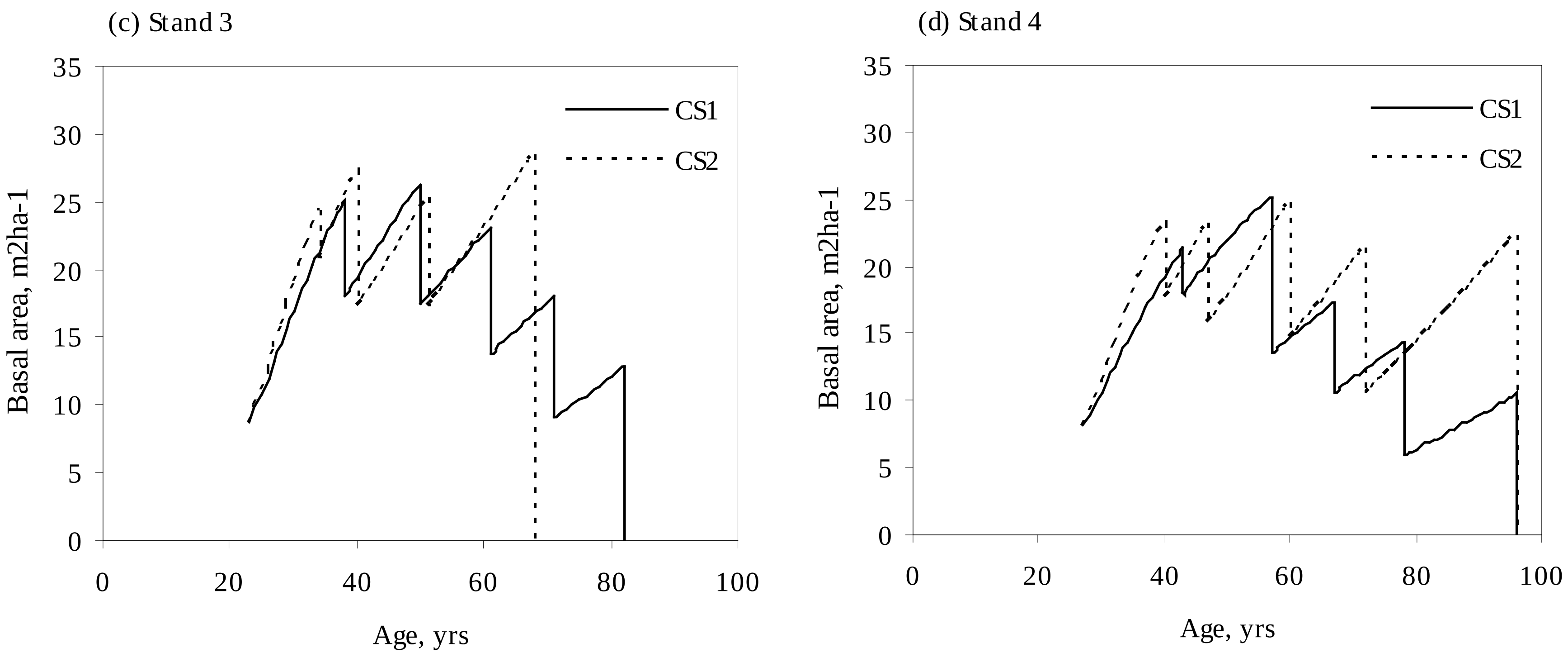


Figure 1. Schematic presentation of the used approach to analyze climate change impacts on forest productivity

Nikinmaa et al. (2011)



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Simulation Optimization Lab

Thank You!

cao@nwafu.edu.cn

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