Complex Forest Ecosystem: Theory and Methodology

Dr. Tianjian Cao
Professor of Forest Management
cao@nwafu.edu.cn
www.optifor.cn
Introduction

- This course is designed for master students who are preparing to engage in an interdisciplinary study in the fields of

  - silviculture,
  - forest health,
  - forest ecology,
  - forest planning,
  - and forest ecosystem management.
Introduction

The aim of this course is to help students acquire skills in predicting and assessing dynamic forest ecosystems in terms of:

- Data
- Models
- DSSs
- Applications
Purposes

- The purpose of this course is to show students how to link their knowledge in forest sciences with an interdisciplinary approach.

- Prerequisites
  - Forest Ecology
  - Forest Mensuration and Management
  - Silviculture
  - Forest Biometrics
  - Operations Research
Grading

- Quiz, 30%
- Final, 70%

- Part I: Forest Inventory
- Part II: Forest Planning
- Lecture hours: 32+32 hrs
- Credits: 2+2 crs

- Instructor: Prof. Dr. Tianjian Cao
- E-mail: cao@nwafu.edu.cn
Simulation Optimization Laboratory

Decision Support Systems for Natural Resources and Ecosystem Management

Assessment System
- Model linkages
- System integration
- Multi-objective assessment

Simulation System
- Empirical methods
- Process-based methods
- Hybrid modelling approach

Optimization System
- Heuristic search
- Stochastic optimization
- Population-based methods

Risk Management System
- Natural disasters
- Human interventions
- Pollution control

Dynamic Observation System
- Ecosystem observation
- Wireless sensor
- Remote sensing

Visualization System
- Scaling
- Spatial data
- GIS

Northwest A&F University
Simulation Optimization Laboratory
Simulation Optimization Laboratory

- EST. since 2008.
  - www. optifor.cn

- Interdisciplinary studies
  - Biometrics, operations research
  - Forest sciences, ecology

- Research interests
  - Uncertainty in dynamic ecosystems
  - Simulation-optimization systems
Research areas

OptiFor Carbon
- Carbon balance

OptiFor Wood
- Wood quality

OptiFor Algorithm
- Artificial intelligence

OptiFor Parameter
- Parameter optimization

OptiFor Structure
- Multi-functional ecosystem

OptiFor Climate
- Climate change

OptiFor Bioenergy
- Bioenergy production

QUASSI
- QUASSI 2.0
  - Process-based
- QUASSI 1.0
  - Empirical
- QUASSI 3.0
  - Hybrid

The OptiFor Family
www.optifor.cn
Research areas, con't

- OptiFor 1.0
  - OptiFor Wood
  - OptiFor Carbon
  - OptiFor Climate
  - OptiFor Bioenergy

- OptiFor 2.0
  - OptiFor Algorithm
  - OptiFor Parameter
  - OptiFor Structure

- QUASSI 1.0
  - Empirical models

- QUASSI 2.0
  - Process-based models

- QUASSI 3.0
  - Hybrid models
Research network

Quang V. Cao
Biometrics

Shuaichao Sun
Competition

Hailian Xue
Algorithms

Tianjian Cao
Simulation-optimization

Xianglin Tian
Hybrid modeling

Bin Wang
Bayesian methods

Mengzhen Kang
Structure-function

Shubin Si
Complex system

Lauri Valsta
Optimization

Kari Hyytiäinen
Model linkages

Timo Pukkala
Forest planning

Jerry Vanclay
Forest modeling

Annikki Mäkelä
Process models
Aims and Objectives

- Long-term and short-term aims
  - Long-term: Uneven-aged stand growth models
  - Short-term: Methodology development

- Theory and application study objectives
  - Theory studies: Complex ecosystem theory and optimization methods
  - Application studies: Adaptive management of natural forest ecosystems
Computer programs to be used in the course

- SPSS
- SigmaPlot
- Solver
- Simile
- OptiFor
- QUASSI
Part I: Forest Inventory
Data and Models
Caveats

- Impossible to validate any model (Popper 1963).
- All models are false, but some models are useful. Our job is to identify the useful ones useful. (G.E.P. Box).
- No model can be evaluated in the absence of some clearly stated objective (Goulding 1979).
- No criterion is universal, so some subjectivity must always remain.
- Empiricism cannot be avoided, and keeps models grounded in reality.

Source: Monserud (2003), presentation in Symposium of Systems Analysis in Forest Resources, October 7-9, 2003, Stevenson, Washington
Contents

- Data collection
- Sampling design
- Calculating supplementary data
- Site and competition variables
- Simulation modeling techniques
- Whole-stand and individual-tree models
- Empirical and mechanistic models
Pre-exam

- Suppose you are running a project concerning forest inventory,

- Try to plan the project, including
  - data requirements,
  - methods applied,
  - budget,
  - timetable,
  - etc..
Why forest inventory?

- Forest managers
- Forest owners
- Forest policy makers
- Investment banks
- Timber industry
- Paper making industry
- Forest ecologists
- Forest parks
- NGOs
Forest inventory systems

- National forest inventory,
- inventory for forest management planning,
- inventory for silvicultural operations

- National and local standards for forest inventory
Ecosystem observation systems

- LTER  www.lternet.edu
- GTOS  www.fao.org/gtos
- ECN   www.ecn.ac.uk
- GEMS  www.unep.org/gemswater
- CNERN www.cnern.org
- CFERN www.cfern.org
Sufficient data

- Field work can provide necessary and sufficient data efficiently.

- However, it may take several years to obtain the necessary data from permanent plots,

- and few of us can wait that long.

- “Nothing perfect except in our memories…”
Procedure of forest inventory

- Definition

  => Collection

  => Validation

  => Storage

  => Analysis

  => Synthesis
Data collection

- The life cycle of a datum spans its definition, collection, validation, storage, analysis and synthesis.

- All stages are equally important, and an efficient data management system requires a healthy balance between them.

- The first step is to define information needs and devise data collection procedures to satisfy those needs.
Differing data needs

- Stem analyses do not provide reliable growth data for many tree species, e.g., in tropical moist forests.
- Permanent plots can never be completely replaced by temporary plots.

- Resource Inventory
- Continuous Forest Inventory for yield control
- Growth modelling
- Long term monitoring of environmental change
IUFRO Division 4


- 4.00.00 - Forest Assessment, Modelling and Management
  - 4.01.00 – Forest mensuration and modelling
  - 4.02.00 – Forest resources inventory and monitoring
  - 4.03.00 – Informatics, modelling and statistics
  - 4.04.00 – Forest management planning
  - 4.05.00 – Managerial economics and accounting
Forest resource monitoring and assessment

Forest Resource Monitoring and Assessment

How are the nation's forests doing? The answer to that question is more important than ever as demand for wood and other forest products grows and land use patterns and public expectations change. Northern Research Station researchers are developing tools to provide more reliable and more consistent answers to questions about forest conditions and the effects of management practices, pests, and changing climate. We are developing techniques to monitor forest ecosystems more closely using scientifically credible methods. We are tracking forest conditions in the Northeast and Midwest through the nationally consistent Forest Inventory and Analysis program. These tools will help federal, state, tribal, and private land managers collect and analyze data that assists their efforts to ensure the sustainability of forests.

Research Studies

Assessment of Forest Site Quality in...
Mechanism

- Monitoring
  - Monitoring systems
  - Monitoring methodology and techniques
- Assessment
  - Assessment index
  - Assessment methodology
  - Single objective vs. multive objective
- Monitoring and Assessment Systems
  - UNDP: United Nations Development Programme
  - UNEP: United Nations Environment Programme
  - IPCC: Intergovernmental Panel on Climate Change
  - FAO: Food and Agriculture Organization
4.01.00 – Forest mensuration and modelling

- 4.01.01 – Design, performance and evaluation of experiments
- 4.01.02 – Growth models for tree and stand simulation
- 4.01.03 – Instruments and methods in forest mensuration
- 4.01.04 – Effects of environmental changes on forest growth
- 4.01.05 – Process-based models for predicting forest growth and timber quality
- 4.01.06 – Analysis and modelling of forest structure
Definitions

- forest resources
- forest inventory
- forest resource monitoring
- forest management planning
Definitions, con’t

- Timber value vs. non-timber value
  - Timber: sawlogs, pulpwood, energy wood, ...
  - Non-timber: mushrooms, hunting, biodiversity, carbon sequestration, amenities, preventing erosion...

- Quantitative method vs Qualitative method
  - Quantitative: connected with the amount or number of sth rather than how good it is
  - Qualitative: connected with how good sth is, rather than how much of it there is
Data requirements

- LUCC: Land Use and Cover Change
- Forest growth and yield
- Forest operations
- Forest biodiversity
- Forest carbon sequestration
- Forest health
- Forest ecosystem management
- etc.
Theory

- Sampling theory and sampling design
- Temporary vs. permanent plots
- Stem analysis
- Empirical vs. mechanistic data
- RS and GIS data
- Forest growth and yield models
Sampling theory

- Sampling plots (Kiaer 18??),
- Stochastic sampling (Bowley 1912)
- Stochastic > Typical, Neyman (1934)
- Confidence interval, Neyman (1934), Bellhouse (1988)
- Reducing inventory cost, Loetsch et al. (1973)
- Early sampling in North America, Germany, Nordic countries, visual inspection
- Since 1900 to 1920, applications of statistics
Sampling theory, con't

- Stripe sampling, 1830s

- Systematic > Stochastic, Hasel (1938), Osborne (1942)

- Stratified random sampling, Finney (1948)

- Point sampling（Bitterlich 1947）

- PPS sampling, Grosenbaugh（1952）
Sampling theory, con't

- CFI (Continues Forest Inventory) system, Scott (1947)
- Permanent + temporary plots, Bickford (1959)
- Satellite images, (Czapelewska 1999)
- Remote sensing, LiDAR
Forest inventory in China

- NFI (National Forest Inventory)
  20m * 30m, 2km * 4km
  tot. 230,000 plots

- Forest management planning
  stand level

- Silvicultural operations
  harvest scheduling
Forest Inventory in Europe

- **Sweden**
  - Since 1923, based on systematic sampling, including permanent plots (since 1983) and temporary plots.

- **Germany**
  - 44000 plots, 150m * 150m, 4km * 4km or 2km * 2km
Forest Inventory in North America

- **Canada**
  - Province government 77%, central government 16%, 7% private
  - Province level, 10-15 years
  - Country level, based on province forest inventory
  - Industrial forests (forest management, silvicultural operations)

- **The United States**
  - FIA, since 1930
  - P1, satellite images, forest or non-forest land
  - P2, one plot every 2439 ha (forest ecosystem)
  - P3, one plot every 39024 ha (ecological data)
Fig. 5.4. Recommended plot layout for permanent sample plots.
<table>
<thead>
<tr>
<th>Field form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plot No ..... Subplot .....</td>
</tr>
<tr>
<td>Subplot Dimensions ..... x .....</td>
</tr>
<tr>
<td>Orientation ..... Coordinates</td>
</tr>
<tr>
<td>Location</td>
</tr>
<tr>
<td>Assessing Officer</td>
</tr>
</tbody>
</table>

| Tree number |
| Coordinates |
| Family |
| Genus |
| Species |
| Common name |
| DBH |
| Point of measure |
| Valid/approx |
| Alive/dead/cut/missing |
| Erect/leaning/fallen |
| Broken/injury |
| Tree height |
| Bole height |
| Crown position |
| Crown form |
| Crown diameter |
| Merchantable length |
| Stem straightness |
| Stem defects |
| Notes: |
| Flower/fruiting |
| Pests/disease |
### Example: Stratified random sampling

<table>
<thead>
<tr>
<th></th>
<th>A (ha)</th>
<th>n</th>
<th>V (m³/ha)</th>
<th>SD (m³/ha)</th>
<th>Vtot (m³)</th>
<th>SD (m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bare land</td>
<td>18.0</td>
<td>18</td>
<td>42</td>
<td>6.7</td>
<td>756</td>
<td>121</td>
</tr>
<tr>
<td>Young stands</td>
<td>33.3</td>
<td>35</td>
<td>167</td>
<td>10.0</td>
<td>5557</td>
<td>344</td>
</tr>
<tr>
<td>Old stands</td>
<td>48.7</td>
<td>49</td>
<td>268</td>
<td>13.3</td>
<td>13033</td>
<td>649</td>
</tr>
</tbody>
</table>

- $V_{ave} = 0.180 \times 41.976 + 0.333 \times 166.84 + 0.487 \times 267.67 = 193 \text{ m}^3/\text{ha}$
- $SD = \sqrt{0.180^2 \times 45.437 + 0.333^2 \times 100.27 + 0.487^2 \times 177.72} = 7.4 \text{ m}^3/\text{ha}$
- $V_{tot} = 755.6 + 5557.1 + 13033.4 = 19346 \text{ m}^3$
- $SD_{tot} = \sqrt{14722 + 111246 + 421372} = 740 \text{ m}^3$
Example: remote sensing as additional info.

- Suppose you have 1000 ha forest, including 5 areas with NIR information, average value 0.2482, SD 0.0364. 1000 observation database

\[ V_i = 322.7473 - 714.951 \times \text{NIR}_i + \text{epsilon}_i \]

- \( \text{epsilon}_i \) SD 38.66m³/ha.
### Example, con't

<table>
<thead>
<tr>
<th>Area</th>
<th>Size (ha)</th>
<th>NIR</th>
<th>V (m³/ha)</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>94</td>
<td>0.22893</td>
<td>155.5</td>
<td>43.82</td>
</tr>
<tr>
<td>2</td>
<td>69</td>
<td>0.25104</td>
<td>140.7</td>
<td>40.43</td>
</tr>
<tr>
<td>3</td>
<td>123</td>
<td>0.26008</td>
<td>139.2</td>
<td>42.34</td>
</tr>
<tr>
<td>4</td>
<td>537</td>
<td>0.28201</td>
<td>120.4</td>
<td>45.40</td>
</tr>
<tr>
<td>5</td>
<td>177</td>
<td>0.31497</td>
<td>92.5</td>
<td>44.35</td>
</tr>
<tr>
<td>mean</td>
<td></td>
<td>0.27802</td>
<td>122.5</td>
<td>47.84</td>
</tr>
<tr>
<td>sum</td>
<td></td>
<td></td>
<td>1000</td>
<td></td>
</tr>
</tbody>
</table>
### Example, con't

<table>
<thead>
<tr>
<th>Area</th>
<th>n</th>
<th>NIR</th>
<th>( y_i )</th>
<th>s_e</th>
<th>( y_{i\text{SYN}} )</th>
<th>( y_{i\text{REG}} )</th>
<th>( y_{i\text{SUR}} )</th>
<th>s_e</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>0.22893</td>
<td>169.2</td>
<td>13.79</td>
<td>115.6</td>
<td>151.4</td>
<td>158.5</td>
<td>11.387</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0.25104</td>
<td>92.5</td>
<td>25.35</td>
<td>115.6</td>
<td>134.1</td>
<td>102.0</td>
<td>31.773</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>0.26008</td>
<td>120.8</td>
<td>20.36</td>
<td>115.6</td>
<td>127.0</td>
<td>123.0</td>
<td>20.389</td>
</tr>
<tr>
<td>4</td>
<td>22</td>
<td>0.28201</td>
<td>113.5</td>
<td>9.45</td>
<td>115.6</td>
<td>109.8</td>
<td>116.3</td>
<td>6.946</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>0.31497</td>
<td>91.4</td>
<td>12.21</td>
<td>115.6</td>
<td>84.0</td>
<td>83.1</td>
<td>8.346</td>
</tr>
<tr>
<td>mean</td>
<td></td>
<td>0.27802</td>
<td>115.6</td>
<td>6.91</td>
<td>115.6</td>
<td>112.9</td>
<td>112.9</td>
<td>5.216</td>
</tr>
<tr>
<td>sum</td>
<td></td>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- \( y_i \) area \( i \), traditional part estimation
- \( y_{i\text{SYN}} \) area \( i \), average estimation
- \( y_{i\text{REG}} \) area \( i \) average value*coefficient
- \( y_{i\text{SUR}} \) tot.
Quiz 2. SD

- average volume per ha:
  \[ y_{ave} = \frac{1}{n} \cdot \sigma_{i=1}^{n}(y_i) \]
  \[ = \frac{1}{102} \cdot \sigma_{i=1}^{102}(y_i) = 193 \text{ m}^3/\text{ha} \]

- \( f = \frac{n}{N} \)
  \[ f = \sigma_{i=1}^{n}(a_i/A) = \sigma_{i=1}^{102}(a_i/100) = 0.02 \]

\[ s_y^2 = \frac{1}{n-1} \left( \sigma_{i=1}^{n}(y_i^2) - \left( \sigma_{i=1}^{n}(y_i) \right)^2/n \right) = 12601 \left( \text{m}^3/\text{ha} \right)^2 \]

Question: \( s_y\hat{} = \sqrt{\left(1 - \frac{n}{N}\right) \left( s_y^2/n \right)} = ? \)
Sigma epsilon, or Pi epsilon?

- Sampling errors
- Design errors
- Inventory errors
- Model errors
- Prediction errors
- Planning errors
- Decision errors

Tot_e = f(e_i, t)
Sampling techniques by data requirements

Table 5.1. Different applications require different sampling techniques.

<table>
<thead>
<tr>
<th>Plot characteristics</th>
<th>Principal objective of permanent plot system</th>
<th>Resource inventory</th>
<th>Continuous forest inventory</th>
<th>Growth modelling</th>
<th>Site monitoring</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Permanence</td>
<td>Temporary</td>
<td>Permanent</td>
<td>Permanent</td>
<td>Permanent</td>
<td>Permanent</td>
</tr>
<tr>
<td>Area</td>
<td>Variable, ( \propto ) tree size</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
</tr>
<tr>
<td>Within-plot variance</td>
<td>Heterogeneous</td>
<td>Homogeneous</td>
<td>Homogeneous</td>
<td>Homogeneous</td>
<td>Homogeneous</td>
</tr>
<tr>
<td>Placement</td>
<td>Stratified random</td>
<td>Systematic</td>
<td>Stratified random</td>
<td>Purposive or systematic</td>
<td></td>
</tr>
<tr>
<td>Sample unit</td>
<td>Plot</td>
<td>Plot</td>
<td>Tree</td>
<td>Plant parts</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 5.2. Efficient placement of ten samples to (A) estimate slope of a straight line, (B) detect curvilinearity where variance $\propto X$, (C) calibrate an optimum, (D) detect a threshold, and (E) fit a curved relationship.
Sampling principles

- **Temporal distribution**
  - Short time periods may give rise to biased growth estimates, and longer periods of observation offer a better basis.

- **Spatial distribution**
  - Permanent plots should sample an adequate geographical range, including latitude, longitude, elevation and other topographical features such as ridge and valley locations.

- **Site factors**
  - The sampling system should ensure that the full range of site factors (e.g. soil type and depth) is included in the permanent plot system.

- **Stand conditions**
  - Stand structure and composition, should be manipulated experimentally to provide the best database.
Interpolation vs. Extrapolation

Fig. 5.1. Interpolation is safer than extrapolation. Both these lines have an $R^2$ better than 0.996, but provide no basis for making a prediction outside the range of the data.
Fig. 5.3. Database weaknesses revealed by comparing dynamic and static inventory data. Five new PSPs (×) would improve the database for modelling (*Callitris* forest in Queensland, redrawn from Beetson *et al.* 1992).
Experiments design

- Passive monitoring data: survey data from forest areas under routine management.
- Treatment response data: from paired treatment and control plots (controlled experiments).
- Number of plots
- Size and shape of plots
- What to measure
- When to remeasure
- Data administration
Number of plots

\[ n_0 = \left( \frac{Z_{\alpha/2}}{d} \right)^2 S^2 \]

\[ n = \frac{n_0}{\left( 1 + \frac{n_0}{N} \right)} \]

- Known:
  - Forest area = 100 ha
  - Inventory aim: \( V_{tot} \) (m³/ha)
  - Confidence interval = 95%
  - \( d = 15 \) m³/ha
  - \( S = 50 \) m³/ha
  - \( z_{\alpha/2} = 1.96 \)

- No. of plots (20m*20m) = ?
Solutions

\[ n_0 = \left(\frac{z_{\alpha/2}}{d}\right)^2 \cdot S^2 = \left(\frac{1.96}{15}\right)^2 \cdot 50^2 \]

\[ N = \frac{100}{\left(20 \cdot 20 / 10000\right)} = 2500 \]

\[ n = \frac{n_0}{1 + n_0 / N} = \frac{42.68}{1 + 42.68 / 2500} = 41.96 \]
Variables should be measured

- At the initial enumeration
  - Plot location, dimensions, orientation and area,
  - Species and coordinates of all trees on the plot,
  - Topographic details, including altitude, aspect, slope, position on slope,
  - Forest type and floristic attributes,
  - Physical soil characteristics (depth, texture, colour, parent material), and
  - Uniformity of the site
- At the first measure, immediately after any harvest, and periodically (e.g. every second or third measure):
  - Sufficient tree heights for the determination of site productivity (or data necessary for alternative estimates of site productivity),
  - Merchantable heights and defect assessments of all stems (including non-commercial species, as utilization standards may change with time),
  - Crown characteristics (position, length, width, form, etc.);
At every measure, assess all stems (including non-commercial; every stem from the previous measure must be reconciled) for:

- Diameter (over bark, breast high or above buttress), height to measure point, and validity (to indicate defects at measure point and anomalous but correct increments),
- Status (alive, dead, harvested, treated) and stance (erect, leaning, fallen, broken), and
- Tree coordinates (recruits only);
Variables, con’t

- As necessary, record the occurrence of:
  - Logging, treatment and other activities, and the prescription used,
  - Scars and other damage with may affect measurements or growth,
  - Meteorological phenomena (drought, flood, etc.),
  - Mast years (heavy seed crops),
  - Pests, diseases, fire, or any other aspect which may affect growth.
Data problems

- The greatest problem facing many agencies is that the data necessary for growth model development are not available.

- Unreliable measurements
- Changes to procedures
- Mistaken or undetermined species identities

- No data set can be perfect, but many will be found to contain deficiencies that will frustrate future analyses.
Density experiment design
Figure 2. Google Earth image of Mt Mee Nelder trial, 20 July 2009 (© 2012 Google, © 2012 GeoEye, 27.096°S, 152.734°E), showing the two species, survival, and proximity of other plantings.
Figure 4. Design for a mixed species trial, showing planting positions for four species (o, +, x and □) in a 20 x 20 grid, showing three different viewpoints: the 3x3 viewpoint with 36 plots (top left), the 4x4 viewpoint with 25 plots (top right), and the 5x5 viewpoint with 16 plots (bottom left).
Figure 5. A possible design for a clinal planting to supplement the design in Figure 1, showing planting positions for 100 trees of each of four species (shown as o, x, + and □).
Regeneration experiment design

- Direct seeding (control)
- Direct seeding + covering
- Direct seeding + liming
- Plots without direct seeding, not considered in this paper
Experimental design
Exercise 1

Design a form to record field measurements during the initial enumeration of a permanent plot in a forest near you. Take it to the field and try it! Enter the data from the form into a text or spreadsheet file on a computer.

What problems did you detect in the field and during data entry, and how would you improve your form next time?

Would you use the same form when the plot was re-measured; if not, what changes would you make?

How could you include some of the data from the initial measure on the remeasure form, so that field crews could cross-check these details?
Exercise 2

- Document ways that the data collected as part of Exercise 1 could be validated on the computer.
- What additional checks could you make when remeasured data become available?
- What errors might remain undetected by these procedures?
- Could these procedures be implemented on an electronic data recorder so that these checks could be made automatically in the field during plot remeasurement?
**V_tot, V_ave, and Standard error**

- Known: 20 ha forests grouped by age stage (young=1, mature=2, old=3), plot size: 20m*30m

<table>
<thead>
<tr>
<th>Age stage</th>
<th>Area, ha</th>
<th>No. of plots, n</th>
<th>Volume/plot, y (m³/ha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young stands</td>
<td>10</td>
<td>8</td>
<td>36 42 41 42 30 35 43 44</td>
</tr>
<tr>
<td>Mature stands</td>
<td>6</td>
<td>6</td>
<td>80 83 77 88 72 68</td>
</tr>
<tr>
<td>Old stands</td>
<td>4</td>
<td>6</td>
<td>122 135 140 127 136 121</td>
</tr>
</tbody>
</table>

- $V_i = ?$, $V_i\_ave = ?$
- $V_{tot} = ?$, $V_{ave} = ?$
- Stand errors = ?
Solutions, $V_{i\text{ ave}}$

\[
\hat{y}_1 = \frac{1}{n_1} \sum_{i=1}^{n_1} y_{1i} = \frac{1}{8} \sum_{i=1}^{8} y_{1i} = \frac{36 + 42 + 41 + 42 + 30 + 35 + 43 + 44}{8} = 39.1 \text{ m}^3/\text{hm}^2
\]

\[
\hat{y}_2 = \frac{1}{n_2} \sum_{i=1}^{n_2} y_{2i} = \frac{1}{6} \sum_{i=1}^{6} y_{2i} = \frac{80 + 83 + 77 + 88 + 72 + 68}{6} = 78.0 \text{ m}^3/\text{hm}^2
\]

\[
\hat{y}_3 = \frac{1}{n_3} \sum_{i=1}^{n_3} y_{3i} = \frac{1}{6} \sum_{i=1}^{6} y_{3i} = \frac{122 + 135 + 140 + 127 + 136 + 121}{6} = 130.2 \text{ m}^3/\text{hm}^2
\]
Variance (SD^2)

\[ s_{y1}^2 = \frac{1}{n_1} \sum_{i=1}^{n_1} (y_{1i} - \hat{y}_1)^2 = 21.1 \]

\[ s_{y2}^2 = \frac{1}{n_2} \sum_{i=1}^{n_2} (y_{2i} - \hat{y}_2)^2 = 44.3 \]

\[ s_{y3}^2 = \frac{1}{n_3} \sum_{i=1}^{n_3} (y_{3i} - \hat{y}_3)^2 = 52.5 \]
Standard error

\[ s_{\bar{y}_1} = \sqrt{\left(1 - \frac{n_1}{N_1}\right)\frac{s_{y1}^2}{n_1}} = \sqrt{\left(1 - \frac{8}{10/0.06}\right)\frac{21.2}{8}} = 1.59 \text{ m}^3/\text{hm}^2 \]

\[ s_{\bar{y}_2} = \sqrt{\left(1 - \frac{n_2}{N_2}\right)\frac{s_{y2}^2}{n_2}} = \sqrt{\left(1 - \frac{6}{6/0.06}\right)\frac{44.3}{6}} = 2.63 \text{ m}^3/\text{hm}^2 \]

\[ s_{\bar{y}_3} = \sqrt{\left(1 - \frac{n_3}{N_3}\right)\frac{s_{y3}^2}{n_3}} = \sqrt{\left(1 - \frac{6}{4/0.06}\right)\frac{52.5}{6}} = 2.83 \text{ m}^3/\text{hm}^2 \]
\( V_i, SE_i \)

- **\( V_i \):**

\[
\hat{T}_1 = A_1 \hat{y}_1 = 10 \times 39.1 = 391.0 \text{m}^3
\]

\[
\hat{T}_2 = A_2 \hat{y}_2 = 6 \times 78.0 = 468.0 \text{m}^3
\]

\[
\hat{T}_3 = A_3 \hat{y}_3 = 4 \times 130.2 = 520.8 \text{m}^3
\]

- **\( SE_i \):**

\[
s_{T_1} = \sqrt{\text{var}(\hat{T}_1)} = \sqrt{A_1^2 \text{var}(\hat{y}_1)} = \sqrt{10^2 \times 1.59^2} = 15.9 \text{m}^3
\]

\[
s_{T_2} = \sqrt{\text{var}(\hat{T}_2)} = \sqrt{A_2^2 \text{var}(\hat{y}_2)} = \sqrt{6^2 \times 2.63^2} = 15.8 \text{m}^3
\]

\[
s_{T_3} = \sqrt{\text{var}(\hat{T}_3)} = \sqrt{A_3^2 \text{var}(\hat{y}_3)} = \sqrt{4^2 \times 2.83^2} = 11.3 \text{m}^3
\]
V\_ave, SE\_ave, V\_tot, SE\_tot

\[
\text{V\_ave: } \hat{y}_{tot} = \sum_{i=1}^{3} \frac{A_i}{A} \hat{y}_i = \frac{10}{20} \times 39.1 + \frac{6}{20} \times 78.0 + \frac{4}{20} \times 130.2 = 69.0 \text{ m}^3/\text{hm}^2
\]

\[
\text{SE\_ave: } s_{\hat{y}_{tot}} = \sqrt{\sum_{i=1}^{3} \left( \frac{A_i}{A} \right)^2 \text{var}(\hat{y}_i)} = \sqrt{\left( \frac{10}{20} \right)^2 \times 1.59^2 + \left( \frac{6}{20} \right)^2 \times 2.63^2 + \left( \frac{4}{20} \right)^2 \times 2.88^2} = 1.25 \text{ m}^3/\text{hm}^2
\]

\[
\text{V\_tot: } T_{tot} = \hat{T}_1 + \hat{T}_2 + \hat{T}_3 = 391.0 + 4680 + 520.8 = 1379.8 \text{ m}^3
\]

\[
\text{SE\_tot: } s_{T_{tot}} = \sqrt{\sum_{i=1}^{3} \text{var}(\hat{T}_i)} = \sqrt{s_{\hat{T}_1}^2 + s_{\hat{T}_2}^2 + s_{\hat{T}_3}^2} = \sqrt{15.9^2 + 15.8^2 + 11.3^2} = 25.1 \text{ m}^3
\]
Forest site evaluation

- Plantations vs. natural forests
- Forest types and forest productivity
- Site index, site form, or site class
- Dynamics of forest site quality
- Multi-data and site evaluation
- Biomass production and forest productivity
# Methods for site evaluation

<table>
<thead>
<tr>
<th></th>
<th>Direct</th>
<th>Indirect</th>
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<tbody>
<tr>
<td><strong>Phytocentric</strong></td>
<td>Wood volume</td>
<td>Tree height</td>
</tr>
<tr>
<td><strong>Geocentric</strong></td>
<td>Soil moisture &amp; nutrient status</td>
<td>Climate</td>
</tr>
<tr>
<td></td>
<td>Photosynthetically active radiation</td>
<td>Land form</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Physiography</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Plant indicators</td>
</tr>
</tbody>
</table>

Source: Vanclay (1994)
The Qinling Mountains

a.s.l. 3767 m
Slope 35°
Soil 50 cm
E-W 1600 km
S-N 10-300 km
Forest types in Qinling

2800-3200 m  Larix chinensis
2400-2800 m  Abies fargesii
2000-2400  m  Betula albo-sinensis,  
    Betula albo-sinensis var. septerionalis
1700-2000 m  Pinus armandii - Betula albo-sinensis
1400-1700 m  Pinus tabulaeformis - Quercus aliena var. acuteserrata
800-1400 m  Quercus aliena, Quercus glandulifera var. brevipetiolata
Forest types, con’t
Classification of forest site

- Based on geo-factors
- Based on soil-factors
- Based on plant indicators
  - Cajander (1926), forest floor vegetation
  - Daubenmire (1968), community
- Based on climate, terrain, soil, and vegetation
Site index, or Site form?

- Site index, even-aged plantations, Hdom
- Site form, uneven-aged forests, D? (Weiskittel et al., 2011)
Site index vs. site class (Wu et al. 2015)

图 3-2 地位级与立地指数曲线簇

Fig.3-2 Curves of site class and site index
Site index vs. site form (Wu et al. 2015)

Fig. 3-1 Examples of site index and site form curves by species.
Site form

GC, Guide Curve
ADA, Algebraic Difference Approach

图 4-1 油松、华山松和锐齿栎立地形曲线簇

Fig.4-1 Curves of site form by species
Site form for pine-oak forest, con’t

- $SF_{PT} = a_1SF_{PA} + e_1$

- $SF_{PA} = a_2SF_{QA} + b \text{ Composition} + e_2$

- $SF_{QA} = a_3SF_{PT} + c \text{ DBH} + d \text{ Density} + e_3$
Factors affecting site form

图 4-2 林分因子对立地形的影响。

Fig.4-2 Influence of stand factors on site form.
Changes on site class

Fig. 5-1 Changes of each site class percentage.
Multi-data and site evaluation
Predicting average height with multi-data

The uncertainty of predictions, 95% Bayesian credible interval, were showed in grey area.
(SCI=11, SDI=600)
### Processing inventory data

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<tr>
<th>Plot no.</th>
<th>Age, yr</th>
<th>H, m</th>
<th>Plot no.</th>
<th>Age, yr</th>
<th>H, m</th>
<th>Plot no.</th>
<th>Age, yr</th>
<th>H, m</th>
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<td>8.4</td>
<td>12</td>
<td>25</td>
<td>6.3</td>
<td>22</td>
<td>39</td>
<td>11.5</td>
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<td>23</td>
<td>22</td>
<td>8.0</td>
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<td>47</td>
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<td>24</td>
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<td>15.7</td>
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<tr>
<td>5</td>
<td>30</td>
<td>9.9</td>
<td>15</td>
<td>67</td>
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<td>25</td>
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<td>14.3</td>
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<td>66</td>
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<td>25</td>
<td>11.8</td>
<td>30</td>
<td>58</td>
<td>16.0</td>
</tr>
</tbody>
</table>
Height vs. Age
Theoretical growth equations

- Assume tree growth $y$ (DBH, H, BA, or V) is a function of time, $t$
- $y = f(t)$

- Schumacher: $y = a \times \exp(-\frac{b}{t})$
- Mitscherlich: $y = a(1 - \exp(-b \times t))$
- Logistic: $y = \frac{a}{1 + c \times \exp(-b \times t)}$
- Gompertz: $y = a \times \exp(-b \times \exp(-c \times t))$
- Korf: $y = a \times \exp(-b \times t^{-c})$
- Richards: $y = a(1 - \exp(-b \times t)^c$

- $a$, $b$, $c$, parameters
Computer programs for biometrics

- Statistical Package for the Social Sciences (SPSS)
- SAS
- R
- Python
Fit-to-curve

- Step 1. select one of equations, e.g. Schumacher
- Step 2. compute initial guesses of parameters

\[ y_{\text{max}} = a \]

- Known:
  - \( y_{\text{max}} = 25 \text{ m} \)
  - \( H = 8 \text{ m}, \text{age} = 20 \text{ yrs} \)
  - \( 8 = 25 \times \exp(-b/20) \)
  - \( \ln8 = \ln25 - b/20 \)
  - \( b = (\ln25 - \ln8) \times 20 = 23 \)
### Iteration History

<table>
<thead>
<tr>
<th>Iteration Number(^a)</th>
<th>Residual Sum of Squares</th>
<th>Parameter</th>
<th>Parameter</th>
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<tr>
<td></td>
<td></td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>1.0</td>
<td>140.767</td>
<td>25.000</td>
<td>23.000</td>
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<tr>
<td>1.1</td>
<td>134.191</td>
<td>25.402</td>
<td>22.273</td>
</tr>
<tr>
<td>2.0</td>
<td>134.191</td>
<td>25.402</td>
<td>22.273</td>
</tr>
<tr>
<td>2.1</td>
<td>134.189</td>
<td>25.369</td>
<td>22.236</td>
</tr>
<tr>
<td>3.0</td>
<td>134.189</td>
<td>25.369</td>
<td>22.236</td>
</tr>
<tr>
<td>3.1</td>
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<td>25.367</td>
<td>22.232</td>
</tr>
<tr>
<td>4.0</td>
<td>134.189</td>
<td>25.367</td>
<td>22.232</td>
</tr>
<tr>
<td>4.1</td>
<td>134.189</td>
<td>25.367</td>
<td>22.232</td>
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</table>
### Parameter estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>95% Confidence Interval</th>
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<td>Lower Bound</td>
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<td>2.013</td>
<td>21.244</td>
</tr>
<tr>
<td>b</td>
<td>22.232</td>
<td>3.243</td>
<td>15.590</td>
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</tbody>
</table>
Predicting height growth

\[ H = 25.367 \times \exp(-22.232/t) \]
## Correlations of parameter estimates

<table>
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<tr>
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<th>a</th>
<th>b</th>
</tr>
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<tr>
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<tr>
<td>b</td>
<td>.938</td>
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ANOVA analysis

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<th>df</th>
<th>Mean Squares</th>
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<tr>
<td>Regression</td>
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<td>3165.235</td>
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<td>Residual</td>
<td>134.189</td>
<td>28</td>
<td>4.792</td>
</tr>
<tr>
<td>Uncorrected Total</td>
<td>6464.660</td>
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<tr>
<td>Corrected Total</td>
<td>438.159</td>
<td>29</td>
<td></td>
</tr>
</tbody>
</table>

Dependent variable: H

R squared = 1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = .694.
## Models for tree height growth

<table>
<thead>
<tr>
<th></th>
<th>Name</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Richard</td>
<td>$H_T = 1.3 + b \times [1 - \exp(-a \times DBH)]^c$</td>
</tr>
<tr>
<td>2</td>
<td>Weibull</td>
<td>$H_T = 1.3 + b \times [1 - \exp(-a \times DBH^c)]$</td>
</tr>
<tr>
<td>3</td>
<td>Logistic</td>
<td>$H_T = 1.3 + b / [1 + a \times \exp(-c \times DBH)]$</td>
</tr>
<tr>
<td>4</td>
<td>Korf</td>
<td>$H_T = 1.3 + b \times \exp(-a / DBH^c)$</td>
</tr>
<tr>
<td>5</td>
<td>Compertz</td>
<td>$H_T = 1.3 + b \times [1 + a \times \exp(-c \times DBH)]$</td>
</tr>
<tr>
<td>6</td>
<td>Quadratic</td>
<td>$H_T = a + b \times \text{Age} + c \times \text{Age}^2$</td>
</tr>
<tr>
<td>7</td>
<td>Inverse</td>
<td>$H_T = a + b / \text{Age}$</td>
</tr>
<tr>
<td>8</td>
<td>Sigmoidal</td>
<td>$H_T = b / [1 + \exp(-a / (\text{Age} - a) / c)]$</td>
</tr>
<tr>
<td>9</td>
<td>Logistic</td>
<td>$H_T = b / [1 + (\text{Age}/a)^c]$</td>
</tr>
<tr>
<td>10</td>
<td>Compertz</td>
<td>$H_T = b \times \exp(- \exp((a - \text{Age}) / c))$</td>
</tr>
<tr>
<td>11</td>
<td>Chapman</td>
<td>$H_T = b \times [1 - \exp(-a \times \text{Age})]^c$</td>
</tr>
<tr>
<td>12</td>
<td>Hill</td>
<td>$H_T = b \times \text{Age}^a / (c^a + \text{Age}^a)$</td>
</tr>
<tr>
<td>13</td>
<td>Hyperbola</td>
<td>$H_T = a - b / (1 + c \times \text{Age})^{1/d}$</td>
</tr>
<tr>
<td>14</td>
<td>Logarithm</td>
<td>$H_T = c + a \times \ln(\text{Age}) + b (\ln(\text{Age}))^2$</td>
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<tr>
<td>15</td>
<td>Power</td>
<td>$H_T = c + a \times \text{Age}^b$</td>
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<tr>
<td>16</td>
<td>Mitscherlich</td>
<td>$H_T = b \times [1 - \exp(-(c - \text{Age}) / a)]$</td>
</tr>
<tr>
<td>17</td>
<td>Modified Gaussian</td>
<td>$H_T = b \times [1 - \exp(-[(\text{Age} - a) / c]^2)]$</td>
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<tr>
<td>18</td>
<td>Richard</td>
<td>$H_T = b \times [1 - \exp(-a \times \text{Age})]^c$</td>
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<tr>
<td>19</td>
<td>Schumacher</td>
<td>$H_T = b \times \exp(-c / (\text{Age} - a))$</td>
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## Fitting results

### Tab. 4 Fitting results of guide curves

<table>
<thead>
<tr>
<th>拟合结果</th>
<th>油松 $P. \text{tabulaeformis}$</th>
<th>华山松 $P. \text{armandii}$</th>
<th>落叶松 $L. \text{principis-rupprechtii}$</th>
<th>锐齿栎 $Q. \text{aliena var. acuteserrata}$</th>
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<td>245</td>
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<td>631</td>
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<td>$b$</td>
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<td>17.391 7</td>
<td>53.928 8</td>
<td>30.772 2</td>
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<tr>
<td>$c$</td>
<td>1.024 3</td>
<td>38.618 9</td>
<td>0.324 8</td>
<td>-0.743 1</td>
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<td>SEE</td>
<td>2.84</td>
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<td>2.40</td>
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</table>

**Notes:**

1. $n$: 建模样本量 Number of observations for modelling; $R^2$: 决定系数 Coefficient of determination; SEE: 标准估计误差 Standard error.
Site class table

- Step 1. data processing
- Step 2. fit-to-curve using nonlinear regression
- Step 3. clean up unusual data with rule of 3*SD
- Step 4. re-fit with remaining data
  - e.g. re-fitted solution, $H = 25.117 \times \exp(-21.391/t)$
- Step 5. compute SD of height residuals by age class
  - e.g. $\sigma = 0.921$
- Step 6. define upper & lower bounds based on 3*SD
- Step 7. average upper & lower bounds of site classes
## Predicted value and Residuals

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<th>H</th>
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<th>RESID</th>
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3\times S.D.
Upper and lower bounds of site class

![Graph showing stand average height (m) vs age (yrs) with curves labeled H, H-3σ, and H+3σ. The graph includes data points from plots.](image)
Site class table for *Pinus tabulaeformis*

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<th>SC IV, m</th>
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<td>16.7–18.4</td>
<td>14.8–16.6</td>
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</table>
Data and Models

- Like chickens and eggs, it is not obvious which comes first.

- Modeling and definition and collection of data should form an iterative process, commencing with the model formulation.

- Most modeling efforts commence with and data available

- The modeling approach often may be dictated by limitations of the data.
Models for forest ecosystem management

- Forest yield models (CACTOS, FPS, FVS, ORGANON, SPS)
- Ecological gap models (JABOWA, FORET)
- Ecological compartment models (CENTURY, FOREST-BGC)
- Process/mechanistic models
  (PnET, PipeStem, ECOPHYS, FOREST-BGC)
- Vegetation distribution models (Monserud et al. 1993)
- Hybrid models (PipeQual/CROBAS, Mäkelä for Finland; 3-PG, Landsberg & Waring; Ågren for Sweden; FOREST 5: Robinson)

Source: Monserud (2003), Pretzsch et al. (2008)
## Model classification by scales and purposes

<table>
<thead>
<tr>
<th>Use</th>
<th>Resolution</th>
<th>Driving variables</th>
<th>Example</th>
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<td>Evapo-transpiration</td>
<td>Lieth &amp; Box (1972)</td>
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<tr>
<td>National forest planning</td>
<td>Stand variables</td>
<td>Age, stand basal area</td>
<td>Clutter (1963)</td>
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<tr>
<td>Regional planning</td>
<td>Individual trees</td>
<td>Tree species &amp; sizes</td>
<td>Prognosis (Stage 1973)</td>
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<tr>
<td>Silvicultural studies</td>
<td>Tree crowns</td>
<td>Tree &amp; branch variables</td>
<td>TASS (Mitchell 1975)</td>
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<td>Wood characteristics</td>
<td>Branches, ring width &amp; density</td>
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<td>Individual trees</td>
<td>Tree species &amp; sizes</td>
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<td>Nutrient cycling</td>
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<td>FORCYTE (Kimmins 1988)</td>
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<td>Physiological studies</td>
<td>Mass of foliage, branches, roots</td>
<td>Biomass, photosynthesis, respiration</td>
<td>Sievänen <em>et al.</em> (1988)</td>
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</table>
Components of forest growth
Yield tables

- A yield table presents the anticipated yields from an even-aged stand at various ages.

- One of the oldest approaches to yield estimation.

- Chinese “Lung Ch’uan codes”, some 350 yrs ago (Vuokila 1965).

- The first yield tables were published in Germany in 1787.

- Various approaches used in Europe (Vuokila 1965) and North America (Spurr 1952)
**Growth and yield tables**

### Yield Table

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</table>

### Yield Equations

Yield Equation: \( \log(V+1) = 3.534 - 14.02/t + 0.2314 \times S/t \)
**Table 1. Experience table for the yield of various species for light thinning (Von Cotta, 1821, p. 34)**

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<th>I</th>
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<th>IV</th>
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Whole stand models

- Whole stand models are those growth and yield models in which the basic units of modelling are stand parameters such as basal area, stocking, stand volume and parameters characterizing the diameter distribution.

- They require relatively little information to simulate the growth of a stand, but consequently yield rather general information about the future stand.
Example, Growth and yield table

- 15 years Chinese fir stand, $H_{ave}$ 12.0 m, $D_{ave}$ 12 cm, $V=150$ m$^3$/ha,
- Try to predict its $H_{ave}$, $D_{ave}$, $V$ at age 30.

- $H_{ave} = 17.0 \times \frac{12}{12.4} = 16.5$ m
- $D_{ave} = 23.0$ cm $\times \frac{12}{14} = 19.78$ cm
- $V = 351 \times \frac{150}{165} = 319.4$ m$^3$/ha
### Stand yield table, Chinese fir, SC II

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<th>Age, yr</th>
<th>H, m</th>
<th>D, cm</th>
<th>N, n/ha</th>
<th>V, m³/ha</th>
<th>V_ave, m³/ha/yr</th>
<th>ΔV, m³/ha/yr</th>
<th>V%, %</th>
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Growth tables and percentages

- Yield tables => require data: stand age, => uneven-aged?

- Growth tables => volume, density, height, average diameter, and crown class, other than age

- Growth percentages => expected growth expressed as a percentage rather than in absolute terms.

- Short-term or long-term?

- Other stands?
Growth and yield equations

- \( \Delta V_n = V_2 - V_1 + V_c \)

- \( \ln(V_t) = 1.34 + 0.394\ln G_0 + 0.346\ln t + 0.00275SC(t^{-1}) \)

- \( \Delta v = \beta_0 + \beta_1(d) + \beta_2(d)^2 \)

- \( \Delta G = \beta_0 + \beta_1*G(t^{-1})+(\beta_2+\beta_3(t^{-1})+\beta_4*SI)*G^2 \)

- \( \ln(\Delta G) = -3.071+1.094\ln G+0.007402G*SF-0.2258G \)
Lab exercise, whole stand model

- Suppose H, D, G growth is a function of t,

- Select the theoretical equation, Schumacher formula, for regression analysis.

- Constructing H-t, D-t, G-t regression model.
**Pinus tabulaeformis, SC II**

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<th>t</th>
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<th>D</th>
<th>G</th>
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<td>31</td>
<td>12.7</td>
<td>16.3</td>
<td>23.3</td>
</tr>
</tbody>
</table>
The whole stand model

- \( H = 22.841 \times \exp(-19.284/t) \)
- \( D = 36.149 \times \exp(-24.507/t) \)
- \( G = 33.767 \times \exp(-15.443/t) \)

- \( N = 40000 \times (G/(\pi \times D^2)) \)
- \( V_{\text{pinus}_t} = 0.33123 \times (D^2) \times H + 0.00805 \times D \times H - 0.00274 \times D^2 + 0.00002 \)
- \( V_{\text{pinus}_t} = (H+3)G \times f_{\text{pinus}_t}, f_{\text{pinus}_t} = 0.41 \)
## The yield table

<table>
<thead>
<tr>
<th>年龄 (a)</th>
<th>树高 (m)</th>
<th>平均胸径 (cm)</th>
<th>林分密度 (株/ha)</th>
<th>林分断面积 (m²/ha)</th>
<th>单木材积 (m³)</th>
<th>林分蓄积 (m³)</th>
</tr>
</thead>
<tbody>
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<td>15</td>
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<td>552</td>
<td>26.6</td>
<td>0.3794</td>
<td>209.4</td>
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</tbody>
</table>
Whole-stand distribution models

Figure 2.3. Possible diameter distributions generated by the Weibull p.d.f. (Eqn 2.9), showing the influence of each parameter (Table 2.1) on the shape of the distribution.

\[ f(x) = \frac{\beta}{\alpha} \left( \frac{x-y}{\alpha} \right)^{\beta-1} \exp \left( -\left( \frac{x-y}{\alpha} \right)^\beta \right) \]
Weibull distributions

<table>
<thead>
<tr>
<th>Example</th>
<th>alpha</th>
<th>beta</th>
<th>gamma</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>4</td>
<td>0.95</td>
<td>0</td>
</tr>
<tr>
<td>b</td>
<td>4</td>
<td>1.6</td>
<td>0</td>
</tr>
<tr>
<td>c</td>
<td>4</td>
<td>3.6</td>
<td>0</td>
</tr>
<tr>
<td>d</td>
<td>4</td>
<td>3.6</td>
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<td>8</td>
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<td>4</td>
</tr>
<tr>
<td>f</td>
<td>18</td>
<td>18.0</td>
<td>0</td>
</tr>
</tbody>
</table>
Diameter distribution by $D_{ave}$ and #tree at stand level
Stand structure development (Cao, 2003)

- Optimal solutions of plot 61 at 3% rate of interest, fit to 4-parameter Weibull distributions.
Optimization of Weibull parameters

Objective function:

$$
\min Z = \frac{\sum (D_i - \hat{D}_i)^2}{\hat{\sigma}_D^2} + \frac{\sum (B_i - \hat{B}_i)^2}{\hat{\sigma}_B^2} + \frac{\sum (N_i - \hat{N}_i)^2}{\hat{\sigma}_N^2}
$$

(3)

where $D$ is diameter, $B$ is basal area, $N$ is density, $\hat{\sigma}$ is estimated value of the standard deviation.

- stand attributes then can be predicted by $m$ tree species and $n$ diameter classes with stand-level data

s.t. $E_{Dg} \leq 0.25$

$E_{BA_i} \leq 0.1$, for all $i = 1, ..., m$

$d_{ij} \geq 0$, for all $i = 1, ..., m$; $j = 1, ..., n$
Whole-stand transition matrices

\[
dh_T / dt = 0.0752 \ h_T \ (3.59 - \ln h_T)^2
\]

\[
dG / dt = 0.0752 \ G \ (4.08 - \ln G) \ (3.59 - \ln h_T)
\]
Fig. 3.1. Stand table projection with movement ratio 0.25, so that 25% of each class moves up to the next class.
Transition Matrices

- Consider a hypothetical system $S$, with $n$ distinct states $S_1, S_2, \ldots, S_n$. If the system starts in state $S_i$, then in a single time interval, it has probability $P_{ij}$ of moving to state $S_j$.

- Provided that these $P_{ij}$ depend only on the current state $S_i$ and not on any historic events,

- These probabilities can be expressed in a square matrix, termed the transition probability matrix or stationary Markov chain.
## Size class model: diameter distribution

<table>
<thead>
<tr>
<th>size class</th>
<th>DBH, cm</th>
<th>N, trees/ha</th>
<th>d_ave, cm</th>
<th>ba_ave, m²</th>
<th>BA, m²/ha</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 [10, 20)</td>
<td>840</td>
<td>15</td>
<td>0.02</td>
<td>14.8</td>
<td></td>
</tr>
<tr>
<td>2 [20, 35)</td>
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<td>0.06</td>
<td>13.4</td>
<td></td>
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<tr>
<td>3 [35, ∞)</td>
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<td>40</td>
<td>0.13</td>
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</tr>
<tr>
<td>tot</td>
<td>1088</td>
<td></td>
<td></td>
<td></td>
<td>30</td>
</tr>
</tbody>
</table>
Transition matrix

- \( y_{1,t+1} = a_1 y_{1t} + R_t \)
- \( y_{2,t+1} = b_1 y_{1t} + a_2 y_{2t} \)
- \( y_{3,t+1} = b_2 y_{2t} + a_3 y_{3t} \)

- \( y_{it} \), number of trees per ha in size class \( i \) at time \( t \)
- \( a_i \), ratio of trees remains in class \( i \) during time \( t \) to \( t+1 \)
- \( b_i \), ratio of trees moves to class \( i+1 \) during time \( t \) to \( t+1 \)
- \( 1-a_i-b_i \), mortality of trees (%) in class \( i \) during time \( t \) to \( t+1 \)
Size class transition and ingrowth

**Known:**
- $R_t = 109 - 9.7G_t + 0.3N_t$
- $N_t = y_{1t} + y_{2t} + y_{3t}$
- $G_t = 0.02y_{1t} + 0.06y_{2t} + 0.13y_{3t}$

**Solutions:**
- $R_t$, recruits during time $t$ to $t+1$
- $R_t = 109 - 9.7(0.02y_{1t} + 0.06y_{2t} + 0.13y_{3t}) + 0.3(y_{1t} + y_{2t} + y_{3t})$
- $R_t = 109 + 0.106y_{1t} - 0.282y_{2t} - 0.961y_{3t}$
- $y_{1,t+1} = 109 + 0.906y_{1t} - 0.282y_{2t} - 0.961y_{3t}$
- $y_{2,t+1} = 0.04y_{1t} + 0.9y_{2t}$
- $y_{3,t+1} = 0.02y_{2t} + 0.9y_{3t}$
### Size class model: transition probability

<table>
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<tr>
<th>size class</th>
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<th>b_i</th>
<th>1-a_i-b_i</th>
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<td>i_1</td>
<td>i_2</td>
<td>i_3</td>
<td>BA</td>
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</table>
Individual-tree models

- $\delta(\text{DBH}) = f(\text{DBH}, \text{BA}, \text{SI}, \ldots)$
- $\delta(\text{HT}) = f(\text{HT}, \text{BA}, \text{SI}, \ldots)$
- $P_s = f(\text{DBH}, \text{HT}, \text{BA}, \text{SI}, \ldots)$
- $R = f(\text{BA}, \text{SI}, \ldots)$

Tree Records

- Spp, DBH, HT, n

- Spp, DBH + $\delta(\text{DBH})$, HT + $\delta(\text{HT})$, $p_s \cdot n$
Fig. 4.3. Tree records representing a forest stand. Growth is modelled by incrementing the diameters in each record \((d+\Delta)\) and mortality is accommodated by reducing expansion factors \((p \times n)\).
Basal area increment model

- $\ln(\text{BAI}) = a + b*\text{SIZE} + c*\text{COMP} + s*\text{SITE}$

- $b*\text{SIZE} = b_1*\ln(\text{DBH}) + b_2*\text{DBH}^2 + b_3*\ln(\text{CR})$

- $c*\text{COMP} = c_1*\text{BAL} + c_2*\text{CCF}$

- $s*\text{SITE} = d*\text{SITE}_1 + e*\text{SITE}_2 + f*\text{SITE}_3$

- $d*\text{SITE}_1 = d_1(\text{ELEV}-d_2)^2 + d_3*\text{SL}^2 + d_4*\text{SL}^2*\sin(\text{AZ}) + d_5*\text{SL}^2*\cos(\text{AZ})$

- $e*\text{SITE}_2 = e_1*\text{HF} + e_2*\text{HH}$

- $f*\text{SITE}_3 = f_1*\text{DP} + f_2*\text{M} + f_3*\text{P} + \sigma(f_4_i*\text{S}_i) + \sigma(f_5_i*\text{V}_i) + \sigma(f_6_i*\text{GD}_i)$
Competition

- Plants modify their environment as they grow, reducing the resources available for other plants.
- The primary mechanism of competition is spatial interaction.
- Plant death due to competition is a delayed reaction to the growth reduction following resource depletion.
- Plants adjust to environmental change, responding to competition and altering the nature of the competition.
- There are species differences in the competition process.
Competition indices include the competitive influence zone (CIZ), area potentially available (APA), horizontal or vertical size–distance (SDh & SDv), sky view (SV) and light interception (LI) approaches.
Process-based models

- Light interception
  \[ LI_N = LI_0 \times \exp(1 - (k \times LAI/N)) \]
- Photosynthesis = \( f(\text{carboxylation, ribulose}) = f(CO2, \text{leaf nutrition, leaf temperature}) \)
- Stomatal conductance
  \[ g_s = g_{\text{max}} \times f_1(\text{APAR}) \times f_2(T) \times f_3(\text{VPD}) \times f_4(C_i) \times f_5(\theta) \]
- Respiration
- Carbon allocation
- Soil water and nutrients
A carbon balance model CROBAS

\[ G = \sum_i G_i = Y^{-1}(P-R) \]

\[ P = P_0(1-e^{-ki})/N \]

\[ R_m = r_1(W_f + W_r) + r_2(W_s + W_b + W_t) \]

Source: Mäkelä (1997)
Applications: even-aged stands

- Characteristics of even-aged stands
- Predicting growth and yield for even-aged stands
- Assessing forest resources for even-aged stands
- Assessing forested land for even-aged stands
Even-aged management

Source: Cao et al. (2006), Fig. 1
Thinning from below was more profitable with interest rates less than 2%, whereas thinning from above was superior with interest rates of 3-5%.
Even-aged forest stand

- Regeneration
  - Natural vs. artificial regenerated
- Plantations
  - Sawlog or pulpwood
  - Energy wood
- Tree species
  - Conifer vs. broadleaf
  - Ever-green vs. deciduous
- Stand structure
  - Diameter distribution
  - Spatial distribution
Predicting even-aged stand growth

- Empirical vs. process-based
- Whole stand vs. individual-tree
- Site quality evaluation
- Competition
- Tree height growth
- Diameter growth
- Basal area growth
- Mortality and self-thinning
- Ingrowth
- Thinning response
Pine-oak forests

- Forests dominated by *Pinus tabuliformis*, *Pinus armandii* and *Quercus aliena* occupy the major area of Qinling Mountains,

- Accurate prediction of the growth and yield for these species has been a problem for many years, due to the diversity in composition and structure of pine-oak forests.

- Generally, forest growth modeling needs a large number of continuous observed data, which is unavailable in Qinling.
A three-stage modeling approach

The purpose is to develop and evaluate diameter increment models for pine-oak forests in Qinling Mountains with different approaches:

- Based on temporary plots,
- Combining the analysis of increment cores,
- And the exploration of diameter structure dynamics.
Diameter increment model

- Age-independent
  - Uneven-aged forest
  - Mixed forest

- Distance-independent
  - Practicability
  - Simplicity
  - Accuracy
Diameter increment model

Model form:

\[ \Delta d_i = f \left( D_i, V_i, C_i, S_i \right) \]  

where \( \Delta d_i \) is the 5-year diameter increment of tree \( i \); \( D_i, V_i, C_i, S_i \) represents tree size factor, historical vigor factor, competition factor and site productivity factor, respectively.

- Tree size: DBH, DBH\(^2\), ln(DBH), 1/DBH, etc.
- Historical vigor: Crown ratio, canopy closure, LAI, etc.
- Competition: BA, BAL, CCF, etc.
- Site productivity: Site index, Forest type, Elevation, etc.
Stand diameter distribution

3-parameter Weibull distribution:

\[ f(x) = \left( \frac{c}{b} \right) \left( \frac{x-a}{b} \right)^{c-1} \exp \left[ -\left( \frac{x-a}{b} \right)^c \right]; \quad x \geq a, b > 0, c > 0 \]

where \( x \) is tree diameter; \( a \) is the location parameter; \( b \) is the scale parameter; \( c \) is the shape parameter.
Optimizing diameter distribution

Objective function:

\[
\min Z = \frac{\sum (D_i - \hat{D}_i)^2}{\hat{\sigma}_D^2} + \frac{\sum (B_i - \hat{B}_i)^2}{\hat{\sigma}_B^2} + \frac{\sum (N_i - \hat{N}_i)^2}{\hat{\sigma}_N^2}
\]  \hspace{1cm} (3)

where \( D \) is diameter, \( B \) is basal area, \( N \) is density, \( \hat{\sigma} \) is estimated value of the standard deviation.
Simulated data => computational experiments

- Increment cores:
  - Collection
  - Pre-Processing
  - Scanning
  - Analysis
  - Cross-Dating

- Temporary plots inventory data

- Weibull distribution:
  - Location parameter
  - Scale parameter
  - Shape parameter

- Tree species
- Tree size
- Historical vigor
- Competition
- Site productivity

- Simulating individual tree data

- Diameter increment model
Even-aged stand management

- Alternative tree species
- Planting density
- Silvicultural treatments
- Thinning and rotation
- Timber production
- Biodiversity
- Carbon sequestration
- Landscape
- Risk management
Characteristics of inventory data

Date types and problems for modeling:

1. Temporary plots (some with increment cores)
   Lacking tree-level information

2. Permanent plots
   Most permanent plots only contain one-time observations

3. Stem analysis
   Accurate but few

Different type of measurement error comparing with plots data
Using Bayesian Calibration

1. Data requirements: small amount of data is acceptable, bridging the gap between models and data.

2. Previous investigation data is good prior information for next experiment, considering that forest inventory is continuous.

3. Bayesian calibration provides a likelihood framework for different types of measurement error.
Each type of model has its advantages and disadvantages on certain situation, especially for thinning modifier. There are different theoretical hypothesis for thinning effects.

Bayesian model averaging is developed as an approach to combine inferences and forecasts from multiple competing modify models. BMA can clearly show the information update process and can also combine predictive distributions from different sources.
A Bayesian framework

1. Uncertainty quantification (UQ)
   Predictive uncertainty can be quantified for each model before any parameter calibration has been carried out (prior UQ), and after calibration (posterior UQ).

2. Bayesian calibration (BC)
   To estimate the posterior distributions, we use a Markov Chain Monte Carlo (MCMC) algorithm.

3. Bayesian model comparison (BMC)
   BMC evaluates models not at one single parameter vector value but takes into account parameter uncertainty.

4. Bayesian model averaging (BMA)
   Bayesian model averaging uses the different model probabilities, derived in preceding BMC, to calculate a weighted probability distribution for model outputs.
Tools for Bayesian Calibration

To estimate the posterior distributions, we use a Markov Chain Monte Carlo (MCMC) algorithm.

1. WinBUGS
   (Bayesian inference Using Gibbs Sampling)

2. R2WinBUGS
   Package linking R with WinBUGS

3. R programming
   (For example, Gibbs within Metropolis)

More R packages see CRAN Task View: Bayesian inference
(https://cran.r-project.org/web/views/Bayesian.html)
To some extent, the parameters were failed to convergence. Maybe that is because the model form is not good enough to describe the data pattern.
The default is a bandwidth computed from the variance of $x$, specifically the ‘oversmoothed bandwidth selector’ of Wand and Jones (1995, page 61)
Model linkages

- Biomass equations
- Taper curve equations
- Timber grading module
- Tracheid properties
- Wind throw
- Forest fire
- Forest biotic damages
Applications: uneven-aged stands

- Characteristics of uneven-aged stands
- Predicting growth and yield for uneven-aged stands
- Assessing forest resources for uneven-aged stands
- Assessing forested land for even-aged stands
Uneven-aged management

- Uneven-aged

- Mixed forest
  - Boreal forest (conifer dominant)
  - Temperate forest (conifer & broadleaf)
  - Tropical forest (broadleaf dominant)

- Diameter distribution: reverse "J" sharp

- Regeneration: natural vs. artificial

- Succession: shade tolerant vs. light-demanding
Uneven-aged management

Source: Pukkala et al. (2009), Fig. 8
Predicting uneven-aged stand growth

- Whole stand models: few
- Transition matrix models: e.g., Buongiorno & Michie (1980)
- Tree list models: e.g., Prognosis (Stage, 1972)
- Individual-tree growth models
  - Spatial models (e.g., Ek and Monserud, 1974)
  - Non-spatial models
    - Stochastic, e.g. JABOWA (Botkin et al., 1972)
    - Pukkala et al. (2009)
QUASSI

- A hybrid modelling approach

- Cooperation
  - UH
  - SCU
  - LSU

QUASSI (Qinling Uneven-Aged Stand Simulator)

1. Forest inventory data at time $i$
2. Calculating supplementary information
   - ST, SI
     - OPT, BPT, HPT
   - Competition
     - BAL
   - Stand structure
     - Weibull distribution
3. Predicting stand dynamics
   - Diameter increment
     - $dbh$, BA, BAL, ST, m.a.s.l.
   - Height model
     - $dbh$, ST, m.a.s.l.
   - Mortality
     - BA, BAL, ST
   - Ingrowth
     - BA, ST
4. Silvicultural treatments
5. Updated stand database at time $i+5$
Modeling uncertainty and risks

- Stochastic vs. deterministic
- Stochastic variables
- Stochastic parameters
- Stochastic simulation
- Stochastic optimization
Risk and uncertainty in forestry

- Economic risk and uncertainty
  - Timber prices and Interest rate
- Management risk and uncertainty
  - Human interventions
  - Natural disasters
  - Forest health

- Ecological stochastic processes
  - Competition and succession
  - Climate change
Competition and succession

- Methods for modeling mortality
- Methods for modeling recruitment
Mortality
Methods for modeling mortality

- Reineke (1933), Yoda et al. (1963), Curtis (1982)
  - Stand density index, self-thinning line, relative density

  - Negative carbon balance

- Haenauer et al. (2001)
  - Neural networks, LOGIT model

- Flewelling and Monsrud (2002)
  - Logit Model for Proportions, Least squares, Walker-Duncan algorithm, Weighted Least Squares, Maximum Likelihood
Recruitment
Methods for modeling recruitment

  - Assume observed reflect long-term ave.
  - Negatively correlated with stand density or BA

- Shifley et al. (1993)
  - Recruitment = f(CCF, diameter threshold)

- Fergusen et al. (1986), Vanclay (1992)
  - 1) Logistic function, 2) conditional function

  - Neural networks
  - Juvenile height growth
Mortality is not a Markov process, however, survival is.

Survival = 1 – Mortality

\[ P_s = 1 - P_m \]

1-year: \[ P_{s1} = 1 - P_{m1} \]

n-year: \[ P_{sn} = (P_{s1})^n = (1 - P_{m1})^n \]

(The Markov property)

1-year: \[ P_{m1} = 1 - (P_{sn})^{1/n} = 1 - (1 - P_{mn})^{1/n} \]

\[ P_m = \frac{\exp(b'X)}{1 + \exp(b'X)} \]

(Flewelling and Monserud 2002)
Neural networks

Advantages:

- Neural networks are a viable alternative to the conventional LOGIT approach for estimating tree mortality.
- Artificial neural networks to be more effective at predicting regeneration establishment than regression equations.
- Artificial neural networks were effective predictors when regeneration data were not available.

Bayesian calibration

- Many parameters
- Many output variables
- Relatively few measured data available

Quantifying uncertainty rather than maximizing fit

\[
p(\theta|D) = c p(D|\theta)p(\theta)
\]

\[
p(D|\theta) = p(E = D - M(\theta))
\]

\[
\theta' = \theta_t + \epsilon
\]

\[
\beta = \frac{p(\theta'|D)}{p(\theta_t|D)} = \frac{p(D|\theta')p(\theta')}{p(D|\theta_t)p(\theta_t)}
\]

(Van Oijen et al. 2005)
Markov Chain Monte Carlo

- Bayesian calibration cannot be performed analytically.
- The posterior parameter distribution must be approximated in the form of a representative sample of parameter values.

\[ \text{MCMC does not require advance knowledge of the shape of the posterior distribution} \]

- MCMC: Metropolis-Hastings sampling

(Van Oijen et al. 2005)
Modeling forest regeneration

- Uncertainty of forest regeneration
- Juvenile tree height growth model
- Forest recruitment simulation
Simulation and uncertainty analysis

■ Methods:

Prior distribution of parameter can be guessed which come from others research. The data which come from sample plots investigation will be used to update the prior information by the computation of the posterior distribution.

\[ P(A_i|B) = \frac{P(A_i)P(B|A_i)}{\sum_{j=1}^{k} P(A_j)P(B|A_j)} \quad i = 1, 2, \ldots, k \]

- \( P(A_i) \): Prior distribution
- \( P(B|A_i) \): likelihood function
- \( P(A_i|B) \): Posterior distribution
Environment and competition factors

- Environment factors
  - elevation,
  - slope,
  - aspect,
  - the interaction of slope and aspect

- Competition factors
  - stand density,
  - stand basal area,
  - stand average diameter (DBH),
  - stand canopy closure
Height growth of juvenile trees

- Explanatory variables:
  - Tree size: height
  - Competition factors: stand density, stand basal area, stand average diameter (DBH), larger trees basal area (BAL)
  - Site factors: site index, slope, the interaction of slope and aspect

- The height growth model of juvenile trees:

\[
HTG = \beta_0 + \beta_1 \times SL \times \cos(ASP) - \beta_2 \times SL \times \sin(ASP) - \beta_3 \times SL \\
+ \beta_4 \times \ln(HT) + \beta_5 \times CCF + \beta_6 \times \left(\frac{BAL}{100}\right) + \epsilon_2
\]
Recruitment models

- Recruitment models predict tree reaching a specified threshold size, usually based on height (breast height) or diameter (5, 7, 9, or 10 cm), a threshold diameter of 5 cm was selected in this study.

- Forest stand recruitment is a complicated stochastic process influenced by several stand characteristics, climatic, geographical factors at a range of spatial and temporal scales.
Recruitment models, con't

- Likelihood function
  - Estimating number of recruitment trees by 5-year juvenile tree height model and 5-year diameter increment model.

- Posterior distribution
  - To update the prior information by the likelihood function, and to obtain Posterior distribution of recruitment model parameter.
ANN vs. Bayesian, 5-yr height growth

**Pinus armandii**

- ANN: $y = 0.4816x + 0.2424$, $R^2 = 0.4816$
- Bayesian method: $y = 0.7017x + 0.128$, $R^2 = 0.7305$

**Pinus tabulaeformis**

- ANN: $y = 0.2967x + 0.4486$, $R^2 = 0.2967$
- Bayesian method: $y = 0.787x + 0.1302$, $R^2 = 0.7753$

- ANN: $y = 0.4809x + 0.5366$, $R^2 = 0.4809$
- Bayesian method: $y = 0.9269x + 0.0573$, $R^2 = 0.8391$
Model linkages with Simile

- http://www.simulistics.com/

- Stage 1: Modelling the growth of a single tree
- Stage 2: Extending the model to represent a population of trees
- Stage 3: Calculating aggregate information
- Stage 4: Visualising the trees in space
Interface of Simile
Simile step-by-step

- Step 1 Add a compartment to the desktop, and rename it size.
- Step 2 Draw a flow into the size compartment, and rename it growth.
- Step 3 Add a variable to the desktop, above the growth flow, and rename it gr.
- Step 4 Draw an influence arrow from both the size compartment and the gr to the growth flow.

The variable gr represents the maximum rate of growth of the tree.
Simile step-by-step, con’t

- Step 5 Enter the following expression for the growth flow:
  \[ gr * (1 - \text{size} / 25) \]
- Step 6 Set the initial value for the size compartment to 3.
- Step 7 Set the value for the variable gr to 0.2.
- Step 8 Prepare the model for running.
- Step 9 Set up a plotter display helper for the size compartment
- Step 10 Run the model
Size growth of single tree
Extending the model to a stand

- Step 1  Draw a submodel box to completely enclose your model diagram.
- Step 2  Rename the submodel tree.
- Step 3  Open up the submodel Properties dialogue window.
- Step 4  Click the radio button labelled “Using population symbols”.
- Step 5  Choose a nice background colour for the submodel.
- Step 6  Close the submodel properties dialogue window.
- Step 7  Change the expression for gr from its current value (0.2) to rand_const(0.1,0.3).
Extending the model, con’t

- Step 8 Add a creation symbol to the model diagram, and give it a value of 5.
- Step 9 Add an immigration symbol to the model diagram, and give it a value of 2.
- Step 10 Add a loss symbol to the model diagram.
- Step 11 Rename the loss symbol death.
- Step 12 Draw an influence arrow from the compartment size to this symbol.
Rebuild the model

- Step 13  Enter the expression size>17 for the symbol labelled death.
- Step 14  Rebuild the model, and run it again.
Calculating aggregate information

- Step 1 Add a variable outside the tree submodel, and rename it total.
- Step 2 Draw an influence arrow from the compartment size inside the tree submodel to the variable total.
Rerun the model, con’t

- Step 3 Enter the expression `sum({size})` for the variable total.
- Step 4 Re-build the model.
- Step 5 Call up a plotter display for the variable total.
- Step 6 Run the model again.
Visualising the trees

- Step 1 Add two variables inside the tree submodel, and rename them x and y.
- Step 2 Enter the following expression for the variable x:
  \text{rand\_const}(0,50)
- Step 3 Enter the following expression for the variable y:
  \text{rand\_const}(0,100).
- Step 4 Re-build the model.
- Step 5 Call up the lollipop display. When it prompts you for the three variables required to set up the display, click on the variable x, the variable y, and the variable size respectively, in that order.
- Step 6 Run the model again.
Visualization, con’t
Part I, the final report

- 1. Field form
- 2. Data processing
- 3. Site conditions
- 4. Diameter distribution
- 5. Stand growth
Part II: Forest Planning
DSSs and Applications
Philosophy of modeling

What lies still is easy to grasp; What lies far off is easy to anticipate; What is brittle is easy to shatter; What is small is easy to disperse.

Laozi (Taoteching, chapter 64)
Interdisciplinary study

- Forest sciences
  - Silviculture
  - Forest ecology
  - Forest biometrics

- Applied mathematics
  - Operations research
  - Nonlinear programming
  - Artificial intelligence
Contents

- Model linkages and applications
- Forest management objectives
- Optimization modeling techniques
- Forest management planning
- Simulation optimization systems
Forest management objectives

- Timber production
- Forest biodiversity
- Carbon sinks
- Forest bioenergy
- Climate mitigation
- Multi-functional services
Model linkages and applications

- Growth model + dynamic carbon model
- Growth model + wood quality module
- Growth model + climate-sensitive module
- Growth model + energy wood module
- Growth model + biodiversity index
Optimization modeling techniques

- Linear programming
- Goal programming
- Integer programming
- Dynamic programming
- Nonlinear programming
- Artificial intelligence
Forest management planning

- Even-aged management
- Uneven-aged management
- Regeneration methods
- Silvicultural operations
- Logging methods
Decision support systems

- Inventory DSS
- Simulation DSS
- Two-level DSS
- Theoretical optimization DSS
- Simulation optimization DSS
Model linkages

All models are false,

but some models are useful.
Model linkages

- Forest growth and yield models
- Wood quality models
- Dynamic carbon models
- Nutrient cycling models
- Water balance models
- Logging models
Forest growth models

- Tree level vs. size class level
- Stand level vs. forest level
- Even-aged vs. uneven-aged
- Empirical vs. mechanistic
- Deterministic vs. stochastic
Simulation models in OPTIFOR 1.0

- **MOTTI** (Hynynen et al. 2002, 2005)
- **Crobas/PipeQual** (Mäkelä 1997, Mäkelä and Mäkinen 2003)
- **Yasso** (Liski et al. 2005)
- **Wood quality** (Mäkinen et al. 2007)
Applications of OPTIFOR 1.0

- 优化间伐 Optimal thinning (Cao et al., 2006)
- 木材质量 OPTIFOR Wood (Cao et al., 2008)
- 森林碳汇 OPTIFOR Carbon (Cao et al., 2010)
- 气候变化 OPTIFOR Climate (Nikinmaa et al. 2011)
- 生物能源 OPTIFOR Energy (Cao et al., 2015)
Optimal stocking control

(a) 1% interest rate
(b) 3% interest rate
(c) 5% interest rate
OPTIFOR Carbon (Cao et al. 2010)

(a) CII

(b) CIII

Biomass production, Mg ha$^{-1}$ yr$^{-1}$
**Figure 4.** Crown ratio of dominant trees and branch thickness (cm) at the second thinning with Policy 0 and Policy II for five Scots pine stands (initial density 3000 trees ha\(^{-1}\)) by \(H_{100}\) (dominant height at 100 yrs, m), energy wood price 15 € m\(^{-3}\), interest rate 3%.
Simulation models in OPTIFOR 2.0

- Yasso 07 (Tuomi et al. 2009)
- Wood quality (Mäkinen and Hynynen 2012)
- PreLES (Mäkelä et al. 2008, Peltoniemi et al. 2012)
- ROMUL (Chertov et al. 2001)
- QUASSI (Liao et al. 2017)
Figure 2. Juvenile wood, heartwood and sapwood in Norway spruce stem (A). Ageing of the xylem along the three major axes of the stem (B): radially from the pith to the bark (1), vertically from the stem base to the stem apex in a given annual ring from the pith (2), and concentrically around the given annual ring from the bark (3) (redrawn and modified from Duff and Nolan 1953, Schweingruber et al. 2006).
CROBAS: a process-based model

Figure 1. Schematic representation of tree structure as applied in the model.
Yasso 07: a simple dynamic carbon model
QUASSI (Qinling Uneven-Aged Stand Simulator)

Collaboration
- LSU
- SCU
- UH

Forest inventory data at time i

Calculating supplementary information
- ST, SI
  - OPT, BPT, HPT
- Competition
  - BAL
- Stand structure
  - Weibull distribution

Predicting stand dynamics
- Diameter increment
  - dbh, BA, BAL, ST, m.a.s.l.
- Height model
  - dbh, ST, m.a.s.l.
- Mortality
  - BA, BAL, ST
- Ingrowth
  - BA, ST

Silvicultural treatments

Updated stand database at time i+5
Fig. 6 Calibration of PRELES on Qinglin site. The predictive uncertainty is higher than any other site due to the limited amount of observations (43 days).
Optimization algorithms in OPTIFOR 2.0

- Differential evolution (Storn and Price 1997)
- Particle swarm (Kennedy and Eberhart 1995)
- Evolution strategy (Bayer and Schwefel 2002)
- The method of Nelder and Mead (Nelder and Mead 1965)
Forest management objectives

Forest management problems

Forest management decisions
Forest management objectives

**Fig. 2.** Scenario analysis with forest stand models. Starting with an initial state of an ecosystem, models display the long-term consequences of the different management options A, B, C and D and the consideration of different objective states.
Definition

- What does sustainable forest management mean anyway?
- Sustainable timber yield?
- Sustainable forest biodiversity?
- Or sustainable forest ecosystem?
Group work

- Sustainable,
- close-to-nature,
- or adaptive forest management:

- does it really matter?
Forest ecosystem functions

- IUFRO Division 8.01
- 8.01.00 – Forest ecosystem functions
  - 8.01.01 – Old growth forests and forest reserves
  - 8.01.02 – Landscape ecology
  - 8.01.03 – Forest soils and nutrient cycles
  - 8.01.04 – Water supply and quality
  - 8.01.05 – Riparian and coastal ecosystems
  - 8.01.06 – Boreal forest ecosystems
  - 8.01.07 – Hydrologic processes and watershed management
Forest biodiversity

- 8.02.01 – Key factors and ecological functions for forest biodiversity
- 8.02.02 – Forest biodiversity and resilience
- 8.02.03 – Humus and soil biodiversity
- 8.02.04 – Ecology of alien invasives
- 8.02.05 – Wildlife conservation and management
- 8.02.06 – Aquatic biodiversity in forests
- 8.02.07 – Bioenergy productions systems and forest biodiversity
- 8.02.08 – African wildlife conservation and management (AWCM)
Ecosystem services

- Humankind benefits from a multitude of resources and processes that are supplied by ecosystems.
- While scientists and environmentalists have discussed ecosystem services for decades, these services were popularized and their definitions formalized by the United Nations 2005 Millennium Ecosystem Assessment (MA).
- This grouped ecosystem services into four broad categories:
  - provisioning
  - regulating
  - supporting
  - cultural
Forest ecosystem services

- Timber
- Fuel wood
- Hunting
- Preventing erosion
- Amenities
- Biodiversity
- Carbon sequestration
- Mushrooms
- etc.

Multiple roles
## Classification of selected forest outcomes

<table>
<thead>
<tr>
<th>Location</th>
<th>Market prices</th>
<th>Administered nominal prices</th>
<th>Not priced</th>
</tr>
</thead>
<tbody>
<tr>
<td>off forest</td>
<td>Timber</td>
<td>Fuelwood</td>
<td>Water</td>
</tr>
<tr>
<td>on forest</td>
<td>Hunting</td>
<td>Forage; developed</td>
<td>Dispersed recreation visual amenities;</td>
</tr>
<tr>
<td></td>
<td>and recreation</td>
<td>recreation</td>
<td>nongame wildlife; endangered species</td>
</tr>
</tbody>
</table>
The protagonist is a congenial poet-forester who lives in the woods of Northern Wisconsin. Some success in his writing allowed him to buy, about ten years ago, a cabin and 90 ha of woods in good productive condition.

The poet needs to walk the beautiful woods to keep his inspiration alive. But the muses do not always respond and he finds that sales from the woods come very handy to replenish a sometimes empty wallet. In fact, times have been somewhat harder than usual lately. He has firmly decided to get the most he can out of his woods.

But the arts must go on. The poet does not want to spend more than half of his time in the woods; the rest is for prose and sonnets.
About 40 ha of the land he owns are covered with red-pine plantations. The other 50 ha contain mixed northern hardwoods.

Having kept a very good record of his time, he figures that since he bought these woods he has spent approximately 800 days managing the red pine and 1500 days on the hardwoods.

The total revenue from his forest during the same period was $36,000 from the red pine land and $60,000 from the northern hardwoods.
Management objective

- The poet’s objective is to maximize his revenues from the property. But this has a meaning only if the revenues are finite; thus he must mean revenues per unit of time, say per year (meaning an average year, like anyone of the past ten enjoyable years that the poet has spent on his property). Formally, we begin to write the objective as:

  - Maximize $Z = $ of revenues per year.

- $X_1 = $ the number of hectares of red pine to manage
- $X_2 = $ the number of hectares of northern hardwoods to manage
The objective function expresses the relationship between Z, the revenues generated by the woods, and the decision variables X1 and X2. Since the poet has earned $36,000 on 40 ha of red pine and $60,000 on 50 ha of northern hardwoods during the past 10 years, the average earnings have been $90 per ha per year (90 $/ha/y) for red pine, and 120 $/ha/y for northern hardwoods. We can now write his objective function as:

$$\max Z = 90 \frac{X_1}{(\text{ha})} + 120 \frac{X_2}{(\text{ha})}$$
Time constraint

- We note that the time he has spent managing red pine during the past 10 years (800 days for 40 ha of land) averages to 2 days per hectare per year (2 d/ha/y). Similarly, he has spent 3 d/ha/y on northern hardwoods (1500 days on 50 ha).
- In terms of the decision variables $X_1$ and $X_2$, the total time spent by the poet-forester to manage his woods is:

$$2 \frac{X_1}{(d/ha/y)(ha)} \quad + \quad 3 \frac{X_2}{(d/ha/y)(ha)} \quad \leq \quad 180 \quad (d/y)$$
Land constraints

Two constraints are very simple. The area managed in each timber type cannot exceed the area available, that is:

- $X_1 \leq 40$ ha of red pine
- $X_2 \leq 50$ ha of northern hardwoods
The optimization problem

we obtain the complete formulation of the poet-forester problem as: Find the variables $X_1$ and $X_2$, which measure the number of hectares of red-pine and of northern hardwoods to manage, such that:

$$\begin{align*}
\text{max } Z &= 90X_1 + 120X_2 \\
\text{subject to } &:\\
X_1 &\leq 40 \\
X_2 &\leq 50 \\
2X_1 + 3X_2 &\leq 180 \\
X_1, X_2 &\geq 0
\end{align*}$$
Graphic solution

\[ Z = 7600 = 90X_1 + 120X_2 \]

\[ Z = 3600 = 90X_1 + 120X_2 \]

\[ Z = 1800 = 90X_1 + 120X_2 \]
Conventional objective: timber production

- maximization of bare land value

\[
\max_{\{h_u, t_u, u=1, \ldots, k|Z_0\}} \ V = \frac{\sum_{u=1}^{k} \left[ \sum_{i=1}^{n} \sum_{j=1}^{2} p_{ij} g_{ij}(Z_{t_u}, h_u) - c_u(Z_{t_u}, h_u) \right] (1+r)^{-t_u} - c_0(Z_0)}{1 - (1+r)^{-t_k}}
\]
Conventional forest management and DP

- In conventional forest management, the dominant model type for growth and yield prediction was whole-stand models.
- This model type is sufficient to achieve the accuracy of volume growth prediction for which timber yield was the main concern of conventional forest management.
- Volume or basal area development is usually simply formulated as a function of time, site type, and stand density in such models.
- In other words, the number of state variables of whole-stand models is limited.
Dynamic programming

- Dynamic programming (DP) was an optimization method widely applied with whole-stand models because of its ability to find a global optimum with a few number of variables.

- Hann and Brodie (1980) reported that DP required more computing hardware capacity or computing time when the number of state variables increased.

- Therefore, earlier studies of stand management optimization concentrated on reducing the dimensionality of optimization problems, either the number of state variables predicted from stand growth models or decision variables optimized by silvicultural treatments (Brodie and Haight 1985).
Intensive forest management and NLP

- Intensive forest management requires detailed information from stand growth predictions with higher resolution, for example, tree diameter distribution, timber assortments and timber grading at tree level.
- It is necessary to apply tree-level growth and yield models, such as tree-list or individual-tree models.
- In the formulation of tree-level growth and yield models, the increment of tree diameter or basal area growth can be modeled as a function of, e.g., diameter at breast height, basal area, site type, site index, basal area in larger trees, crown ratio, crown competition factor, etc.
- Tree-level growth and yield models clearly increase the number of state variables.
Intensive forest management, con’t

- Meanwhile, intensive forest management means more intensive silvicultural treatments, such as fertilization, pruning, pre-commercial thinning, and commercial thinning in terms of thinning intensity, thinning type, thinning frequency, and timing of thinning.

- This type of stand management problem is typically non-smooth or non-differentiable because of timber assortments and thinning interventions, and may contribute to more number of state and decision variables formulated in stand management optimization (Hyytiäinen et al. 2004).
Nonlinear programming (NLP) turns out to be an effective tool handling such complicated optimization problems. Roise (1986) and Valsta (1990) both reported that NLP algorithms were more efficient than those of DP. Most of conventional and intensive forest management studies assume that stand dynamics can be predicted based on deterministic empirical growth models. This model type heavily depends on empirical observed data. As a matter of fact, collecting long-term re-measured data for modeling impacts of thinning or climate effects on regeneration, in-growth, and mortality by various stand density and site conditions often is inefficient and difficult.
Sustainable forest management, gap models

- Applying more detailed succession or process models to explain uncertainties and biological reasons becomes a helpful alternative in contrast to empirical models.
- Gap models perhaps would be one of alternatives to explain succession based on ecological population theory.
- But most of gap models rely on expert opinion for predicting species succession patterns rather than observed data.
- Even calibrated with long-term observed data, individual tree dimensions and stand structure predicted by gap models were still unrealistic (Lindner et al., 1997, Monserud, 2003).
Sustainable forest management, algorithms

- The dimensionality of variables is a choice of a robust and accurate algorithm in stand management optimization in addition to the convexity of the objective function (Cao 2010).
- The HJ algorithm has been well demonstrated earlier with various empirical stand growth models, such as whole-stand models (e.g., Roise 1986), and individual-tree models (e.g., Haight and Monserud 1990).
- As well as heuristic algorithms, such as genetic algorithm (Lu and Eriksson 2000), tabu search (Wikström and Erikson 2000), and simulated annealing (Lockwood and Moore 1993).
Adaptive forest management, process models

Based on physiological theory, process or mechanistic growth models intend to include key growth processes and underlying causes of forest productivity, for example, photosynthesis and respiration, nitrogen cycles, water balance, and carbon balance, and climate effects.

Although the common purpose of process-based models is to explain scientific reasons rather than growth prediction,

efforts have also been made to build management-oriented hybrid models by linking process-based and empirical growth models.
it is possible to apply detailed process-based growth models for more complicated forest management situation where is a lack of observed data.

However, taking stochastic effects into consideration significantly increase the complexity of process-based growth models with a number of parameters,

for example, 48 parameters for 3-PG (Landsberg and Waring, 1997),

and 39 parameters for CROBAS (Mäkelä, 1997).
Adaptive forest management, AI algorithms

- With more detailed process-based models (thousands of state variables),
- and more complicated optimization problems (a number of decision variables)
- Differential evolution (Storn and Price 1997),
- Particle swarm optimization (Kennedy and Eberhart 1995),
- Evolution strategy (Bayer and Schwefel 2002)
- Direct and random search (Osyczka 1984)
- Hybrid algorithms
Faustmann equation (1849)

- Bare land value

\[
P V_{\text{timber}} = \frac{e^{-rt}[B(T) - c]}{1 - e^{-rt}}
\]

- Optimalitiy condition

\[
\frac{B'(T)}{B(T) - c} = \frac{r}{1 - e^{-rt}}
\]
Hartman (1976) model

- The discounted perpetual value of the externality

\[ PV_{ext} = \frac{Q(T)}{1 - e^{-rt}} \]

- The optimality condition

\[ \frac{Q'(T)}{Q(T)} = \frac{re^{-rt}}{1 - e^{-rt}} \]

- The joint production problem

\[ PV_{tot} = PV_{timber} + PV_{ext} \]
Clark (1976) modified model (thinning)

- **Common**
  \[
  \frac{dV}{dt} = g(t)f(V) \quad V(t_0) = V_0
  \]

- **Modified**
  \[
  \frac{dV}{dt} = g(t)f(V) - h(t) \quad t \geq t_0 \quad h(t) \geq 0 \quad V(t_0) = V_0
  \]

- **Objective function**
  \[
  PV = \int_{t_0}^{\infty} e^{-rt} [p - c(V)]h(t)dt
  \]

- **Optimality condition**
  \[
  r = \frac{\partial}{\partial V} [g(t)f(V)]
  \]
To sustain Eco-econ-soci values

- Multiple parties
- Multiple objectives
- Solving one problem often creates new problems

- Contract theory: Focus on one (or a few) problems
  - The agency
  - The principle

- Transaction cost theory
  - Min. the costs of planning, monitoring, motivating
  - Min. the cost of risk and uncertainty
Social criteria

■ Community stability
  ■ e.g., the national land management policy shift noted above changed harvest levels, which, in turn, affected the local and regional economies as well as the social dimensions of impacted communities.

■ Community resilience
  ■ Horned and Haynes (1999) from Shannon and Weaver (1949)
  ■ \( D = \sum_{i=1}^{n} (E_i \log(E_i)) \)
  ■ \( E_i \) = the proportion of total employment in the area located in the ith industry

■ Political considerations
Ecological criteria

- Ecological and environmental goals
  - Maintaining and enhancing forest productivity
  - Conservation of biological diversity
  - Protecting and enhancing environmental conditions

- Biological diversity
  - The landscape level
  - The species level, e.g. Shannon and Weaver (1949)
    \[ D = \sum_{i=1}^{n} (E_i \log(E_i)) \]
    \[ E_i = \text{the proportion of total individuals in the area of the } \text{ith species} \]

- Environmental protection (short/long term)
  - Vegetation, soil, and watercourses
Economic criteria

- Economic equity
  - e.g., while the nation as a whole may have benefited from improved forestland conditions, sawmill employees and loggers, and the rural communities in which they lived, bore costs of the logging-ban policy

- Regional economics
  - Defining the region
  - Regional goals and criteria

- Economic efficiency
  - "The greatest good for the greatest number in the long run." -- Gifford Pinchot
Multi-criteria decision methods

- Multi-Objective Programming (MOP)
- Goal programming (GP)
- Compromise Programming (CP)
- Multi-attribute utility theory (MAUT)
- Fuzzy Mult-Criteria Programming (FMCP)
- Analytic hierarchy process (AHP)
- Other Discrete Methods (ODM)
- Data envelopment analysis (DEA)
- Group Decision Making Techniques (GDM)
MCDM papers by foestry topics


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<th>C</th>
<th>D</th>
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255 28 45 10 43 12 56 29 33 46
Analytic hierarchy process (AHP)

- Multifunctional forest management

- Define Criteria
  - NPV, forest health index, PM2.5

- Alternatives
  - timber, carbon balance, biodiversity, recreation
Analytic Hierarchy Process (AHP)

Problem to be Solved

CRITERION 1
Weight

CRITERION 2
Weight

CRITERION 3
Weight

Alternatives

1 2 3
AHP

- Developed by Thomas L. Saaty

- Hierarchy Tree (Decomposition)

- Determining Weights using Pairwise Comparisons

- Synthesizing

- Consistency of Evaluations
Weights

- Scale [1, ..., 9]:
  - 1 Equal Importance
  - 3 Weak Importance of one over Another
  - 5 Essential or Strong Importance
  - 7 Very Strong or Demonstrated Importance
  - 9 Absolute Importance
Multifunctional forest

NPV
- timber
- carbon
- biodiv.
- recrea.

FH index
- timber
- carbon
- biodiv.
- recrea.

PM2.5
- timber
- carbon
- biodiv.
- recrea.

Overall Goal

Criteria

Alternatives
Pairwise comparison

- FH index is slightly more important (2) than NPV
- NPV is more important (3) than PM2.5
- FH index is clearly more important (4) than PM2.5

<table>
<thead>
<tr>
<th></th>
<th>NPV</th>
<th>FH index</th>
<th>PM2.5</th>
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<td>3/1</td>
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<tr>
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<td>2/1</td>
<td>1/1</td>
<td>4/1</td>
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<tr>
<td>PM2.5</td>
<td>1/3</td>
<td>1/4</td>
<td>1/1</td>
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Eigenvector of Pairwise Comparison Matrix

NPV \begin{bmatrix} 0.3196 \end{bmatrix} \quad \text{Next important}

FH index \begin{bmatrix} 0.5584 \end{bmatrix} \quad \text{Most important}

PM2.5 \begin{bmatrix} 0.1220 \end{bmatrix} \quad \text{Least important}
How about the rank order of Alternatives?
# Pairwise comparison

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<th>Carbon</th>
<th>Biod.</th>
<th>Recr.</th>
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<td>1/6</td>
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<td>1/4</td>
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<td>1/1</td>
<td>1/5</td>
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<tr>
<td>Recreation</td>
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<td>4/1</td>
<td>5/1</td>
<td>1/1</td>
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<td>1/1</td>
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<td>1/1</td>
<td>3/1</td>
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<td>Biod.</td>
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<td>1/2</td>
<td>4/1</td>
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<td>NPV</td>
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<tr>
<td>Timber</td>
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# PM2.5

<table>
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<td>Recr.</td>
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Normalizing

\[
\begin{align*}
0,4752 & \quad 1,0000
\end{align*}
\]
Final hierarchy

Multifunctional forest 1.0

NPV 0.3196
- Tim. 0.1160
- Car. 0.2470
- Bio. 0.0600
- Rec. 0.5770

FH index 0.5584
- Tim. 0.3790
- Car. 0.2900
- Bio. 0.0740
- Rec. 0.2570

PM2.5 0.1220
- Tim. 0.3010
- Car. 0.2390
- Bio. 0.2120
- Rec. 0.2480

What to choose?
Final score

1. Recreation 0,3582
2. Timber 0,2854
3. Carbon 0,2700
4. Biodiversity 0,0864
PuMe II demo

- Student version
- Process-based
- Wood quality
- Soil carbon
- Water use
PuMe II simulator is developed for forestry studies at university, polytechnic and vocational school level in Finland. PuMe II contains forest growth simulator (pine and spruce), based on PipeQual model developed by University of Helsinki, and information packages providing information on natural and commercial forests in Finland as well as on how natural and scenic values are taken into account in commercial forestry. You can also watch videos highlighting the different development stages of forests.

Information on Finnish forests:
- Typical tree species
- Forest site types
- Commercial forests
- Natural forests

Additional information and links:
- PuMe project’s web page
- PipeQual growth model
- Links on Finnish forestry
- Information about forest mensuration equipment

1. Initial situation
   - Tree species: Pine (Pinus sylvestris)
   - Site type: Mytilus site type
   - Regeneration method: Planting
   - Planted seedlings/ha: 2000
   - Natural seedlings/ha: 2000

2. Forest management
   - Pre-commercial thinning: Remaining trees/ha: 2200
   - Pruning
     - Age (yrs): 25
     - Height (m): 4
   - Thinnings: No thinnings

3. Other factors affecting the growth
   - Fertilization: Age (yrs): 50
   - Needle damage: Age (yrs): 50, Wideness(%): 50

4. How long the growth will be simulated?
   - Rotation length (yrs): 100
   - Final cutting, remaining trees/ha: 0

5. Simulate the forest growth
   - Simulate the growth
Silvicultural recommendations

**Traditional stand characteristics**

- **Basal area**
  - Stand basal area (m²/ha)
    - Show tree classes: □ Dominant □ Intermediate □ Suppressed
    - Basal area (m²/ha)
    - Stand basal area (m²/ha) is the cross-sectional area of a tree with bark at breast height. The sum of tree basal areas in one hectare.
    - Stand basal area is measured with a basal area gauge. On a basal area gauge test plot, a full round is done and all trees filling the opening of the device are included and counted as corresponding to (usually) 1 m²/ha. Within a stand, several basal area gauge measurements are done, from which the average is calculated.

- **Increment of stand basal area (m²/ha/year)**
  - Annual growth (m²/year)
  - Increment of stand basal area (m²/ha/year)
Biomass production
Wood quality

Zone of living branches
Radial growth in branches ceases on average in 20 years from the birth of the branch. In older trees, radial growth continues longer than in young trees. The branches are estimated to live another 7 years after the growth has finished.

Zone of dead branches
After the branches have died, they will be healed over, which takes about 40 years. Thick branches heal over slowly but on the other hand, branch remains in thick trees occlude quickly.

Branchless zone
Branchless zone is that part of the stem in which branches have already healed over.
Carbon balance and water use

**Carbon Balance**

Carbon production in a forest stand is the amount of carbon assimilated in photosynthesis annually. The part of the assimilation, which is not consumed by growth and maintenance respiration, is used for annual growth. Carbon makes up a constant 48% of wood biomass.

Forest growth fixes atmospheric carbon which is stored in the forest ecosystem. Deforestation in turn increases the amount of carbon in the atmosphere. Forestry thus plays an important role in combatting the global change.

**Photosynthesis**

1. Diffusion of carbon dioxide from the air into the chloroplasts
2. Binding of light energy in a photo-chemical process
3. Conversion of carbon dioxide into carbohydrates and other vital compounds

**Light** is the single most important environmental factor affecting the rate of photosynthesis. Other factors are temperature, atmospheric carbon dioxide content, and water availability. The rate of photosynthesis varies.
Optimization modeling techniques

- Linear vs. Nonlinear
- Convex vs. Non-convex
- Strictly convex vs. Pseudo-convex vs. Quasi-convex
- One-dimensional vs. Multidimensional
- Deterministic vs. Stochastic
- Constrained vs. Unconstrained
- Single-objective vs. Multi-objective

NLP: theory and algorithms

- Overview
- Mathematical background
- Convex sets
- Convex functions
- The FJ and KKT optimility conditions
- Lagrangian duality and saddle point
- Algorithms
- Unconstraint optimization
- Penalty and barrier functions
- Interior point method
Overview

Optimization problem has the form

\[
\begin{align*}
\text{minimize} & \quad f_0(x) \\
\text{subject to} & \quad f_i(x) \leq b_i, \ i = 1, \ldots, m.
\end{align*}
\]

where \( x \in \mathbb{R}^n \) is the optimization variable, \( f_0(x) : \mathbb{R}^n \rightarrow \mathbb{R} \) is the objective function, \( f_i : \mathbb{R}^n \rightarrow \mathbb{R} \) are (inequality) constraint functions and \( b_i \) are the limits, or bounds, for the constraints.

A vector \( x^* \) is called optimal, or a solution if it has the smallest objective value among all vectors \( z \) that satisfy the constraints: for any \( z \) with \( f_1(z) \leq b_1, f_2(z) \leq b_2, \ldots, f_m(z) \leq b_m \) we have \( f_0(z) \geq f_0(x^*) \).
Optimization classifications

Optimization problems can be divided into classes based on the properties of the objective and constraint functions.

A linear problem satisfies

\[ f_i(\alpha x + \beta y) = \alpha f_i(x) + \beta f_i(y), \]

for all \( x, y \in \mathbb{R}^n \) and for all \( \alpha, \beta \in \mathbb{R} \). Problems that do not satisfy the above are called non-linear.

A convex problem satisfies

\[ f_i(\alpha x + \beta y) \leq \alpha f_i(x) + \beta f_i(y), \]

for all \( x, y \in \mathbb{R}^n \) and for all \( \alpha, \beta \geq 0 \) and \( \alpha + \beta = 1 \). Problems that do not satisfy the above are called non-convex.

Note: convexity is more general than linearity: instead of an equality, we only require an inequality to be satisfied, and only for a certain values of \( \alpha \) and \( \beta \).
Historical notes

- The traditional division in optimization literature was linear vs. non-linear problems as the former were thought to be "easier" to solve than the latter.
- The more recent division is between convex and non-convex problems, as it has been found that non-linear problems that are convex are often "almost as easy" to solve as linear problems, while non-convex non-linear problems often pose problems.
- One of the aims of this course is to cover a significant portion of these efficient NLP techniques.
Application -- facility location

- In facility location problems one considers a set of facilities that need to be placed optimally with respect to some a priori fixed locations.
- In minimax delay placement, one has a directed graph of nodes on a plane with some of the node locations fixed and some being free.
- The aim is to minimize the longest path between a source and sink node.
Application -- support vector classification

Given a set of labeled points
\((x, y), x \in \mathbb{R}^n, y \in \{-1, +1\}\), a support vector machine aims at finding the hyperplane separating the differently labeled points in a way that the minimum distance (or margin) from a point to the hyperplane is maximized.

This maximization problem is a convex optimization problem, a convex quadratic programme to be exact.
Application -- portfolio optimization

- We have a set of assets or stocks held over a period of time.
- Assume prices change according to a probability distribution with known mean and variance. Then the return of the portfolio is a random variable as well.
- The classical Markovitz portfolio optimization problem of finding the distribution of investments among the set of assets that minimizes the variance of the return, subject to the return being above given threshold, is a convex (quadratic) optimization problem.
Concepts

- Vectors and matrices
- Norms
- Open and closed sets, supremum and infimum
- Functions
- Derivatives
Set notation, supremum, infimum

- Set membership $x \in X$, set union $X_1 \cup X_2$, set intersection $X_1 \cap X_2$, quantifiers $\exists$ (there exists), $\forall$ (for all), empty set $\emptyset$.

- Set of real numbers $\mathbb{R}$, extended real numbers $\mathbb{R} \cup \{\infty\} \cup \{-\infty\}$

- Intervals: Open $(a, b) = \{x | a < x < b\}$, closed $[a, b] = \{x | a \leq x \leq b\}$, and half-open $[a, b) = \{x | a \leq x < b\}$, $(a, b] = \{x | a < x \leq b\}$.

- Supremum of a non-empty set of real numbers $X$, denoted $\sup X$ is the least number $y$ that satisfies $x \leq y$ for all $x \in X$. Infimum of a non-empty set, denoted $\inf X$ is the largest number $y$ such that $x \geq y$ for all $x \in X$. If $X$ is unbounded from above, $\sup X = \infty$. If $X$ is unbounded from below $\inf X = -\infty$.

- For empty sets we use the conventions $\inf \emptyset = \infty$ and $\sup \emptyset = -\infty$.

- If the supremum (resp. infimum) belongs to the set, supremum equals the maximum element (resp. minimum element) of the set.
Vectors

- We denote by \( \mathbb{R}^n \) the set of \( n \)-dimensional real vectors. For any \( x \in \mathbb{R}^n \) we use \( x_i \) to indicate the \( i \)'th coordinate (also called component), we also write \( x = (x_1, \ldots, x_n) \).

- Vectors will be viewed as column vectors. For any vector \( x \in \mathbb{R}^n \), the transpose \( x^T \) denotes the \( n \)-dimensional row vector.

- The inner product of two vectors \( x, y \in \mathbb{R}^n \) is defined by

\[
x^T y = \sum_{i=1}^{n} x_i y_i,
\]

- Vectors \( x, y \) and called orthogonal if \( x^T y = 0 \).

- For vector \( x \) the notation \( x > 0 \) and \( x \geq 0 \) denote that all its components are positive and nonnegative, respectively. For any two vectors \( x, y \) the notation \( x > y \) denotes that the difference vector \( d = x - y \) satisfies \( d > 0 \). The notation \( x \geq y, x < y \) and \( x \leq y \) is defined analogously.
Vector sum, Cartesian product

If $X$ is a set and $\lambda$ is a scalar, we denote by $\lambda X$ the set $\{\lambda x|x \in X\}$.

If $X_1$ and $X_2$ are two subsets of $\mathbb{R}^n$, we denote by $X_1 + X_2$ the set

$$\{x_1 + x_2|x_1 \in X_1, x_2 \in X_2\},$$

which is referred to as the vector sum of $X_1$ and $X_2$.

When one of the sets consists of a single vector $x$ we use the simplified notation $\bar{x} + X$ to denote $\{\bar{x} + x|x \in X\}$. We also denote by $X_1 - X_2$ the set

$$\{x_1 - x_2|x_1 \in X_1, x_2 \in X_2\}$$

Given sets $X_i \subset \mathbb{R}^{n_i}$, $i = 1, \ldots, m$, the Cartesian product of the sets, denoted by $X_1 \times \cdots \times X_m$ is the set

$$\{(x_1, \ldots, x_m)|x_i \in X_i, i = 1, \ldots, m\},$$

which is a subset of $\mathbb{R}^{n_1 + \cdots + n_m}$. 
Subspaces and linear independence

- A non-empty subset $S$ of $\mathbb{R}^n$ is a subspace if $ax + by \in S$ for every $x, y \in S$ and every $a, b \in \mathbb{R}$.

- An affine set or linear manifold in $\mathbb{R}^n$ is a translated subspace, i.e., a set $X$ of the form $X = \bar{x} + S = \{\bar{x} + x \mid x \in S\}$ where $\bar{x} \in \mathbb{R}^n$ and $S \subset \mathbb{R}^n$. $S$ is called the subspace parallel to $X$.

- The span of a finite collection $\{x_1, \ldots, x_m\}$ of elements or $\mathbb{R}^n$ is the subspace consisting of all vectors $y$ of the form $y = \sum_{k=1}^{m} \alpha_k x_k$, where $\alpha_k \in \mathbb{R}$.

- The vectors $x_1, \ldots, x_k$ are called linearly independent if there exists no set of scalars $\alpha_1, \alpha_k$, at least one of which is non-zero, such that $\sum_{k=1}^{m} \alpha_k x_k = 0$. 
A set of linearly independent vectors in subspace $S$ whose span is equal to $S$ is called a basis for $S$.

Every basis of a given subspace has the same number of vectors, this number is called the dimension of $S$.

The dimension of an affine set $x + S$ is the dimension of the corresponding subspace $S$.

Every subspace of non-zero dimension has a basis that is orthogonal, i.e. $x^T z = 0$ for any two vectors $x, z$ of the basis.

Given any set $X$, the set of vectors that are orthogonal to all elements of $X$, 

$$X^\perp = \{ y | y^T x = 0, \forall x \in X \},$$

is called the orthogonal complement of $X$. Any vector can be uniquely decomposed as a sum of a vector from a subspace and its orthogonal complement.
Matrix range, null space and rank

We denote by $\mathbb{R}^{m \times n}$ the set of real matrices with $m$ rows and $n$ columns. For any $A \in \mathbb{R}^{m \times n}$ we use $a_{ij}$ to indicate the component on the $i$’th row and $j$’th column. We may also write $[A]_{ij}$ to denote the same element.

The range space of $A$, denoted $\mathcal{R}(A)$, is the set of all vectors $y \in \mathbb{R}^m$ such that $Ax = y, \forall x \in \mathbb{R}^n$.

The null space of $A$, denoted $\mathcal{N}(A)$, is the set of all $x \in \mathbb{R}^n$ such that $Ax = 0$.

The rank of $A$ is the dimension of $\mathcal{R}(A)$.

The range space and null space of a matrix are tied together via:

$$\mathcal{R}(A) = \mathcal{N}(A^T)^\perp$$
Euclidean norm and angle

The Euclidean norm, or $\ell_2$-norm, of a vector $x \in \mathbb{R}^n$ is defined as

$$||x||_2 = (x^T x)^{1/2} = (x_1^2 + x_2^2 + \cdots + x_n^2)^{1/2},$$

i.e. the square root of the inner product of the vector with itself.

The cosine angle between $x$ and $y$ is given by

$$\cos \angle(x, y) = \frac{x^T y}{||x||_2 ||y||_2} = \frac{x^T y}{x^T y},$$

where $x = x/||x||_2$ and $y = y/||y||_2$.

The distance of two vectors is defined as the norm of their difference vector:

$$\text{dist}(x, y) = ||x - y||$$
Norm equivalence

The Euclidean norm belongs to the family of $p$-norms

$$
\|x\|_p = \left( |x_1|^p + \cdots + |x_n|^p \right)^{1/p}
$$

Other important $p$-norms for vectors are the $\ell_1$-norm

$$
\|x\|_1 = \sum_{i=1}^{n} |x_i|
$$

and the $\ell_\infty$ norm

$$
\|x\|_\infty = \max_i |x_i|
$$

The norms are equivalent in the following way. Suppose that $\|\cdot\|_a$ and $\|\cdot\|_b$ are norms on $\mathbb{R}^n$. Then it can be proven that there exist positive constants $\alpha$ and $\beta$ such that for all $x \in \mathbb{R}^n$,

$$
\alpha \|x\|_a \leq \|x\|_b \leq \beta \|x\|_a
$$
Matrix norms

As well as vectors, norms can be defined for matrices.

Perhaps the most important is the Frobenius norm:

\[ \| X \|_F = \left( \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}^2 \right)^{1/2}, \]

that can be seen as the counterpart of Euclidean norm for matrices.

Other matrix norms include the sum-absolute-value norm (cf. \( \ell_1 \)-norm for vectors)

\[ \| X \|_{sav} = \sum_{i=1}^{m} \sum_{j=1}^{n} |x_{ij}|, \]

and the maximum-absolute-value norm (cf. \( \ell_{inf} \)-norm)

\[ \| X \|_{max} = \max x_{ij} \quad i = 1, \ldots, m, j = 1, \ldots, n \]
Symmetric eigenvalue decomposition and definiteness

Let $S^n$ denote the set of symmetric matrices in $\mathbb{R}^{n \times n}$. Every $A \in S^n$ can be decomposed as

$$A = Q\Lambda Q^T,$$

where $Q$ is an orthogonal matrix (columns are orthogonal and have Euclidean norm of 1) and $\Lambda$ is a diagonal matrix, with the eigenvalues $(\lambda_1, \lambda_n)$ of $A$ as the diagonal elements.

A matrix $A \in S^n$ is called positive definite (pos. semi-definite), if all its eigenvalues are positive (non-negative). Similarly a matrix $A \in S^n$ is called negative definite (neg. semi-definite) if all its eigenvalues are negative (non-positive).

Alternatively, the matrix $A$ is seen to be positive definite, if the quadratic form $x^T A x > 0$ for all $x > 0$. The other definiteness classes can be defined analogously.
Interior of a set

An element $x \in C \subseteq \mathbb{R}^n$ is called an interior point of $C$ if there exists an $\epsilon > 0$ for which

$$\{y | \|y - x\|_2 \leq \epsilon\} \subseteq C,$$

in other words, there exists a ball centered at $x$ that lies entirely in $C$. The set of all points interior to $C$ is called the interior of $C$ and denoted $\text{int } C$.

All norms generate the same set of interior points (this is a consequence of the equivalence of the norms)
A set $C$ is *open* if $\text{int} \ C = C$, that is, every point in $C$ is an interior point. A set $C \subseteq \mathbb{R}^n$ is *closed* if its complement $\mathbb{R}^n \setminus C$ is open.

The *closure* of a set $C$ is defined as

$$\text{cl} \ C = \mathbb{R}^n \setminus \text{int}(\mathbb{R} \setminus C),$$

in other words, as the complement of the interior of the complement of $C$.

A point $x$ is in the closure of $C$ if for every $\epsilon > 0$, there is a $y \in C$ with $\|x - y\|_2 \leq \epsilon$.

The *boundary* of the set $C$ is defined as

$$\text{bd} \ C = \text{cl} \ C \setminus \text{int} \ C$$

A point $x \in \text{bd} \ C$ is called a *boundary point*. 
Characterizing open and closed sets

Characterization using convergent sequences and limit points: A set $C$ is closed if and only if contains the limit point of every convergent sequence in it. The boundary of the set is the set of all limit points of convergent sequences in $C$.

Characterization using the boundary concept: $C$ is closed if $\text{bd } \in C$. It is open if it contains no boundary points $C \cap \text{bd}C = \emptyset$.
Functions and continuity

- $f : X \mapsto Y$ denotes a function that has a domain $X$, denoted $\text{dom} f$ and range $Y$.

- If $U$ and $V$ are subsets of $X$ and $Y$, respectively, the set $\{f(x) | x \in U\}$ is the image of $U$ (under $f$) and the set $\{x | f(x) \in V\}$ is the inverse image (or preimage) of set $V$.

- A function $f : \mathbb{R}^n \mapsto \mathbb{R}^m$ is continuous at $x \in \text{dom} f$ if for all $\epsilon > 0$ there exists a $\delta$ such that

$$y \in \text{dom} f, ||y - x|| \leq \delta \Rightarrow ||f(y) - f(x)|| \leq \epsilon$$

- Continuity can be expressed in terms of limits: whenever the sequence $x_1, x_2, \cdots \in \text{dom} f$ converges to a point in $x \in \text{dom} f$, the sequence $f(x_1), f(x_2), \cdots$ converges to $f(x)$

- A function is continuous if it is continuous in every point of its domain
Closed functions

A function $f : \mathbb{R}^n \to \mathbb{R}$ is said to be closed if, for each $\alpha \in \mathbb{R}$, the sublevel set

$$\{ x \in \text{dom } f \mid f(x) \leq \alpha \}$$

is closed.

If $f : \mathbb{R}^n \to \mathbb{R}$ is continuous, and $\text{dom } f$ is closed, then $f$ is closed.

If $f : \mathbb{R}^n \to \mathbb{R}$ is continuous, and $\text{dom } f$ is open, then $f$ is closed if and only if $f$ converges to $\infty$ along every sequence converging to a boundary point of $\text{dom } f$. 
Examples

- The function $f : \mathbb{R} \mapsto \mathbb{R}$ with $f(x) = x \log x$, $\text{dom } f = \mathbb{R}_{++}$, is not closed.

- The function $f : \mathbb{R} \mapsto \mathbb{R}$ with

  \[ f(x) = \begin{cases} 
  x \log x, & x > 0 \\
  0, & x = 0 
  \end{cases} \]

  $\text{dom } f = \mathbb{R}_+$, is closed.

- The function $f(x) = -\log x$, $\text{dom } f = \mathbb{R}_{++}$, is closed.
Suppose $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $x \in \text{int dom } f$. The derivative, or Jacobian, of $f$ at $x$ is the matrix $Df(x) \in \mathbb{R}^{m \times n}$, given by

$$Df(x)_{ij} = \frac{\partial f_i(x)}{\partial x_j}, i = 1 \ldots m, j = 1 \ldots, n.$$ 

provided the partial derivatives exist, in which case, $f$ is said to be differentiable at $x$.

The function $f$ is differentiable if $\text{dom } f$ is open, and it is differentiable at every point in its domain.

The affine function of $z$ given by

$$f(x) + Df(x)(z - x)$$

is called the first order approximation of $f$ at (or near) $x$. 
Gradient

When $f$ is real-valued $f: \mathbb{R}^n \to \mathbb{R}$, the Jacobian is a $1 \times n$ matrix, that is a row vector. Its transpose is called the gradient of $f$:

$$\nabla f(x) = Df(x)^T.$$ 

Example: Consider the quadratic function $f: \mathbb{R}^n \to \mathbb{R}$,

$$f(x) = 1/2x^T H x + q^T x + r,$$

where $H \in S^n$, $q \in \mathbb{R}^n$, and $r \in \mathbb{R}^n$.

The gradient of $f$ is

$$\nabla f(x) = Hx + q$$
Second derivative

The second derivative, or Hessian matrix of $f : \mathbb{R}^n \to \mathbb{R}$ at $x \in \text{int dom } f$, is given by

$$[\nabla^2 f(x)]_{ij} = \frac{\partial^2 f(x)}{\partial x_i \partial x_j}, \quad i = 1, \ldots, n, \quad j = 1, \ldots, n,$$

provided that $f$ is twice differentiable at $x$

The second order approximation of $f$, at or near $x$, is the quadratic function of $z$, given by

$$\hat{f}(z) = f(x) + \nabla f(x)^T(z - x) + 1/2(z - x)^T \nabla^2 f(x)(z - x).$$

Example: the second derivative of

$$f(x) = 1/2 x^T H x + q^T x + r,$$

is the matrix $H$. 
The H&J Algorithm

Initialization Step
- Choose a scalar \( \varepsilon > 0 \) to be used in terminating the algorithm.
- Choose a starting point \( x_1 \), let \( y_1 = x_1 \), let \( k = j = 1 \), and got to the main step.

Main step
- Let \( \lambda_j \) be an optimal solution to the problem to minimize \( f(y_j + \lambda d_j) \) s.t. \( \lambda \) in \( E^1 \), and let \( y_{j+1} = y_j + \lambda_j d_j \). If \( j < n \), replace \( j \) by \( j+1 \), and repeat step 1. Otherwise, if \( j = n \), let \( x_{k+1} = y_{n+1} \). If \( \|x_{k+1} - x_k\| < \varepsilon \), stop; otherwise, go to step 2.
- Let \( d = x_{k+1} - x_k \), and let \( \lambda_{\text{hat}} \) be an optimal solution to the problem to minimize \( f(x_{k+1} + \lambda d) \) s.t. \( \lambda \) in \( E^1 \). Let \( y_1 = x_{k+1} + \lambda_{\text{hat}} d \), let \( j = 1 \), replace \( k \) by \( k+1 \), and repeat step 1.
Method of Hooke and Jeeves

Figure 8.10 Method of Hooke and Jeeves using line searches. Method of Hooke and Jeeves with Discrete Steps

The Osyczka’s (1984) direct and random search algorithm (DRS) is a hybrid algorithm based on neighborhood search, shotgun search and Hooke and Jeeves’ direct search.

By integrating neighborhood search and random search into direct search phases, DRS has proved to be a successful method for solving forest management problems.

The details of DRS were explained in Valsta (1992).
Particle Swarm Optimization

- Particle Swarm Optimization (PSO) is a stochastic global optimization algorithm inspired by swarm behavior in birds, insects, fish, even human behavior (Kennedy and Eberhart, 1995).
- In PSO, each particle (individual) adjusts its position and velocity, moves to some global objective through information exchange between its neighbor particles and the whole swarm (population).
- PSO carries out a five steps search:
1) Randomly generate initial swarm (population) which consists of \( m \) particles (individuals), each particle \( \mathbf{x}_i=(x_{i1}, x_{i2}, \ldots, x_{in}) \) has velocity \( \mathbf{v}_i=(v_{i1}, v_{i2}, \ldots, v_{in}) \).

2) Evaluate each particle, store the previous best position for each particle \( \mathbf{p}_{best_i}=(p_{i1}, p_{i2}, \ldots, p_{in}) \), and find the global best for the entire population \( \mathbf{g}_{best}=(g_1, g_2, \ldots, g_n) \).

3) Update the \( i+1 \)th generation \( \mathbf{x}_{i+1} = \mathbf{x}_i + \mathbf{v}_{i+1} \), where \( \mathbf{v}_{i+1} \) is updated as

\[
\mathbf{v}_{i+1} = w\mathbf{v}_i + c_1r_1(\mathbf{p}_{best_i} - \mathbf{x}_i) + c_2r_2(\mathbf{g}_{best} - \mathbf{x}_i),
\]

where \( w=(0.4+(0.9-0.4)(80-i)/80) \) is the inertia factor decreased linearly from 0.9 to 0.4, \( c_1 \) and \( c_2 \) are constants, \( r_1 \), \( r_2 \) are random values between \([0,1]\).
PSO, con’t

4) Evaluate every new particle, $x_i = x_{i+1}$ if $f(x_{i+1}) < f(x_i)$, otherwise $x_{i+1} = x_i$. Compare the best value $f(x_{i+1})$ with $f(pbest_i)$ and $f(gbest)$, if $f(x_{i+1}) < f(pbest_i)$, $pbest_{i+1} = x_{i+1}$, if $f(x_{i+1}) < f(gbest)$, $gbest = x_{i+1}$.

5) Check whether the number of iterations reaches up its maximum limit. If not, go to step 3.
Differential Evolution

- Storn and Price (1995) proposed Differential Evolution (DE), a stochastic evolutionary algorithm to solve global optimization problems.
- In DE an offspring individual (candidate solution) is generated through mutation and crossover with the weighted difference of parent solutions.
- The offspring may replace its parent through competitive selection.
- The most applied mutation strategies are rand/1, best/1, current to best/1, best/2, and rand/2 schemes (for details, see Liu et al., 2010).
- In this version we used the mutation strategy of current to best/1 scheme rather than the rand/1 scheme used in Pukkala (2009). The method of Differential Evolution (DE) performs a six steps search:
DE, con’t

1) Randomly generate initial population (initial parent individuals) $x_i=(x_{i1}, x_{i2}, \ldots x_{in})$.

2) Evaluate the initial population, calculate every individual function value $f(x_i)$, and record the optimized value and the previous best individual $pbest_i$.

3) Randomly select two remainder individuals $x_{r1}$ and $x_{r2}$, and calculate for the mutant individual $y_i=(y_{i1}, y_{i2}, \ldots y_{in})$ using the current to best/1 scheme,

$$y_i = x_i + \alpha (pbest_i - x_i) + \beta (x_{r1} - x_{r2}),$$

where $\alpha$ is a random number between $[0,1]$, $\beta=0.8$. 
4) Generate the offspring individual $x_i'$ by a crossover operation on $x_i$ and $y_i$ with a crossover probability parameter (in this study $CR=0.5$) determining the genes of $x_i'$ are inherited from $x_i$ or $y_i$. Let $x_{ij}'=y_{ij}$, if a random real number from $[0,1]$ is less than $CR$, otherwise, $x_{ij}'=x_{ij}$.

5) Select the best individual for the next generation $x_{i+1}$ by the competition between the offspring individual $x_i'$ and the parent individual $x_i$. If $f(x_i') \leq f(x_i)$, $x_{i+1}=x_i'$, otherwise, $x_{i+1}=x_i$.

6) Check whether the number of iterations reaches the maximum limit. If not, go to step 3.
Evolution Strategy

- The Evolution Strategy (ES) uses strategy parameters to determine how a recombinant is mutated.
- ES generates an offspring as a mutated recombination from two parents. One of the parents is the previous best individual, and another one is randomly drawn from the remaining individuals.
- The offspring then compete with the parents. If the offspring is better, the mutated solution replaces the worst solution of the parent population. The best solution at the last generation is the optimal solution.
- ES conducts a five steps search:
1) Randomly generate initial population $x_i$. The initial strategy parameters $\sigma_i$ was calculated from $\sigma_i=\alpha x_i$, where $\alpha=0.2$.

2) Obtain the previous best individual $p_{best_i}$ and $\sigma_{best_i}$ strategy parameter values. Recombine the selected parents, the best individual $p_{best_i}$ and random individual $x_r$, obtain the recombined individual $x_m=0.5(p_{best_i}+x_r)$, and strategy parameters

$$\sigma_m=0.5(\sigma_{best_i}+\sigma_r)\times \exp(\tau_g \times N(0,1)+\tau_l \times N(0,1)),$$

where the global study parameter $\tau_g$ is $1/\sqrt{(2 \times n_d)}$, the local study parameter $\tau_l$ is $1/\sqrt{(2\sqrt{n_d})}$, $N(0,1)$ is a normally distributed random number.
ES, con’t

3) Mutate an offspring individual $x' = x_m + \sigma_m \times N(0,1)$.

4) Evaluate the new individual $x'$. Replace the worst solution of the parent generation if $f(x')$ is less than the worst function value.

5) Check whether the number of iterations reaches its maximum limit. If not, go to step 2.
Nelder-Mead

- Similar to ES, Nelder-Mead (NM) also uses a new candidate solution to replace the worst solution of all solutions at every iteration.
- In NM the new candidate solution is calculated based on the centroid solution and the best solution through reflection, expansion, and contraction operations.
- In case none of better new candidate solutions can be found in the reflection, expansion, and contraction operations, NM carries out an additional shrinking operation for a new iteration by updating all candidate solutions except the best solution.
- In NM all operations are calculated without stochasticity.
- NM implements a six steps search (Lagarias et al. 1998):
1) Randomly generate initial population. Select the best, the worst, the second worst solutions $x_b$, $x_w$, $x_{sw}$ from all candidate solutions by their function values $f(x_b)$, $f(x_w)$, $f(x_{sw})$.

2) Calculate the reflection point $x_{rf} = (1+\rho)x_m - \rho x_w$, where the reflection parameter $\rho=1.4$, and the centroid point (average except the worst point $x_w$) $x_m = \sum_{i \neq w} (x_i / (n_d - 1))$. If $f(x_b) < f(x_{rf}) < f(x_{sw})$, replace $x_w$ with $x_{rf}$ and terminate the iteration.

3) If $f(x_{rf}) < f(x_b)$, compute expansion point $x_e = \chi x_{rf} + (1-\chi)x_m$, where the expansion parameter $\chi=2.5$. If $f(x_e) \leq f(x_b)$, replace $x_w$ with $x_e$ and terminate the iteration; else replace $x_w$ with $x_{rf}$ and terminate the iteration.
Number of function evaluations
Effects of decision variables
Robustness of algorithms
4) If \( f(x_{rf}) > f(x_{sw}) \), compute inside contraction point \( x_c = \gamma x_w + (1-\gamma)x_m \), where the contract parameter \( \gamma = 0.5 \). If \( f(x_c) \leq f(x_w) \), replace \( x_w \) with \( x_c \) and terminate the iteration, else go to step 5. If \( f(x_{sw}) \leq f(x_{rf}) < f(x_w) \), compute outside contraction point \( x_c = \gamma x_{rf} + (1-\gamma)x_m \). If \( f(x_c) \leq f(x_{rf}) \), replace \( x_w \) with \( x_c \) and terminate the iteration, else go to step 5.

5) Compute \( x_i \) (\( i \neq b \)) with \( x_b \) and shrinkage parameter \( \delta = 0.8 \) for the new generation \( x_i' = x_b + \delta (x_i - x_b) \), and begin a new iteration.

6) Check whether the number of iterations reaches its maximum limit. If not, go to step 2.
Therefore deal with things before they happen; Create order before there is confusion.
Observation data

- Which type of information is more important in forestry?

- Forest stand data from
  - Earlier inventory
  - Later inventory
4.04.00 – Forest management planning

- 4.04.02 – Planning and economics of fast-growing plantation forests
- 4.04.03 – SilvaPlan: Forest management planning terminology
- 4.04.04 – Sustainable forest management scheduling
- 4.04.06 – Nature conservation planning
- 4.04.07 – Risk analysis
- 4.04.08 – Adaptation to climate change
Even-aged management

- Even-aged management deals with forests composed of even-aged stands.
- In such stands, individual trees originate at about the same time, either naturally or artificially.
- In addition, stands have a specific termination date at which time all remaining trees are cut.
- This complete harvest is called a clear-cut.
Even-aged management

Source: Cao et al. (2006), Fig. 1
Uneven-aged management

Source: Pukkala et al. (2009), Fig. 8
Regeneration methods

- Regeneration of even-aged stands may be done by planting or seeding. The latter may be natural. For example, in a *shelterwood* system a few old trees are left during the period of regeneration to provide seed and protect the young seedlings.
- Natural regeneration may continue for a few years after initial planting or seeding.
- Nevertheless, the basic management remains the same, it leads to a total harvest and a main crop when the stand has reached rotation age.
- Light cuts called “thinnings” are sometimes done in even-aged stands before the final harvest.
EXAMPLE: Converting forests

**Figure 4.1** Hardwood forest with (a) two initial age classes, converted to a regulated pine plantation with (b) three age classes.
The even-aged management problem

- There are only two compartments on this forest, labeled 1 and 2. Compartment 1 has an area of 120 ha, compartment 2 has 180 ha. Southern hardwoods of low quality currently cover the two compartments. However, they are on distinct soils and timber grows better on compartment 1 than on compartment 2.

- One objective of the owner of this property is to convert the entire area to a pine plantation during a period of 15 years. The forest created at the end of this period should be regulated, with a rotation age of 15 years.

- That is to say, one third of the forest should be covered with trees 0 to 5 years old, a third with trees 6 to 10 years old and another third with trees 11 to 15 years old. This would lead to a pattern of age-classes like that shown in Fig. (b).
Even-aged management, con’t

- Finally, the owner desires to maximize the amount of wood that will be produced from his forest during the period of conversion to pine.
- However, the owner will not cut any of the pine stands before they are 15 years old.
- We shall learn how to represent this problem as a linear program with decision variables, constraints, and an objective function, and how to solve it to find the best solution.
Model formulation

- Let $X_{ij}$ be the area to be cut from compartment $i$, in period $j$, where $i$ and $j$ are integer subscripts. Here, $i$ may take the value 1 or 2 since there are only two compartments in the initial forest.

- Let us use a time unit of 5 years in this example. Therefore, $j$ can take the values 1, 2 or 3 depending on whether a cut occurs during the first 5 years of the plan, the second 5 years, or the third. As soon as an area is cut over, it is immediately replanted with pine trees, so $X_{ij}$ is also the area replanted in compartment $i$ during period $j$.

- Thus, all the possible harvests and reforestations in compartment 1 are defined by the three decision variables: $X_{11}$, $X_{12}$, and $X_{13}$, while those possible on compartment 2 are: $X_{21}$, $X_{22}$, and $X_{23}$.
The optimization problem

- Objective function:
  \[ Z = 16X_{11} + 23X_{12} + 33X_{13} + 24X_{21} + 32X_{22} + 45X_{23} \text{ tons} \]
  where \(16X_{11} + 23X_{12} + 33X_{13}\), and \(24X_{21} + 32X_{22} + 45X_{23}\)
  the tonnage of hardwood cut from compartments 1, and 2.

- Constraints:
  \[
  \begin{align*}
  X_{11} + X_{12} + X_{13} &= 120 \text{ ha}, \\
  X_{21} + X_{22} + X_{23} &= 180 \text{ ha} \\
  X_{11} + X_{21} &= 100 \text{ ha} \\
  X_{12} + X_{22} &= 100 \text{ ha} \\
  X_{13} + X_{23} &= 100 \text{ ha}
  \end{align*}
  \]
# OPTIMIZED HARVEST PLAN: AREA DATA

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<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>300</td>
</tr>
</tbody>
</table>
## Optimized Harvest Plan: Tonnage Data

<table>
<thead>
<tr>
<th>Compartment</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Total tons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1600</td>
<td>460</td>
<td>0</td>
<td>2060</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2560</td>
<td>4500</td>
<td>7060</td>
</tr>
<tr>
<td>Total</td>
<td>1600</td>
<td>3020</td>
<td>4500</td>
<td>9120</td>
</tr>
</tbody>
</table>
Reforestation

- Silvicultural treatments
- Logging
- Administration
- Financial costs
- Taxes
Regeneration

- Clearing and soil preparation (scarification, harrowing, mounding, ploughing, 0 year after clearcut, by tree species, and site types);
- Seeding or planting (1st year after clearcut, by tree species, and site types);
- Tending (7-14 years after clearcut, by tree species, and site types);
- Pruning, fertilization, and forest road construction.
### Natural vs. artificial regeneration

#### Table 3. Unit costs, timing, and probability of occurrence of stand establishment and silvicultural activities.

<table>
<thead>
<tr>
<th>Type of work</th>
<th>Timing, year</th>
<th>Total area, 1000ha</th>
<th>Total cost, €1000</th>
<th>Unit cost, €/ha</th>
<th>Probability of occurrence in H100 class</th>
<th>Probability of occurrence in H50 class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural regeneration</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(%)</td>
<td>(%)</td>
</tr>
<tr>
<td>Clearing</td>
<td>0</td>
<td></td>
<td>113</td>
<td>60</td>
<td>60</td>
<td>75</td>
</tr>
<tr>
<td>Soil preparation</td>
<td>0</td>
<td></td>
<td>189</td>
<td>40</td>
<td>20</td>
<td>75</td>
</tr>
<tr>
<td>Artificial regeneration</td>
<td></td>
<td></td>
<td>119</td>
<td></td>
<td>(%)</td>
<td>(%)</td>
</tr>
<tr>
<td>Clearing</td>
<td>0</td>
<td></td>
<td>113</td>
<td>60</td>
<td>60</td>
<td>75</td>
</tr>
<tr>
<td>Soil preparation</td>
<td>0</td>
<td></td>
<td>189</td>
<td>40</td>
<td>20</td>
<td>75</td>
</tr>
<tr>
<td>Seeding</td>
<td>1</td>
<td>32</td>
<td>5858</td>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Planting</td>
<td>1</td>
<td>87</td>
<td>51740</td>
<td></td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Weeding (1, 2, 3)</td>
<td>2, 2, 3</td>
<td>6</td>
<td>841</td>
<td>140</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Tending (1, 2, 3)</td>
<td>7, 11, 14</td>
<td>138</td>
<td>41786</td>
<td>303</td>
<td>90</td>
<td>80</td>
</tr>
<tr>
<td>Fixed annual cost (€/ha/yr)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Note: Fixed annual cost and probability of occurrence are extended from Hyytiäinen and Tahvonen (2001), Salminen (1993), and Hyytiäinen et al. (2005).

Logging

- Logging method
- Terrain class
- Average travel distance
- Thinnings/partial cut (mean tree size, by tree species, and site types);
- Final harvesting (mean tree size, by tree species, and site types).
A simple logging model

- The absolute effect of logging conditions on harvest revenues, $bih ($/m^3), is a function of the average size of the removed trunks in liters, $vi$, and the removed total volume, $xi$, expressed in m^3.

- The logging cost for thinning and clearcutting. The equations are, for thinning (equation 1), ($h = 1$):

$$bih_1 = (1.413 + 4.005 \ln(0.008908 \, xi) + 4.634 \ln(0.0422 \, vi) - 2622/\, vi) \, xi,$$

and for clearcutting (equation 2), ($h = 2$):

$$bih_2 = (-0.3191 + 1.602 \ln(0.004259 \, xi) + 2.408 \ln(9.579 \, vi) - 2642/\, vi) \, xi.$$
The logging model

Logging Parameters

- Terrain Class (1-3): 1
- Ave. Travel, m: 200.00
- Cfell, euro/hour: 75.67
- Ctrans, euro/hour: 53.35
- Fixed Cost, euro: 100.00
- Logistics cost: 6.00 euro/m³
Fig.1. Basal area development by optimal solutions for stands 1-4 (a) and stands 5-7 (b).
Type of thinning

(a) 1st thinning
(b) 2nd thinning
(c) 3rd thinning
(d) 4th thinning
Simulation optimization systems

Yet a tree broader than a man can embrace is born of a tiny shoot; A dam greater than a river can overflow starts with a clod of earth; A journey of a thousand miles begins at the spot under one's feet.
Decision support systems

- Inventory DSS
- Simulation DSS
- Two-level DSS
- Theoretical optimization DSS
- Simulation optimization DSS
A simulation optimization approach

- **Sweden** (e.g., Eriksson 1994, 1997; Gong, 1998; Wikström 2000, 2001; Lu and Gong, 2003)
- **Norway** (e.g., Risvand, 1969; Solberg and Haight, 1991; Hoen and Solberg, 1994)
- **Denmark** (e.g., Thorsen and Helles, 1998; Strange et al., 1999)
- **The Netherlands** (e.g., Brazee and Bulte, 2000)
- **Finland** (e.g., Kilkki and Väisänen, 1969; Valsta, 1986, 1990, 1992; Salminen, 1993; Miina, 1996; Pukkala and Miina, 1997, 1998; Hyytiäinen et al., 2003, 2004; Cao et al., 2006; Pukkala, 2009; Cao et al., 2010)
Structure of simulation-optimization system

Valsta (1992)
Forest decision-support systems

- EFISCEN
  http://www.efi.int/portal/completed_projects/efiscen/
- FVS http://www.fs.fed.us/fmsc/fvs/
- MOTTI, SIMO, MONSU, SMA
  www.simo-project.org
  http://fp0804.emu.ee/wiki/index.php/Monsu
  http://www.helsinki.fi/forestsciences/research/projects/sma/programme/index.htm
- OptiFor
  http://www.optifor.cn
The optimization problem

- Forest management objectives
  - Problems to be solved

- Silvicultural operations
  - Decision variables

- Stand simulator
  - State variables

- Biological and economic data
  - Initial states
OptiFor simulation-optimization system

Inputs (Initial guess)
- Optimal harvesting
- Biodiversity
- Carbon accounting
- Timber grading
- Energy wood

Main Program (Interface)
1. Optimization
   - Controller (Optimizer)
   - Measurement (Convergence)
2. Simulation
   - Process (Simulator)

Inputs (Initial state)
- Growth and yield
- Biomass
- Soil carbon
- Wood properties
- Tree structure

Outputs
Decision levels

Tree level
Stand level
Forest level
Enterprise level
Region/sector level

Source: Valsta (1993)
Decision makers

- Decision makers categories
  - risk averse,
  - risk neutral
  - risk preferring.
- Maximizing expected value
  - risk neutraler
- Maximizing the maximum value (Maxmax)
  - extreme risk lover
- Maximizing the minimum value (Maxmin)
  - extreme risk averter
Decision-making under risks

- Production risk
  - Silvicultural operations
  - Biotic and Abiotic risks
- Price risk
  - Timber market
  - Financial market
  - Long-term investment
- Institutional risk
  - Common forestry policy
- Risk and insurance
  - Forest fire insurance
Risk analysis

- The degree of risk in a revenue is the amount of variation in its possible outcomes. A risk-free return has no variation.
- The expected value of any risky variable is the sum of the possible values multiplied by their probabilities of occurrence.
- Most people are risk-averse: they prefer less variation in revenues.
- The certainty-equivalent of a risky revenue is a sure dollar amount giving the investor the same satisfaction as the risky revenue.
Risk analysis, con't

- For a risk-averse person, the certainty-equivalent of a risky revenue will be less than its expected value.
- Risk-free revenues and certainty-equivalents should be discounted with a risk-free interest rate to arrive at the correct present value.
- The correct present value of a risky revenue is its expected value discounted with a risk-adjusted discount rate (RADR). For a risk-averse investor, this RADR exceeds the risk-free discount rate.
- Make sure you're discounting expected values, not optimistic values.
Risk-adjusted discount rate

- The further in the future a risky revenue is, the lower the correct RADR is, given the same degree of risk and risk aversion.
- Thus, forestry's long production periods may often require lower RADRs than average short-term industrial RADRs.
- There's no such thing as "the" correct RADR for forestry's expected values. In reality, a different RADR should be used for each cash flow, depending on its probability distribution, on its time from the present, and on the decision maker.
- We can hope to give only rough guidelines for different situations.
RADA, con't

- If data are available, try, using computer simulations, to construct probability distributions of net present value or internal rate of return for projects, and let decision makers compare them.

- While it's correct for a risk-averse investor to increase the discount rate for risky future revenues, the discount rate should be lowered for risky future costs that are independent of revenues.
Predicting biotic forest damages

- Mycelium originating from old-growth stumps may be viable for up to 60-120 yrs, and spread after felling of butt-rotted trees (Stenlid & Redfern 1998).

- Frequent summer thinnings without stump treatment are the most important operation increasing the proportion of trees with butt rot at the end of rotation (Swedjemark & Stenlid 1993, Venn & Solheim 1994, Vollbrecht & Agestam 1995).
Optimal harvesting with butt rot effects


- A comparison of optimal harvesting with and without butt rot effects
Basal area development in optimal solutions, with or without butt rot effects, 3% rate of interest (Cao 2003).
Basal area development in optimal solutions, with or without butt rot effects, 3% rate of interest (Cao 2003).
Modelling the spread of butt rot


A simulation model was developed to predict the growth of a Norway spruce stand under risk of butt rot caused by *Heterobasidion annosum* stump infection and logging injuries.

The simulation model was distance-dependent. The spread of butt rot through root contacts depended on tree location. Infection of stumps and injured trees, the spread of butt rot in the stand were stochastic processes whereas tree growth and mortality were treated as deterministic processes.
Simulation of spread of butt rot

- The progress of decay in the root system was 30 cm per year (ISOMA KI and KALLIO 1974; STENLID 1987; SWEDJEMARK and STENLID 1993).

- \[ h_{\text{rot}} = 4.78 \times t_d^{0.5} + 1.59 \times t_d^{0.5} \times d_d \]
- \[ d_{\text{rot}} = 0.05 \times h_{\text{rot}} \]

- For example, a tree infected 10 years ago with d.b.h. of 2 dm has a decay cone which is 25.2 dm high with a diameter of 1.26 dm at the stump level.
OptiFor demo

- A simulation-optimization tool for forest resources management
- Website: www.optifor.cn
- Documentation
- Student version
- Scientific version
OptiFor

Command Window

BLV = 215.94, r = 0.04
39.70 0.46
30.12 0.64
69.04 0.32
70.12 0.00
BLV = 234.01, r = 0.04
39.70 0.51
40.64 0.58
68.65 0.35
80.62 0.00
BLV = 212.90, r = 0.04
39.70 0.49
40.64 0.62
69.00 0.35
70.12 0.00
BLV = 224.93, r = 0.04
*** End of job, thank you ***

Decision Variables

Harvest Scheduling

<table>
<thead>
<tr>
<th>Final telling</th>
<th>Thinning time, yrs</th>
<th>Thinning intensity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>75.00 yrs</td>
<td>1st</td>
<td>30.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a 30.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b 30.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c 30.00</td>
</tr>
<tr>
<td>2nd</td>
<td>45.00</td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td>55.00</td>
<td></td>
</tr>
<tr>
<td>4th</td>
<td>101.00</td>
<td></td>
</tr>
<tr>
<td>5th</td>
<td>120.00</td>
<td></td>
</tr>
<tr>
<td>6th</td>
<td>151.00</td>
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</table>

OK  Reset  Cancel
HJ algorithm parameters
Comparison of optimization algorithms
## Harvest scheduling

### Control Variables

<table>
<thead>
<tr>
<th>Rotation, yr</th>
<th>Thinnings</th>
<th>Type</th>
<th>Bound</th>
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<tbody>
<tr>
<td>75.00</td>
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</tbody>
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### Thinning Regime

<table>
<thead>
<tr>
<th>Timing, yr</th>
<th>Intensity, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>30.00</td>
</tr>
<tr>
<td>2nd</td>
<td>45.00</td>
</tr>
<tr>
<td>3rd</td>
<td>55.00</td>
</tr>
<tr>
<td>4th</td>
<td>101.00</td>
</tr>
<tr>
<td>5th</td>
<td>120.00</td>
</tr>
<tr>
<td>6th</td>
<td>151.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>30.00</th>
<th>30.00</th>
<th>30.00</th>
<th>30.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2nd</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>4th</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>5th</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6th</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Buttons

- Reset
- OK
- Cancel
OptiFor Wood

Grading Systems
1. Timber grading
2. Sawn wood grading

Log prices
Log A: 71.52  Log C: 45.03
Log B: 52.98  Log D: 39.73

Sawn wood prices
A center: 221.52  side: 175.00
B center: 190.98  side: 155.00
CD center: 175.00  side: 135.00

OK  Cancel
Cao et al. (2008)

Initial stand states

Process-based growth model

Models for predicting wood and tracheid properties

Residual wood volume
Pulpwood volume
Sawlog volume
Outwood volume

Timber assortments

Wood density
Tracheid length
Latewood proportion
Tracheid width

Economic returns

Wood and tracheid properties

Logging cost model; Timber price; Interest rate
Fig. 4. Removals (open bars) in size classes (a–c) and yields of end products in thinnings and final felling in the QPC on MTS\textsubscript{1500} (d).
OptiFor Bioenergy

Energy wood options

1. Profitable
2. Compulsory
3. Superior to commercial thinning

Option of energy wood from precommercial thinning: 0
Energy wood price: 15.00

OK Cancel
Cao et al. (2015)

(a) Energy wood harvesting  (b) Conventional thinning
OptiFor Carbon

Carbon assessment methods

1. Stem carbon
2. Biomass expansion factors
3. Process-based model

Carbon assessment method: 0
Carbon price: 10.00

OK  Cancel
Cao et al. (2010)

Graphs showing carbon stock over age and time for different categories:
- **Carbon stock, Mg ha⁻¹**
  - CI
  - CII
  - CIII

- **Carbon stock, 100%**
  - Sawlogs
  - Pulpwood
  - Residues
OptiFor Climate

Climate Scenarios

1. Climate scenario I
2. Climate scenario II
3. Climate scenario III
4. Climate scenario IV

Climate scenario: 0
Changes in costs (0, 1): 0

[OK, Cancel]
Climate-sensitive model

Response of GPP and NPP to climate change
- 15 a observations
- response studies

Productivity changes

Process-based growth model
- Microforest
- C and N cycling
- SOM decomposition
- tested in present climate

Fertility changes

Growth changes

Site index changes
- dominant height vs age
- different scenarios

Site index changes

Regional growth trends

Management simulator
- Simo

Process-based growth model
- pipeQual
- dynamic growth allocation
- extended testing

Literature comparisons

Figure 1. Schematic presentation of the used approach to analyze climate change impacts on forest productivity
References

Thank You!

cao@nwafu.edu.cn

www.optifor.cn